

Pensieve header: Roland checking Joe Boninger's conjecture that for a closure of a positive braid the coefficients of ρ_1 are all positive. Continued pensieve://Talks/Oaxaca-2210/Rho_d-Positivity.nb.

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```
In[*]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\APAI"];
Once[<< KnotTheory` ; << Rot.m];
```

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Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at <http://katlas.org/wiki/KnotTheory>.

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Loading Rot.m from <http://drorbn.net/APAI> to compute rotation numbers.

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```
In[*]:= R1[s_, i_, j_] := s (g_{ji} (g_{j+1,j} + g_{j,j+1} - g_{ij}) - g_{ii} (g_{j,j+1} - 1) - 1 / 2);
rho[K_] := rho[K] = Module[{Cs, phi, n, A, s, i, j, k, Delta, G, rho1},
  {Cs, phi} = Rot[K]; n = Length[Cs];
  A = IdentityMatrix[2 n + 1];
  Cases[Cs, {s_, i_, j_} -> (A[[{i, j}, {i + 1, j + 1}]] += ( -T^s T^s - 1 ))];
  Delta = T^(-Total[phi] - Total[Cs[[All, 1]]) / 2) Det[A];
  G = Inverse[A];
  rho1 = Sum_{k=1}^n R1 @@ Cs[[k]] - Sum_{k=1}^{2n} phi[[k]] (g_{kk} - 1 / 2);
  Factor@{Delta, Delta^2 rho1 /. g_{alpha, beta} -> G[[alpha, beta]]};
```

```
In[*]:= Mat[u_] :=
  Transpose@Table[Prepend[Table[Coefficient[(x^{1/2} - x^{-1/2})^{2j} // Expand, x^k], {k, 1, u}],
    (Expand[(x^{1/2} - x^{-1/2})^{2j}] /. {x^Integer -> 0, x -> 0})], {j, 0, u}]
  ToConway[P_] :=
  If[P === 0, 0, Module[{CC, deg = Exponent[P, T], M}, M = Inverse[Mat[deg]];
    CC = Expand[P] /. {T^j ->
      If[j < 0, 0, T^j]};
    Total[CoefficientRules[CC, T]] /.
      {({a_} -> b_) -> b M[[ ; ; , a + 1]].Table[z^{2^i}, {i, 0, deg}]} // Expand
  ]]
  ToConway[7 + 3 / T^2 + 3 T^2 - 101 T^4 - 101 / T^4]
  % /. {z -> x^{1/2} - x^{-1/2}} // Expand
```

Out[*]=

$$-189 - 1604 z^2 - 2017 z^4 - 808 z^6 - 101 z^8$$

Out[*]=

$$7 - \frac{101}{x^4} + \frac{3}{x^2} + 3 x^2 - 101 x^4$$

```
In[*]:= Cρ1[K_] := ToConway[Expand@Together[ρ[K][[2]] (-T) / (-1 + T)2]]
Cρ0[K_] := ToConway[Expand@Together[ρ[K][[1]]]]
CheckForConstantSigns[P_] :=
  If[Length@Union[Sign /@ (MonomialList[P, z] /. z -> 1)] == 1, True, False]
PosKnotsi[n_] :=
  Table[If[CheckForConstantSigns[Cρi[K]], K, Nothing], {K, AllKnots[{3, n}]}
```

```
In[*]:= PosKnots1[10];
```

 KnotTheory: Loading precomputed data in PD4Knots`.

```
In[*]:= PosKnots0[10];
```

```
In[*]:= (*False Alexander positives detected by ρ1,
i.e. knots that are not positive braid closures where the Alex is positive.*)
Complement[PosKnots0[10], PosKnots1[10]]
```

```
Out[*]=
```

```
{Knot[9, 26], Knot[9, 29], Knot[9, 41], Knot[9, 44], Knot[10, 93],
Knot[10, 108], Knot[10, 113], Knot[10, 129], Knot[10, 146], Knot[10, 164]}
```

```
In[*]:= {Cρ1[#], Cρ0[#]} &@Knot[9, 41]
```

```
Out[*]=
```

```
{2 - 17 z2 + 2 z4 - 3 z6, 1 + 3 z4}
```

```
In[*]:= (*Copied from KnotInfo*)
```

```
PosBraidKnots = {Knot[3, 1], Knot[5, 1], Knot[7, 1], Knot[8, 19], Knot[9, 1],
Knot[10, 124], Knot[10, 139], Knot[10, 152], Knot[11, Alternating, 367],
Knot[11, NonAlternating, 77], Knot[12, NonAlternating, 242],
Knot[12, NonAlternating, 472], Knot[12, NonAlternating, 574],
Knot[12, NonAlternating, 679], Knot[12, NonAlternating, 688],
Knot[12, NonAlternating, 725], Knot[12, NonAlternating, 888]};
```

```
In[*]:= (*Checking Boninger's conjecture up to 12 crossings*)
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```
CheckForConstantSigns[Cρ1[#]] & /@ PosBraidKnots
```

 KnotTheory: Loading precomputed data in DTCode4KnotsTo11`.

 KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

 KnotTheory: Loading precomputed data in KnotTheory/12N.dts.

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Out[*]=
```

```
{True, True, True, True, True, True, True,
True, True, True, True, True, True, True, True}
```