Pensieve header: Computing and playing with ρ_1 in the language of perturbed Gaussian Integration.

Programs

In[∘]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\APAI"];
Once[<< KnotTheory`; << Rot.m];
```

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
Loading Rot.m from http://drorbn.net/APAI to compute rotation numbers.
```

In[∘]:=
```
CCF[ℰ_] := ExpandDenominator@ExpandNumerator@Together[ℰ];
CF[ℰ_List] := CF /@ ℰ; CF[ℰ_EPD] := CF /@ ℰ;
CF[ℰ_] := Module[{vs = Cases[ℰ, (x | p)_, ∞] ⋃ {x, p}, ps, c},
   Total[CoefficientRules[Expand[ℰ], vs] /. (ps_ → c_) :> CCF[c] (Times @@ vs^ps)]];
CF[eqp_EQP] := CF /@ eqp
```

In[∘]:=
```
EQP /: c_ * EQP[Q_, P_] := EQP[Q, CF[c P]];
```

In[∘]:=
```
{p*, x*} = {π, ξ}; (z_i__)* := (z*)_i; vs_List* := (v ↦ v*) /@ vs;
Zip_{}[ℰ_] := ℰ;
Zip_{z_,zs___}[ℰ_] := (Collect[ℰ // Zip_{zs}, z] /. f_. z^d_. :> (D[f, {z*, d}])) /. z* → 0
```

In[∘]:=
```
FI[EQP[Q_, P_]] := FI[EQP[Q, P], Union@Cases[Q, p_, ∞], Union@Cases[Q, x_, ∞]];
FI[EQP[Q_, P_], ps_List, xs_List] := Module[{u, v},
  A = Table[∂_{u,v} Q, {u, ps}, {v, xs}];
  Factor[Det[A]^-1 Zip_{ps⋃xs}[P ⅇ^(-xs*.Inverse[A].ps*)]]]
```

ρ_0 Tests

In[∘]:=
```
ρ0i[K_] := ρ0i[K, False]; ρ0i[Flip@K_] := ρ0i[K, True];
ρ0i[K_, flip_] := Module[{Cs, φ, n, s, i, j, k, vs, Q, Qp},
   {Cs, φ} = Rot[K]; n = Length[Cs];
   If[flip, Cs = Cs[[All, {1, 3, 2}]]; φ = -φ];
   Q = -p_{2 n+1} x_{2 n+1}; Qp = 0;
   Cases[Cs, {s_, i_, j_} :>
     (Q -= x_i (p_i - T^s p_{i+1} + (T^s - 1) p_{j+1}) + x_j (p_j - p_{j+1}); Qp -= s T^(s-1) x_i (p_{j+1} - p_{i+1}))];
   EQP[Q, -T^((Total[φ]+Total[Cs[[All,1]]])/2) Qp -
     (Total[φ] + Total[Cs[[All, 1]]]) T^((Total[φ]+Total[Cs[[All,1]]])/2-1) / 2]
  ];
```

In[∘]:=
```
K = Knot[8, 17];
Factor[∂_T (Alexander[K][T]^-1)]
```

KnotTheory: Loading precomputed data in PD4Knots`.

Out[∘]=

$$\frac{(-1+T)\,T^2\,(1+T)\,\left(1-T+T^2\right)\,\left(3-5\,T+3\,T^2\right)}{\left(1-4\,T+8\,T^2-11\,T^3+8\,T^4-4\,T^5+T^6\right)^2}$$

In[∘]:=
```
K = Knot[3, 1]; {Cs, φ} = Rot[K]; n = Length[Cs];
ν = {lν = 0};
writhe = Total@Cs[[All, 1]];
Do[Cs /. {{s_, k, j_} :→ AppendTo[ν, lν += s], {s_, i_, k} :→ AppendTo[ν, lν -= s]}, {k, 2 n}];
eqp = ρ0i[K];
eqp1 = T^-writhe CF[eqp /. Flatten@{{x_{2 n+1} → p_1, p_{2 n+1} → x_{2 n+1}},
        Cs /. {s_Integer, i_, j_} :→ {x_j → -T^ν[[i]] p_{j+1} + (1 - T^s) T^ν[[i]] p_{i+1} + T^{s+ν[[i]]} p_1,
           x_i → -T^ν[[i]] p_{i+1} + T^ν[[i]] p_1, p_i → T^-ν[[i]] x_i, p_j → T^{-ν[[i]]-s} x_j}}];
eqp2 = CF[ρ0i[Flip@K] /. T → T^-1];
FI /@ {eqp, eqp1, eqp2}
```

Out[∘]=

$$\left\{-\frac{(-1+T)\,(1+T)}{\left(1-T+T^2\right)^2},\ -\frac{(-1+T)\,(1+T)}{\left(1-T+T^2\right)^2},\ \frac{(-1+T)\,T^2\,(1+T)}{\left(1-T+T^2\right)^2}\right\}$$

In[∘]:=
```
CF[eqp1[[1]] - eqp2[[1]]]
```

Out[∘]=

0

In[∘]:=
```
CF[T^2 eqp1[[2]] + eqp2[[2]]]
```

Out[∘]=

$$3\,T^2+T^3\,p_2\,x_1-T^3\,p_5\,x_1-T\,p_1\,x_2+T\,p_5\,x_2+T^2\,p_1\,x_3-T^2\,p_3\,x_3+T^3\,p_4\,x_3-T^3\,p_7\,x_3-$$
$$T\,p_1\,x_4+T\,p_7\,x_4+T^2\,p_1\,x_5-T^3\,p_3\,x_5-T^2\,p_5\,x_5+T^3\,p_6\,x_5-T\,p_1\,x_6+T\,p_3\,x_6+T^2\,p_1\,x_7-T^2\,p_7\,x_7$$

In[∘]:=
```
FI@EQP[eqp1[[1]], T^2 eqp1[[2]] + eqp2[[2]]]
```

Out[∘]=

0

ρ_1 Tests

In[∘]:=
```
r1[s_, i_, j_] :=
  s (-1 + 2 p_i x_i - 2 p_j x_i + (-1 + T^s) p_i p_j x_i^2 + (1 - T^s) p_j^2 x_i^2 - 2 p_i p_j x_i x_j + 2 p_j^2 x_i x_j) / 2;
γ1[φ_, k_] := φ (1 / 2 - p_k x_k);
ρ1i[K_] := ρ1i[K, False]; ρ1i[Flip@K_] := ρ1i[K, True];
ρ1i[K_, flip_] := Module[{Cs, φ, n, s, i, j, k, vs, Q, P},
   {Cs, φ} = Rot[K]; n = Length[Cs];
   If[flip, Cs = Cs[[All, {1, 3, 2}]]; φ = -φ];
   Q = -x_(2 n+1) p_(2 n+1);
   Cases[Cs, {s_, i_, j_} :> (Q -= x_i (p_i - T^s p_(i+1) + (T^s - 1) p_(j+1)) + x_j (p_j - p_(j+1)))];
   P = Sum[r1 @@ Cs[[k]], {k, n}] + Sum[γ1[φ[[k]], k], {k, 2 n}];
   CF@EQP[Q, T^((Total[φ]+Total[Cs[[All,1]]])/2) P]
  ];
```

In[∘]:=
```
K = Knot[5, 2];
ρ1i[K]
```

Out[∘]=

$$\mathrm{EQP}\Big[-((p_1-p_2)\,x_1)-\left(p_2-\frac{p_3}{T}+\left(-1+\frac{1}{T}\right)p_8\right)x_2-(p_3-p_4)\,x_3-\left(\left(-1+\frac{1}{T}\right)p_2+p_4-\frac{p_5}{T}\right)x_4-(p_5-p_6)\,x_5-\left(p_6-\frac{p_7}{T}+\left(-1+\frac{1}{T}\right)p_{10}\right)x_6-(p_7-p_8)\,x_7-\left(\left(-1+\frac{1}{T}\right)p_4+p_8-\frac{p_9}{T}\right)x_8-(p_9-p_{10})\,x_9-\left(\left(-1+\frac{1}{T}\right)p_6+p_{10}-\frac{p_{11}}{T}\right)x_{10}-p_{11}\,x_{11},$$
$$\frac{1}{T^3}\left(-\frac{1}{2}+p_4 x_4+\frac{1}{2}\left(1+2p_1x_4-2p_4x_4-2p_1^2x_1x_4+2p_1p_4x_1x_4-\left(1-\frac{1}{T}\right)p_1^2x_4^2-\left(-1+\frac{1}{T}\right)p_1p_4x_4^2\right)+\right.$$
$$\frac{1}{2}\left(1-2p_2x_2+2p_7x_2-\left(-1+\frac{1}{T}\right)p_2p_7x_2^2-\left(1-\frac{1}{T}\right)p_7^2x_2^2+2p_2p_7x_2x_7-2p_7^2x_2x_7\right)+$$
$$\frac{1}{2}\left(1+2p_3x_8-2p_8x_8-2p_3^2x_3x_8+2p_3p_8x_3x_8-\left(1-\frac{1}{T}\right)p_3^2x_8^2-\left(-1+\frac{1}{T}\right)p_3p_8x_8^2\right)-p_9x_9+$$
$$\frac{1}{2}\left(1-2p_6x_6+2p_9x_6-\left(-1+\frac{1}{T}\right)p_6p_9x_6^2-\left(1-\frac{1}{T}\right)p_9^2x_6^2+2p_6p_9x_6x_9-2p_9^2x_6x_9\right)+p_{10}x_{10}+$$
$$\left.\frac{1}{2}\left(1+2p_5x_{10}-2p_{10}x_{10}-2p_5^2x_5x_{10}+2p_5p_{10}x_5x_{10}-\left(1-\frac{1}{T}\right)p_5^2x_{10}^2-\left(-1+\frac{1}{T}\right)p_5p_{10}x_{10}^2\right)\right)\Big]$$

In[∘]:=
```
Factor@Together[FI@ρ1i[K]]
```

Out[∘]=

$$-\frac{(-1+T)^2\,T\,\left(5-4T+5T^2\right)}{\left(2-3T+2T^2\right)^3}$$

In[∘]:=
```
K = Knot[3, 1]; {Cs, φ} = Rot[K]; n = Length[Cs];
ν = {lν = 0}; writhe = Total@Cs⟦All, 1⟧;
Do[Cs /. {{s_, k, j_} :> AppendTo[ν, lν += s], {s_, i_, k} :> AppendTo[ν, lν -= s]}, {k, 2 n}];
eqp = ρ1i[K];
eqp1 = T^-writhe CF[eqp /. Flatten@{{x_{2n+1} → p_1, p_{2n+1} → x_{2n+1}},
    Cs /. {s_Integer, i_, j_} :> {x_j → -T^ν⟦i⟧ p_{j+1} + (1 - T^s) T^ν⟦i⟧ p_{i+1} + T^{s+ν⟦i⟧} p_1,
      x_i → -T^ν⟦i⟧ p_{i+1} + T^ν⟦i⟧ p_1, p_i → T^-ν⟦i⟧ x_i, p_j → T^{-ν⟦i⟧-s} x_j}}];
eqp2 = CF[ρ1i[Flip@K] /. T → T^-1];
FI /@ {eqp, eqp1, eqp2}
```

Out[∘]=

$$\left\{-\frac{(-1+T)^2\,T\,\left(1+T^2\right)}{\left(1-T+T^2\right)^3},\ -\frac{(-1+T)^2\,T\,\left(1+T^2\right)}{\left(1-T+T^2\right)^3},\ -\frac{(-1+T)^2\,T\,\left(1+T^2\right)}{\left(1-T+T^2\right)^3}\right\}$$

In[∘]:=
```
CF[eqp1⟦1⟧ - eqp2⟦1⟧]
```

Out[∘]=

0

In[∘]:=
```
CF[eqp1⟦2⟧ - eqp2⟦2⟧]
```

Out[∘]=

$$-T+\left(T+T^2\right) p_1 x_1-T p_4 x_1-T^2 p_5 x_1+\frac{1}{2}\left(-T^2-T^3\right) p_1^2 x_1^2+T^3 p_1 p_2 x_1^2+\frac{1}{2}\left(-T+T^2\right) p_1 p_4 x_1^2+$$
$$\frac{1}{2}\left(T-T^2\right) p_4^2 x_1^2+T^2 p_1 p_5 x_1^2-T^3 p_2 p_5 x_1^2+\frac{1}{2}\left(-T^2+T^3\right) p_5^2 x_1^2-T p_1 x_2+T p_3 x_2+T^2 p_1 x_3+T p_3 x_3-$$
$$T p_6 x_3-T^2 p_7 x_3+\frac{1}{2}\left(-T^2-T^3\right) p_1^2 x_3^2+T^3 p_1 p_4 x_3^2+\frac{1}{2}\left(-T+T^2\right) p_3 p_6 x_3^2+\frac{1}{2}\left(T-T^2\right) p_6^2 x_3^2+$$
$$T^2 p_1 p_7 x_3^2-T^3 p_4 p_7 x_3^2+\frac{1}{2}\left(-T^2+T^3\right) p_7^2 x_3^2+T p_4 x_4+\frac{1}{2}\left(T+T^2\right) p_1^2 x_1 x_4-T^2 p_1 p_2 x_1 x_4-$$
$$T p_1 p_4 x_1 x_4+T p_4^2 x_1 x_4-T p_1 p_5 x_1 x_4+T^2 p_2 p_5 x_1 x_4+\frac{1}{2}\left(T-T^2\right) p_5^2 x_1 x_4+T^2 p_1 x_5-T p_2 x_5-$$
$$T^2 p_3 x_5+T p_5 x_5+\frac{1}{2}\left(T+T^2\right) p_1^2 x_2 x_5+T p_2^2 x_2 x_5-T p_1 p_3 x_2 x_5+\frac{1}{2}\left(T-T^2\right) p_3^2 x_2 x_5-T p_2 p_5 x_2 x_5-$$
$$T^2 p_1 p_6 x_2 x_5+T^2 p_3 p_6 x_2 x_5+\frac{1}{2}\left(-T^2-T^3\right) p_1^2 x_5^2+\frac{1}{2}\left(T-T^2\right) p_2^2 x_5^2+T^2 p_1 p_3 x_5^2+\frac{1}{2}\left(-T^2+T^3\right) p_3^2 x_5^2+$$
$$\frac{1}{2}\left(-T+T^2\right) p_2 p_5 x_5^2+T^3 p_1 p_6 x_5^2-T^3 p_3 p_6 x_5^2-T p_1 x_6+T p_7 x_6+\frac{1}{2}\left(T+T^2\right) p_1^2 x_3 x_6-$$
$$T^2 p_1 p_4 x_3 x_6-T p_3 p_6 x_3 x_6+T p_6^2 x_3 x_6-T p_1 p_7 x_3 x_6+T^2 p_4 p_7 x_3 x_6+\frac{1}{2}\left(T-T^2\right) p_7^2 x_3 x_6$$

In[∘]:=
```
FI@EQP[eqp1⟦1⟧, eqp1⟦2⟧ - eqp2⟦2⟧]
```

Out[∘]=

0

In[∘]:=
```
Monitor[sum = 0; Do[
  {Cs, φ} = Rot[K]; n = Length[Cs];
  ν = {lν = 0}; writhe = Total@Cs〚All, 1〛;
  Do[Cs /.
    {{s_, k, j_} :> AppendTo[ν, lν += s], {s_, i_, k} :> AppendTo[ν, lν -= s]}, {k, 2 n}];
  eqp1 = T^-writhe CF[ρ1i[K] /. Flatten@{{x_{2 n+1} → p_1, p_{2 n+1} → x_{2 n+1}},
        Cs /. {s_Integer, i_, j_} :> {x_j → -T^ν〚i〛 p_{j+1} + (1 - T^s) T^ν〚i〛 p_{i+1} + T^(s+ν〚i〛) p_1,
          x_i → -T^ν〚i〛 p_{i+1} + T^ν〚i〛 p_1, p_i → T^-ν〚i〛 x_i, p_j → T^(-ν〚i〛-s) x_j}}];
  eqp2 = CF[ρ1i[Flip@K] /. T → T^-1];
  sum += Simplify[eqp1〚1〛 == eqp2〚1〛] ∧ FI@eqp1 == FI@eqp2,
  {K, AllKnots[{3, 10}]}
 ], {K, sum}]; sum
```

Out[∘]=

249 True

In[∘]:=
```
Monitor[sum = 0; Do[
  {Cs, φ} = Rot[K]; n = Length[Cs];
  ν = {lν = 0}; writhe = Total@Cs〚All, 1〛;
  Do[Cs /.
    {{s_, k, j_} :> AppendTo[ν, lν += s], {s_, i_, k} :> AppendTo[ν, lν -= s]}, {k, 2 n}];
  eqp1 = T^-writhe CF[ρ1i[K] /. Flatten@{{x_{2 n+1} → p_1, p_{2 n+1} → x_{2 n+1}},
        Cs /. {s_Integer, i_, j_} :> {x_j → -T^ν〚i〛 p_{j+1} + (1 - T^s) T^ν〚i〛 p_{i+1} + T^(s+ν〚i〛) p_1,
          x_i → -T^ν〚i〛 p_{i+1} + T^ν〚i〛 p_1, p_i → T^-ν〚i〛 x_i, p_j → T^(-ν〚i〛-s) x_j}}];
  eqp2 = CF[ρ1i[Flip@K] /. T → T^-1];
  sum += Simplify[eqp1〚1〛 == eqp2〚1〛] ∧ FI@EQP[eqp1〚1〛, eqp1〚2〛 - eqp2〚2〛] == 0,
  {K, AllKnots[{3, 10}]}
 ], {K, sum}]; sum
```

Out[∘]=

249 True

Palindromicity for ρ_1

In[∘]:=
```
CF[T^-s r_1[s, i, j]]
```

Out[∘]=

$$-\frac{1}{2} s T^{-s} + s T^{-s} p_i x_i - s T^{-s} p_j x_i + \frac{1}{2} T^{-s} (-s + s T^s) p_i p_j x_i^2 + \frac{1}{2} T^{-s} (s - s T^s) p_j^2 x_i^2 - s T^{-s} p_i p_j x_i x_j + s T^{-s} p_j^2 x_i x_j$$

In[∘]:=
```
CF@PowerExpand@Plus[r1[s, i, j] /. {xj → -T^(νi) pj+1 + (1 - T^s) T^(νi) pi+1 + T^(s+νi) p1,
    xi → -T^(νi) pi+1 + T^(νi) p1, pi → T^(-νi) xi, pj → T^(-νi-s) xj},
  r1[s, j, i] /. T → T^-1
 ]
```

Out[∘]=

$$-s + s p_1 x_i - s p_{1+i} x_i - s T^{-s} p_1 x_j - s p_i x_j + s T^{-s} p_{1+i} x_j + s p_j x_j + \frac{1}{2} T^{-s} (-s - s T^s) p_1^2 x_i x_j + s p_i^2 x_i x_j + s p_1 p_{1+i} x_i x_j + \frac{1}{2} T^{-s} (s - s T^s) p_{1+i}^2 x_i x_j - s p_i p_j x_i x_j + s T^{-s} p_1 p_{1+j} x_i x_j - s T^{-s} p_{1+i} p_{1+j} x_i x_j + \frac{1}{2} T^{-2s} (s + s T^s) p_1^2 x_j^2 + \frac{1}{2} T^{-s} (-s + s T^s) p_i^2 x_j^2 - s T^{-s} p_1 p_{1+i} x_j^2 + \frac{1}{2} T^{-2s} (-s + s T^s) p_{1+i}^2 x_j^2 + \frac{1}{2} T^{-s} (s - s T^s) p_i p_j x_j^2 - s T^{-2s} p_1 p_{1+j} x_j^2 + s T^{-2s} p_{1+i} p_{1+j} x_j^2$$

In[∘]:=
```
K = Knot[3, 1]; {Cs, φ} = Rot[K]; n = Length[Cs];
ν = {lν = 0}; writhe = Total@Cs⟦All, 1⟧;
Do[Cs /. {{s_, k, j_} ↦ AppendTo[ν, lν += s], {s_, i_, k} ↦ AppendTo[ν, lν -= s]}, {k, 2 n}];
eqp = ρ1i[K];
eqp1 = T^-writhe CF[eqp /. Flatten@{{x2n+1 → p1, p2n+1 → x2n+1},
      Cs /. {s_Integer, i_, j_} ↦ {xj → -T^(ν⟦i⟧) pj+1 + (1 - T^s) T^(ν⟦i⟧) pi+1 + T^(s+ν⟦i⟧) p1,
         xi → -T^(ν⟦i⟧) pi+1 + T^(ν⟦i⟧) p1, pi → T^(-ν⟦i⟧) xi, pj → T^(-ν⟦i⟧-s) xj}}];
eqp2 = CF[ρ1i[Flip@K] /. T → T^-1];
FI@EQP[eqp1⟦1⟧, eqp1⟦2⟧ - eqp2⟦2⟧]
CF@EQP[eqp1⟦1⟧, eqp1⟦2⟧ - eqp2⟦2⟧]
CF@Plus[Sum[{s, i, j} = Cs⟦k⟧;
    -s + s p1 xi - s p1+i xi - s T^-s p1 xj - s pi xj + s T^-s p1+i xj + s pj xj + 1/2 T^-s (-s - s T^s) p1^2 xi xj +
     s pi^2 xi xj + s p1 p1+i xi xj + 1/2 T^-s (s - s T^s) p1+i^2 xi xj - s pi pj xi xj + s T^-s p1 p1+j xi xj -
     s T^-s p1+i p1+j xi xj + 1/2 T^-2s (s + s T^s) p1^2 xj^2 + 1/2 T^-s (-s + s T^s) pi^2 xj^2 - s T^-s p1 p1+i xj^2 +
     1/2 T^-2s (-s + s T^s) p1+i^2 xj^2 + 1/2 T^-s (s - s T^s) pi pj xj^2 - s T^-2s p1 p1+j xj^2 + s T^-2s p1+i p1+j xj^2,
    {k, n}],
  Sum[γ1[φ⟦k⟧, k], {k, 2 n}]
 ]
```

Out[∘]=

$$0$$

Out[∘]=

$$\begin{aligned}
&\mathrm{EQP}\Big[-p_1 x_1+T p_2 x_1+(1-T)\, p_5 x_1-p_2 x_2+p_3 x_2-p_3 x_3+T p_4 x_3+\\
&\quad (1-T)\, p_7 x_3-p_4 x_4+p_5 x_4+(1-T)\, p_3 x_5-p_5 x_5+T p_6 x_5-p_6 x_6+p_7 x_6-p_7 x_7,\\
&\quad -T+\left(T+T^2\right) p_1 x_1-T p_4 x_1-T^2 p_5 x_1+\frac{1}{2}\left(-T^2-T^3\right) p_1^2 x_1^2+T^3 p_1 p_2 x_1^2+\frac{1}{2}\left(-T+T^2\right) p_1 p_4 x_1^2+\\
&\quad \frac{1}{2}\left(T-T^2\right) p_4^2 x_1^2+T^2 p_1 p_5 x_1^2-T^3 p_2 p_5 x_1^2+\frac{1}{2}\left(-T^2+T^3\right) p_5^2 x_1^2-T p_1 x_2+T p_3 x_2+T^2 p_1 x_3+T p_3 x_3-\\
&\quad T p_6 x_3-T^2 p_7 x_3+\frac{1}{2}\left(-T^2-T^3\right) p_1^2 x_3^2+T^3 p_1 p_4 x_3^2+\frac{1}{2}\left(-T+T^2\right) p_3 p_6 x_3^2+\frac{1}{2}\left(T-T^2\right) p_6^2 x_3^2+\\
&\quad T^2 p_1 p_7 x_3^2-T^3 p_4 p_7 x_3^2+\frac{1}{2}\left(-T^2+T^3\right) p_7^2 x_3^2+T p_4 x_4+\frac{1}{2}\left(T+T^2\right) p_1^2 x_1 x_4-T^2 p_1 p_2 x_1 x_4-\\
&\quad T p_1 p_4 x_1 x_4+T p_4^2 x_1 x_4-T p_1 p_5 x_1 x_4+T^2 p_2 p_5 x_1 x_4+\frac{1}{2}\left(T-T^2\right) p_5^2 x_1 x_4+T^2 p_1 x_5-T p_2 x_5-\\
&\quad T^2 p_3 x_5+T p_5 x_5+\frac{1}{2}\left(T+T^2\right) p_1^2 x_2 x_5+T p_2^2 x_2 x_5-T p_1 p_3 x_2 x_5+\frac{1}{2}\left(T-T^2\right) p_3^2 x_2 x_5-T p_2 p_5 x_2 x_5-\\
&\quad T^2 p_1 p_6 x_2 x_5+T^2 p_3 p_6 x_2 x_5+\frac{1}{2}\left(-T^2-T^3\right) p_1^2 x_5^2+\frac{1}{2}\left(T-T^2\right) p_2^2 x_5^2+T^2 p_1 p_3 x_5^2+\frac{1}{2}\left(-T^2+T^3\right) p_3^2 x_5^2+\\
&\quad \frac{1}{2}\left(-T+T^2\right) p_2 p_5 x_5^2+T^3 p_1 p_6 x_5^2-T^3 p_3 p_6 x_5^2-T p_1 x_6+T p_7 x_6+\frac{1}{2}\left(T+T^2\right) p_1^2 x_3 x_6-\\
&\quad T^2 p_1 p_4 x_3 x_6-T p_3 p_6 x_3 x_6+T p_6^2 x_3 x_6-T p_1 p_7 x_3 x_6+T^2 p_4 p_7 x_3 x_6+\frac{1}{2}\left(T-T^2\right) p_7^2 x_3 x_6\Big]
\end{aligned}$$

Out[∘]=

$$\begin{aligned}
&\frac{5}{2}+(-1+T)\, p_1 x_1+p_4 x_1-T p_5 x_1+\frac{1}{2}\left(-T-T^2\right) p_1^2 x_1^2+T^2 p_1 p_2 x_1^2+\frac{1}{2}(1-T)\, p_1 p_4 x_1^2+\\
&\frac{1}{2}(-1+T)\, p_4^2 x_1^2+T p_1 p_5 x_1^2-T^2 p_2 p_5 x_1^2+\frac{1}{2}\left(-T+T^2\right) p_5^2 x_1^2-p_1 x_2+p_3 x_2+T p_1 x_3-p_3 x_3+\\
&p_6 x_3-T p_7 x_3+\frac{1}{2}\left(-T-T^2\right) p_1^2 x_3^2+T^2 p_1 p_4 x_3^2+\frac{1}{2}(1-T)\, p_3 p_6 x_3^2+\frac{1}{2}(-1+T)\, p_6^2 x_3^2+\\
&T p_1 p_7 x_3^2-T^2 p_4 p_7 x_3^2+\frac{1}{2}\left(-T+T^2\right) p_7^2 x_3^2-p_1 x_4+p_4 x_4+p_5 x_4+\frac{1}{2}(1+T)\, p_1^2 x_1 x_4-\\
&T p_1 p_2 x_1 x_4+p_1 p_4 x_1 x_4-p_4^2 x_1 x_4-p_1 p_5 x_1 x_4+T p_2 p_5 x_1 x_4+\frac{1}{2}(1-T)\, p_5^2 x_1 x_4+T p_1 x_5+\\
&p_2 x_5-T p_3 x_5-p_5 x_5+\frac{1}{2}(1+T)\, p_1^2 x_2 x_5-p_2^2 x_2 x_5-p_1 p_3 x_2 x_5+\frac{1}{2}(1-T)\, p_3^2 x_2 x_5+\\
&p_2 p_5 x_2 x_5-T p_1 p_6 x_2 x_5+T p_3 p_6 x_2 x_5+\frac{1}{2}\left(-T-T^2\right) p_1^2 x_5^2+\frac{1}{2}(-1+T)\, p_2^2 x_5^2+T p_1 p_3 x_5^2+\\
&\frac{1}{2}\left(-T+T^2\right) p_3^2 x_5^2+\frac{1}{2}(1-T)\, p_2 p_5 x_5^2+T^2 p_1 p_6 x_5^2-T^2 p_3 p_6 x_5^2-p_1 x_6+p_7 x_6+\frac{1}{2}(1+T)\, p_1^2 x_3 x_6-\\
&T p_1 p_4 x_3 x_6+p_3 p_6 x_3 x_6-p_6^2 x_3 x_6-p_1 p_7 x_3 x_6+T p_4 p_7 x_3 x_6+\frac{1}{2}(1-T)\, p_7^2 x_3 x_6
\end{aligned}$$