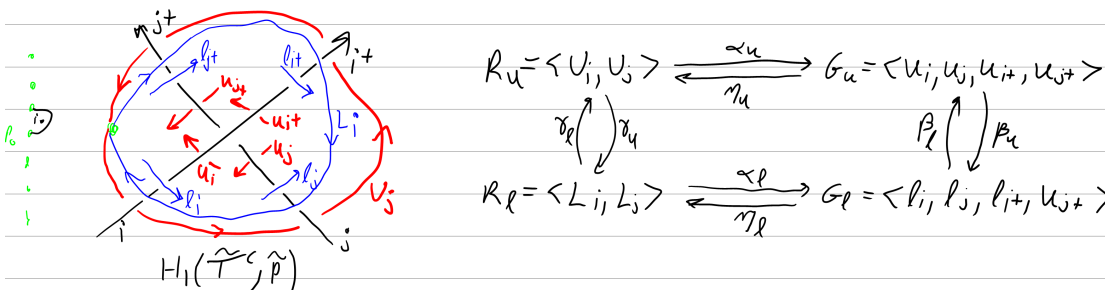


Pensieve header: Comparing the upper and the lower Wirtinger presentations.

The case of the positive crossing



In[*]:= $R_u = \{U_i, U_j\}; G_u = \{u_i, u_j, u_{i+}, u_{j+}\};$
 $R_l = \{L_i, L_j\}; G_l = \{l_i, l_j, l_{i+}, l_{j+}\};$

In[*]:= $\{\alpha_u, \alpha_l\} = \left(\begin{array}{cc} U_i \rightarrow u_i - u_{i+} & U_j \rightarrow -T^{-1} u_i - T^{-2} u_j + T^{-2} u_{i+} + T^{-1} u_{j+} \\ L_i \rightarrow l_j + T l_{i+} - T l_j - l_i & L_j \rightarrow l_j - l_{j+} \end{array} \right);$

$U_i \rightarrow u_i - u_{i+}$	$U_j \rightarrow -T^{-1} u_i - T^{-2} u_j + T^{-2} u_{i+} + T^{-1} u_{j+}$	The ba
$U_i \rightarrow -T^{-1} u_j - T^{-2} u_i + T^{-2} u_{j+} + T^{-1} u_{i+}$	$U_j \rightarrow u_j - u_{j+}$	Same, evaluate
$U_i \rightarrow T^{-2} u_{j+} + T^{-1} u_{i+} - T^{-1} u_j - T^{-2} u_i$	$U_j \rightarrow u_j - u_{j+}$	Re - arrar
$U_i \rightarrow u_{j+} + T u_{i+} - T u_j - u_i = (1 - T) u_{j+} + T u_{i+} - u_i$	$U_j \rightarrow u_j - u_{j+}$	Multiply r
$L_i \rightarrow l_{j+} + T l_{i+} - T l_j - l_i$	$L_j \rightarrow l_j - l_{j+}$	The targ
$U_i \rightarrow u_i - u_{i+}$	$U_j \rightarrow -T^{-1} u_i - T^{-2} u_j + T^{-2} u_{i+} + T^{-1} u_{j+}$	The base α_u
$U_i \rightarrow u_i - u_{i+}$	$U_j \rightarrow (1 - T^{-1}) u_i + T^{-1} u_j - u_{j+}$	Using $u_i = u_{i+}$ within U_j and mul
$u_i \rightarrow U_i + (1 - T^{-1}) U_j - U_{i+}$	$u_j \rightarrow T^{-1} U_j - U_{j+}$	Taking the transpose
$U_i \rightarrow u_i + (1 - T^{-1}) u_j - u_{i+}$	$U_j \rightarrow T^{-1} u_j - u_{j+}$	Taking the transpose
$U_i \rightarrow u_{i+} + (1 - T^{-1}) u_{j+} - u_i$	$U_j \rightarrow T^{-1} u_{j+} - u_j$	Shifting the colu
$U_i \rightarrow u_{i+} + (T - 1) u_{j+} - T u_i$	$U_j \rightarrow u_{j+} - u_j$	Multiplying each column by
$U_i \rightarrow T^{-1} u_{i+} + (1 - T^{-1}) u_{j+} - u_i$	$U_j \rightarrow u_{j+} - u_j$	Divide U_i by T
$U_i \rightarrow T u_{i+} + (1 - T) u_{j+} - u_i$	$U_j \rightarrow u_{j+} - u_j$	Replace $T \rightarrow T^{-1}$; Bi

In[*]:= $\beta_u = \{u_i \rightarrow l_i, u_j \rightarrow l_i + T l_j - T l_i, u_{i+} \rightarrow l_{j+} + T l_{i+} - T l_{j+}, u_{j+} \rightarrow l_{j+}\};$
 $\beta_l = \{l_i \rightarrow u_i, l_j \rightarrow -T^{-1} u_i + T^{-1} u_j + u_i, l_{i+} \rightarrow -T^{-1} u_{j+} + T^{-1} u_{i+} + u_{j+}, l_{j+} \rightarrow u_{j+}\};$

In[*]:= $\gamma_u = \{U_i \rightarrow -L_i - T L_j, U_j \rightarrow T^{-2} L_i\};$
 $\gamma_l = \{L_i \rightarrow T^2 U_j, L_j \rightarrow -T^{-1} U_i - T U_j\};$

In[*]:= $\{\text{Expand}[G_u /. \beta_u /. \beta_l], \text{Expand}[G_l /. \beta_l /. \beta_u]\}$

Out[*]:= $\{\{u_i, u_j, u_{i+}, u_{j+}\}, \{l_i, l_j, l_{i+}, l_{j+}\}\}$

In[*]:= $\text{Simplify}[\{(R_u /. \alpha_u /. \beta_u) - (R_u /. \gamma_u /. \alpha_l), (R_l /. \alpha_l /. \beta_l) - (R_l /. \gamma_l /. \alpha_u)\}]$

Out[*]:= $\{\{0, 0\}, \{0, 0\}\}$