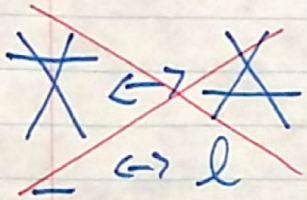
Abstract: Goal is to study FT invariants of doodles (certain equivalence classes of plane curves), construct UFTI.

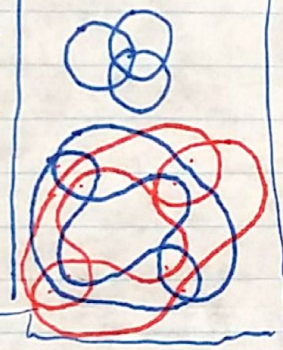
While the topology is not difficult, the FT theory parallels the case of FT for knots.

Doodles: $K = \{$  $\} / \chi = \mathcal{C}$

oriented plane curves



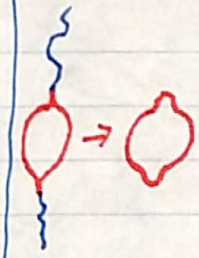
Examples:



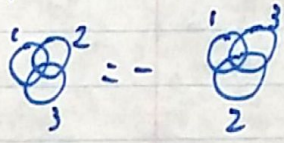
$K_n / \delta K_{n+1} = \{$  $\} / \bigcirc = \bigcirc\bigcirc$

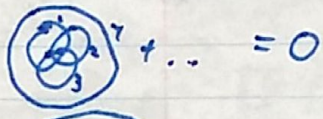
cyclically ordered n-component doodles, ± 1 winding number

(Modulo δK_{n+1}, forget where blue is embedded)



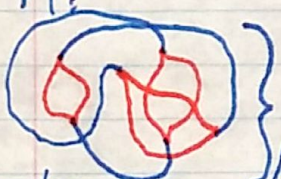
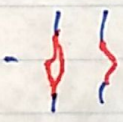
$\cong_{1SAS} \langle$  elementary doodles $\rangle /$

AS 

Tetrahedron 

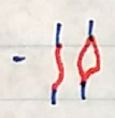
Ring exchange 

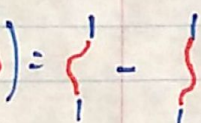
Gaussarov FT,

$K_n = \{$  $\} /$ 

n-bracelets

"doodles with defects"

$= \{$ 

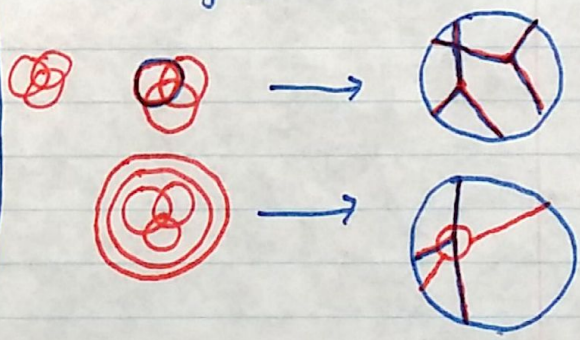
$\delta($  $) =$ 

v is of type $\geq n$ if v vanishes on $\delta^{\wedge}(K_n)$

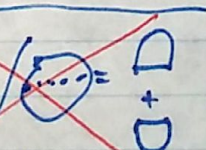
* examples

* Forget δ?

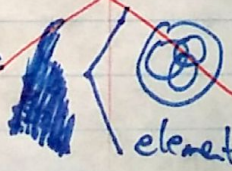
Chord diagrams



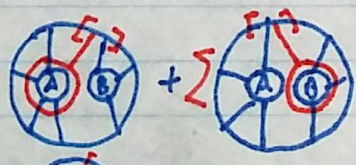
* Colour arms/legs differently?

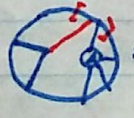
$K_n / \delta K_{n+1} = \{$  $\} /$ 

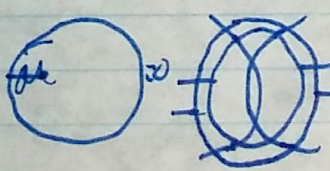
cyclically ordered n-component doodles, winding number ± 1

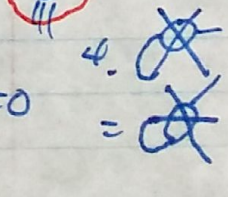
$= \{$  elementary doodles $\} /$ AS ring exchange.

Relations on chord diagrams

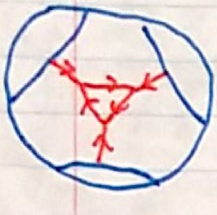
1. Σ 

2. Σ 

3. $\Sigma_{2^{\wedge}\text{loops}}$ 

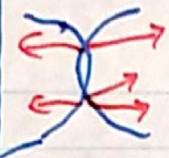
4. 

Arrow Diagrams



Gauss diagram skeleton
2-in-one-out
internal vertices

R2 invariance



$\Rightarrow \sum_{2^n \text{ diagrams}}$

$= 0$

Configuration space integrals

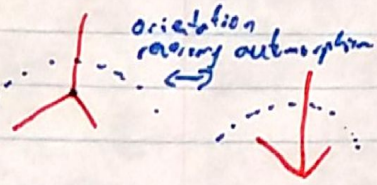
$$Z_{guess} = \sum_{\substack{\text{arrow} \\ \text{diagram}}} \sum_{\substack{\text{skeleton} \\ \text{embeddings} \\ D}} \int_{\text{Conf}(D)} \Phi^*(\omega)^{\#\text{arrows}} \cdot D$$

Φ

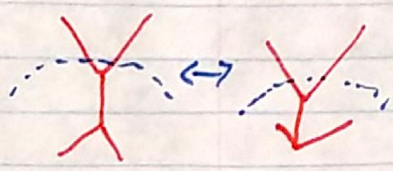
$\rightarrow \times \nearrow \times \downarrow \in (S^1)^3$

Hidden Faces Vanish

Usual trick:

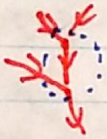


insert 2 S^1 factors and swap



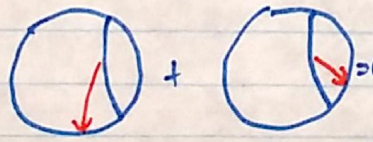
Primary Faces $\Rightarrow$ Relations on Arrow Diagrams

$\Rightarrow$ STU

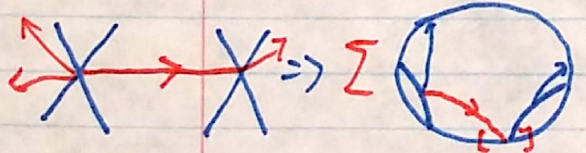


$\Rightarrow$ IHX

$\Rightarrow$



$= 0$



$\Rightarrow \sum$

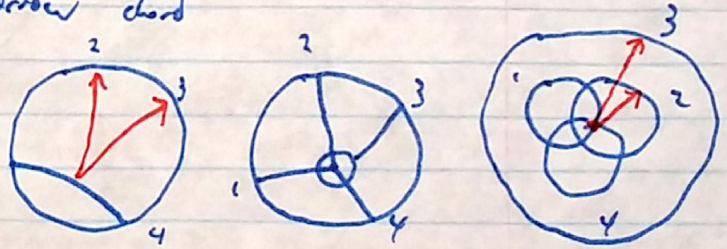
$+ \sum$

+ same with double points switched $= 0$

Evaluating Arrow Diagrams Integrals on n-bracelets

$I(D)(C) = \#$ "embeddings" of D in C

D: arrow diagram, C: chord diagram



Relations on Arrow diagrams
Pull back to relations on chord diagrams under I^*