

$$R_1[s_, i_, j_] := s (g_{ji} (g_{j^+, j} + g_{j, j^+} - g_{ij}) - g_{ii} (g_{j, j^+} - 1) - 1/2);$$

$$\rho[K_] := \rho[K] = \text{Module}\left[\{Cs, \varphi, n, A, s, i, j, k, \Delta, G, \rho1\},\right.$$

$$\{Cs, \varphi\} = \text{Rot}[K]; \quad n = \text{Length}[Cs];$$

$$A = \text{IdentityMatrix}[2n + 1];$$

$$\text{Cases}\left[Cs, \{s_, i_, j_\} \Rightarrow \left(A[\{i, j\}, \{i + 1, j + 1\}] += \begin{pmatrix} -T^s & T^s & -1 \\ 0 & -1 & \end{pmatrix}\right)\right];$$

$$\Delta = T^{(-\text{Total}[\varphi] - \text{Total}[Cs[\text{All}, 1]])/2} \text{Det}[A];$$

$$G = \text{Inverse}[A];$$

$$\rho1 = \sum_{k=1}^n R_1 @@ Cs[[k]] - \sum_{k=1}^{2n} \varphi[[k]] (g_{kk} - 1/2);$$

$$\text{Factor}@ \left\{ \Delta, \Delta^2 \rho1 /. \alpha_-^+ \Rightarrow \alpha + 1 /. g_{\alpha_-, \beta_-} \Rightarrow G[\alpha, \beta] \right\};$$