$$P = \epsilon\left(P_2 + P_4\right)$$

$$\begin{pmatrix} 1 & 1-t \\ 0 & t \end{pmatrix} \qquad \sigma_{ij} \longmapsto \begin{pmatrix} 1 & & & & 0 \\ & 1 & & & \\ & & 1 & 1-t & \\ 0 & & 0 & t & \\ & & & & 1 \\ & & 0 & & & 1 \end{pmatrix}$$

$$\downarrow$$

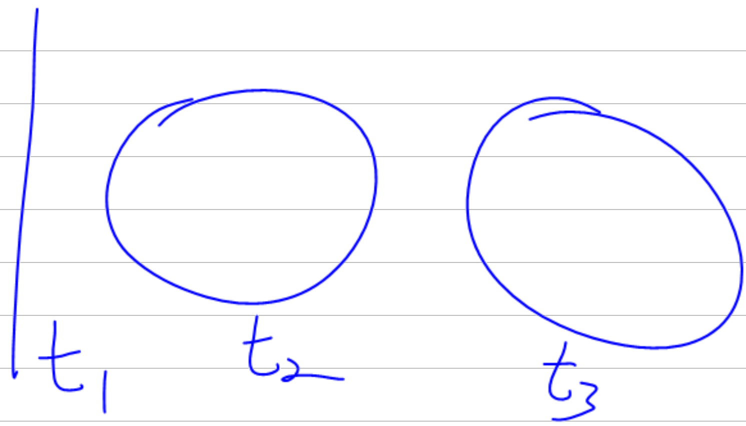
$$\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \qquad t_{ij} = \begin{pmatrix} 0 & 0 & \\ & 1 & -1 \\ & & 0 & \\ & -1 & & 1 \\ 0 & & & & 0 \end{pmatrix}$$

$$R = \mathbb{E}\left(\left(e^{t+\epsilon} - 1\right)\left(P_1 - P_2\right) \supset G_2\right)$$

$$R \sim e^{yx+ba}$$

$e^W =$ 

$\to \mathbb{E}\left(yx+ba+a^b_{\text{stuff}}+p\right)$



$$[x, ay] = xy + at$$
$$\|$$
$$[a, xy]$$

$$W = ta + yx \qquad [x, y] = t$$

$$a = \frac{W}{t} - \underbrace{\frac{y}{t}}_{p}x \qquad [x, p] = 1$$

$$F(\epsilon) = \int_0^\infty \frac{e^{-x}}{1+\epsilon x}\,dx \sim \int_0^\infty dx \sum_{n=0}^{\infty} e^{-x}(-\epsilon x)^n$$

$\epsilon \geq 0$ $\qquad\qquad$ $|x| < \frac{1}{\epsilon}$

$$= \sum_{n=0}^{\infty} (-\epsilon)^n \int_0^\infty e^{-x} x^n\,dx$$

$$= \sum_{n=0}^{\infty} (-1)^n n!\, \epsilon^n \qquad \sum \frac{1}{n!}\epsilon^n$$

$\sin 100$

Aug 11, 2020

Q If P_1 is $2k+2$-docile, show that so is

$$\log(\Phi(e^{P_1})) \in \mathcal{U}(-)[[\epsilon]]$$

If P_2 is $4k$-docile, so is

$$\log(\Phi(P_2))$$

$$\begin{tikzcd}
\mathcal{S} \arrow[r, "?"] \arrow[d, "\Phi"] & \mathcal{S} \arrow[d, "\Phi"] \\
\mathcal{U} \arrow[r, "\log"] & \mathcal{U}
\end{tikzcd}$$

$$\mathcal{D}(e^{\mu xy}) = \sum \frac{\mu^{B} x^n y^n}{n!} \xrightarrow{\log} \log(\mu+1)\cdot xy$$

$$\text{in } \mathcal{U} \quad e^{\lambda xy} = \mathcal{D}\left(e^{(e^\lambda - 1)xy}\right)$$

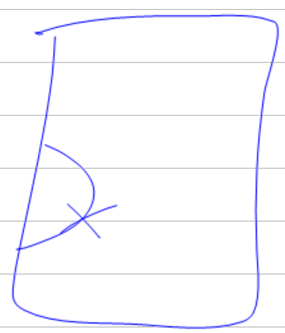
$\log_u \mathcal{D}(e_s^{\lambda Q})$ is quadratic?

$$\partial_\lambda : \mathcal{D}(e^{\lambda Q})^{-1}\,\mathcal{D}(Q e^{\lambda Q})$$

$$e^{P_0 + tP_1 + t^2 P_2}$$

where $wt(P_k) \le 2k+2$ — linear

at $k=0$ $wt(P_0) \le 2$ — $e(t)a$, $wt\ 2$



$$\begin{tikzcd} & \text{EDDO} \arrow[dl, dash] \arrow[dr, dash] & \\ \text{no } t^2 a \arrow[dr, dash] & & \text{no } \alpha\beta \arrow[dl, dash] \\ & \text{neither} & \end{tikzcd}$$

Aug 17, 2020

$$Q \longmapsto Q'$$

$$\underline{\underline{L}} + \underline{\underline{Q}} + \underline{\underline{P}}$$

no $\quad (n-k)k$

$$\underline{\underline{e^{Q} p}} \sim \underline{\underline{e^{Q+p}}}$$

$$\langle a_1 a_2 a_3, \; e^{q_{ij} x_i y_j} p(\bar{x}, \bar{y}) \rangle$$

n^2

$$\frac{\partial}{\partial \alpha} \rho(A) \qquad \deg \rho = n$$

$$\frac{\partial}{\partial \alpha}\left(\sum_{k=-n}^{n} \lambda_k e^{k\alpha} \right) = \sum_{k=-n}^{n} k \lambda_k e^{k\alpha}$$

$$\frac{d}{dx} f = \lim \frac{f(x+h) - f(x)}{h}$$

$$\frac{c}{cx} f = f(x+1) - f(x)$$

$y^n \to n y^{n-1}$

$$\operatorname{ad}_x f(y) = b f'(y)$$

$$\operatorname{ad}_x f(a) = \cancel{a} f(a+1) - f(a)$$

$$a^{(n)} = a(a-1)(a-2) \ldots (a-n+1)$$

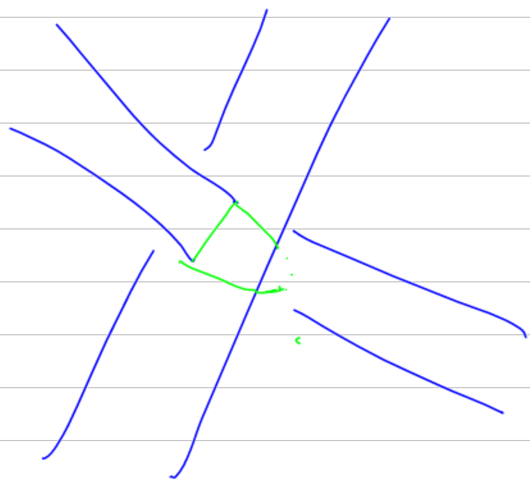
$$L : a^n \longmapsto a^{(n)}$$

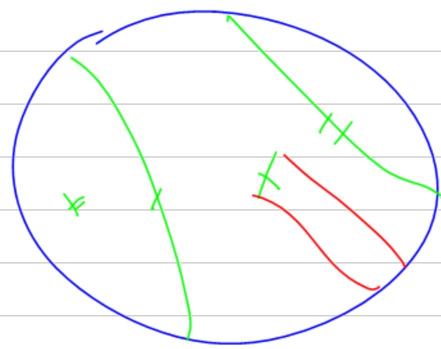
$$g(L) = \sum \frac{1}{n!} \alpha^n a^{(n)}$$

$$= (1+\alpha)^{\wedge}$$

$$+ : \mathbb{R}^2 \to \mathbb{R}$$

$$D+ = + \qquad (+)' = (1 \quad 1)$$





$$u = ABS(u) \Rightarrow u \geq 0$$

$$\frac{1}{1-\alpha} = \frac{1}{-(\alpha + \frac{\alpha^2}{2} +}$$

$$= \frac{-1}{\alpha}\left(\frac{1}{1+\frac{\alpha}{2}}\right)$$

$$\left\langle \frac{1}{\alpha}, \alpha^n \right\rangle = \partial_\alpha^n \frac{1}{\alpha}\Big|_{\alpha=0}$$ Boom!

$$\left\langle \frac{1}{\alpha}, e^{\lambda a} \right\rangle = e^{\lambda \partial_\alpha} \frac{1}{\alpha}\Big|_{\alpha=0}$$

$$= \frac{1}{\alpha+\lambda}\Big|_{\alpha=0} = \frac{1}{\lambda} \underset{\lambda<0}{=} \int_0^\infty e^{\lambda a}\, da$$

$$\left\langle \frac{1}{\alpha}, f \right\rangle \rightsquigarrow \int_{-\infty}^{\infty} \frac{\hat{f}(\lambda)}{i\lambda}\, d\lambda$$

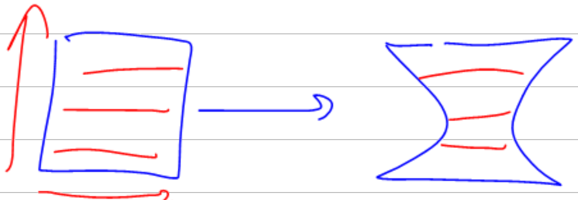
$$e^{Q_1} /\!/ e^{Q_2} = e^{Q} \qquad \frac{1}{1-FG} = \sum (FG)^n$$

$$e^{Q} \qquad \rho$$

---

cov if g is l.p.

$$g\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} \cancel{x} \\ z \\ x_n \end{pmatrix} \qquad \begin{pmatrix} \boxed{n-1} & \\ & \end{pmatrix}$$



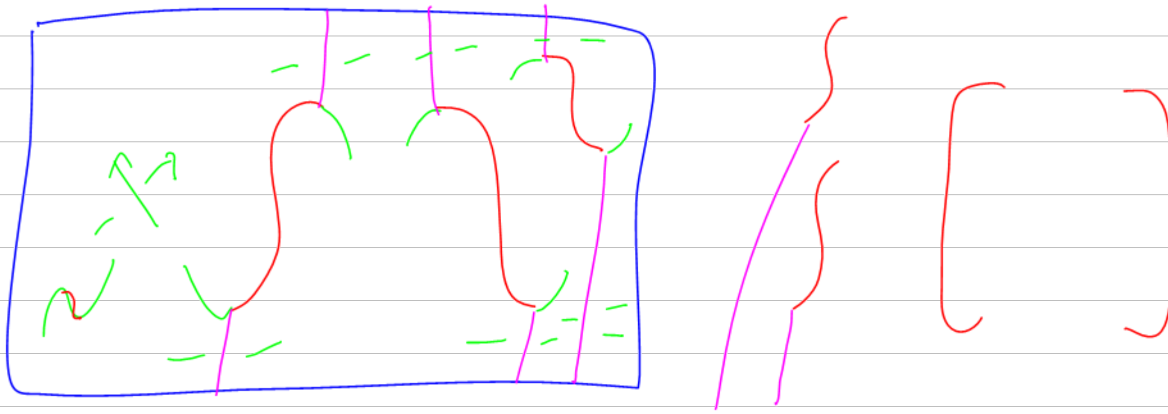
$\mathbb{R}^{n-1}$

$$(x_1 \ldots x_n) \longmapsto (y_1 \ldots y_n)$$

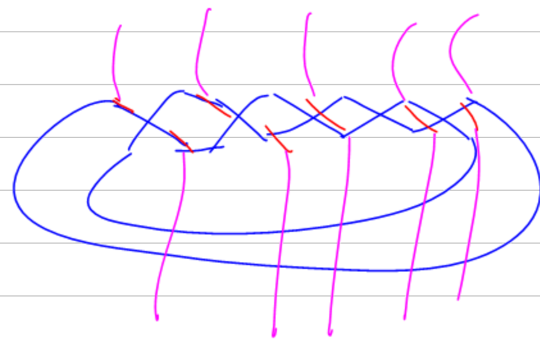
$$(x_1 \ldots x_{n-1}, y_k)$$

$$\underline{\text{Knots}} \xrightarrow{\text{Alex}} \underline{\text{Braids}}$$

$$\text{R-moves} \longleftarrow \text{M-moves}$$



$n\cdot\sqrt{n}$



linear

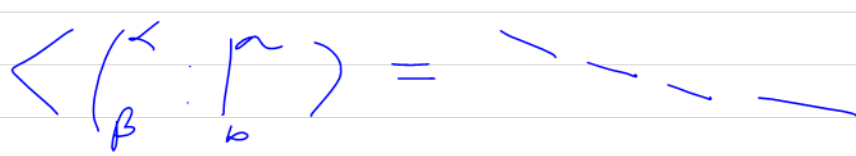
$$e^{L+\lambda_8} = F_L(\lambda_8) \qquad \text{to deg } 8$$

$x \quad \bar{x} \quad \tilde{x}$

$R \quad \bar{R} \quad \tilde{R}$

$$\partial_x \partial_y e^Z = \partial_x\left((\partial_y z) e^z\right)$$

$$\begin{array}{ccc} & & \\ \alpha & \multicolumn{2}{c}{\multirow{2}{*}{$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$}} \\ \beta & & \\ & \alpha & \beta \end{array}$$



$$(w, A) \longmapsto w \Lambda^*\left(\frac{A}{w}\right)$$

1. no dens.

2.

$$R \times M_{n\times n}(R) \xrightarrow[\text{linear}]{\text{non}} \mathrm{End}(\Lambda^*(R^n))$$

$\circlearrowleft m^{ij}_k$ $\quad \circlearrowleft tr_i$ $\qquad\qquad \circlearrowleft m^{ij}_k$ $\quad \circlearrowright_{tr_i}$ linear.

$$\mathbb{R}^n \longrightarrow S^*(\mathbb{R}^n)$$

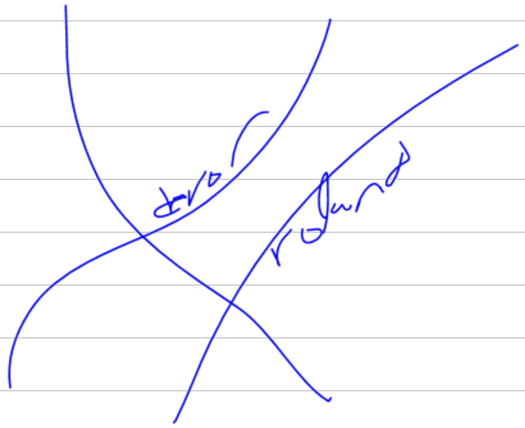
$\circlearrowleft_p$ $\qquad\qquad \circlearrowleft_L$

$$\mathrm{Hom}(SX, SY) \sim S(X^*, Y)$$
$$\mathrm{Hom}(\Lambda X, \Lambda Y) \sim \Lambda(X^*, Y)$$

$$R \times M_{s\times s}(R) \longrightarrow \mathrm{Hom}(\Lambda X, \Lambda X) = \Lambda(X^*, X)$$

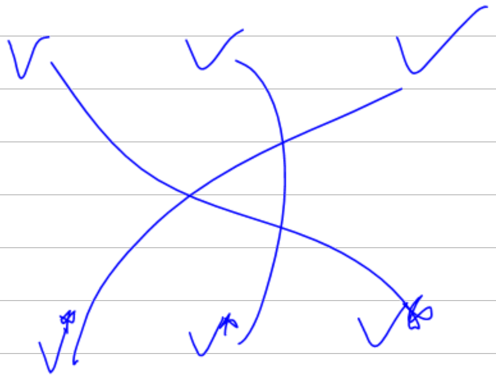
$\circlearrowleft m^{ij}_k$ $\qquad\qquad \circlearrowleft$ zip w/ x^n

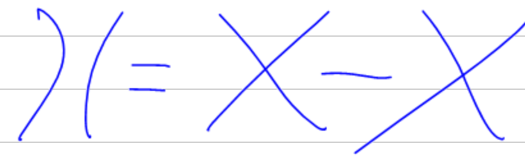
$$(w, A) \longmapsto w e^{(x^*)^T A x}$$

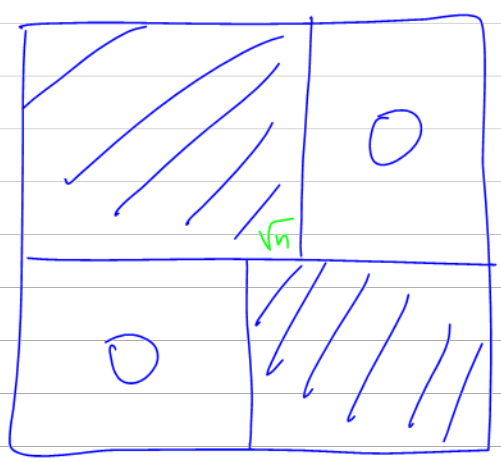


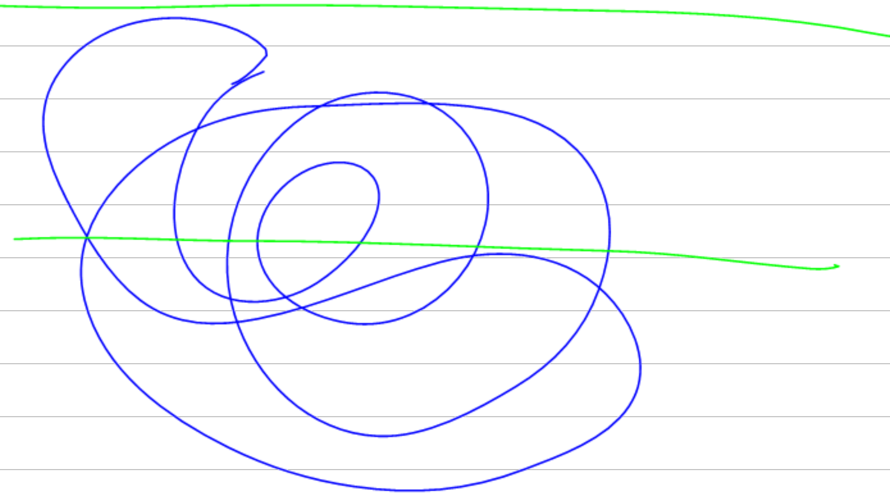
$$A^{\otimes 3} \xrightarrow{\rho} V^{\otimes 3} \otimes (V^*)^{\otimes 3}$$

$$A \to V^* \otimes V$$

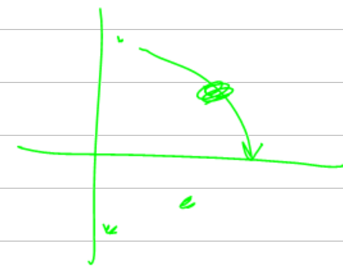




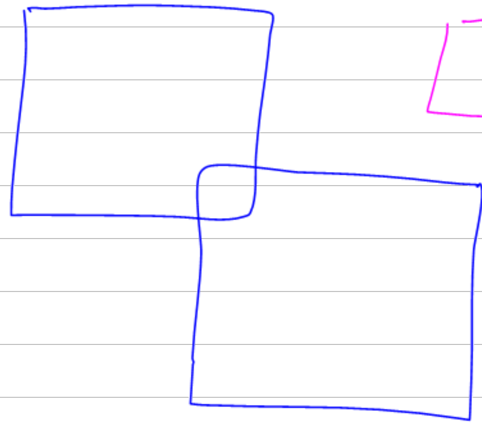


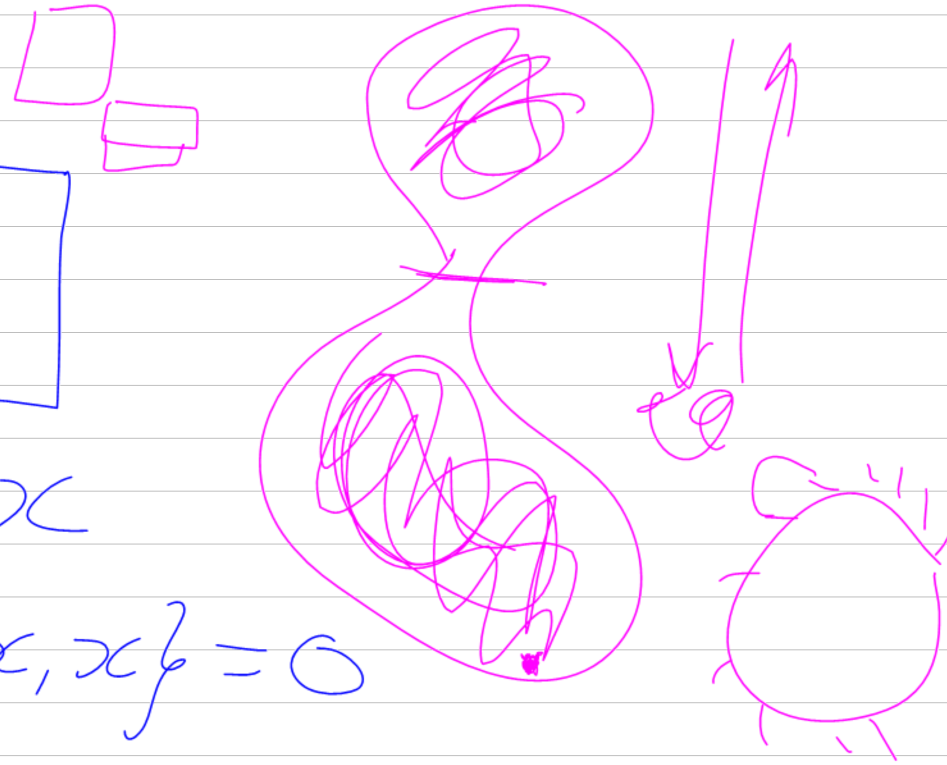


$$\frac{1}{(2\pi)^{n/2}}\int e^{-\frac{1}{2}\lambda_{ij}x^i x^j} = \frac{1}{\sqrt{\det \Lambda}}$$



$$\frac{1}{\sqrt{2\pi}}\int e^{\frac{i\lambda}{2}x^2 - \epsilon x^2} = \frac{1}{\sqrt{i\lambda}} \to \frac{1}{\sqrt{\lambda}}\, e^{\mp i\frac{\pi}{4}\,\mathrm{sign}\,\lambda}$$





$$[a, x] = x$$

even odd

$$\{x, x\} = 0$$

$$x^2 = 0$$

$$Q[a] \otimes \langle 1, x \rangle$$

$$\frac{e^b - 1}{}$$

$$y \; b \; a \; x$$

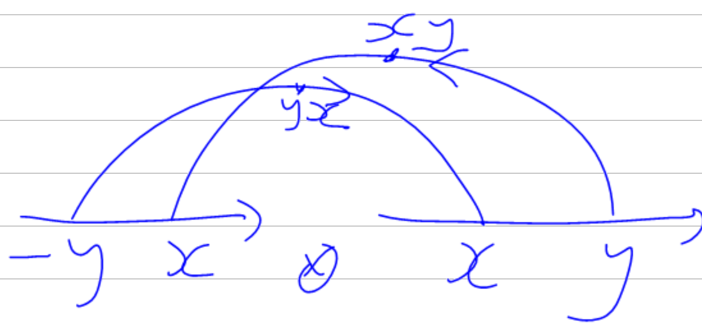
even

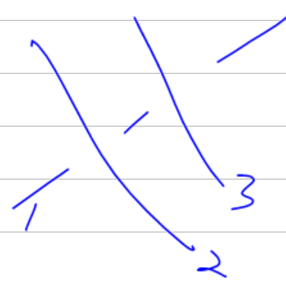
odd

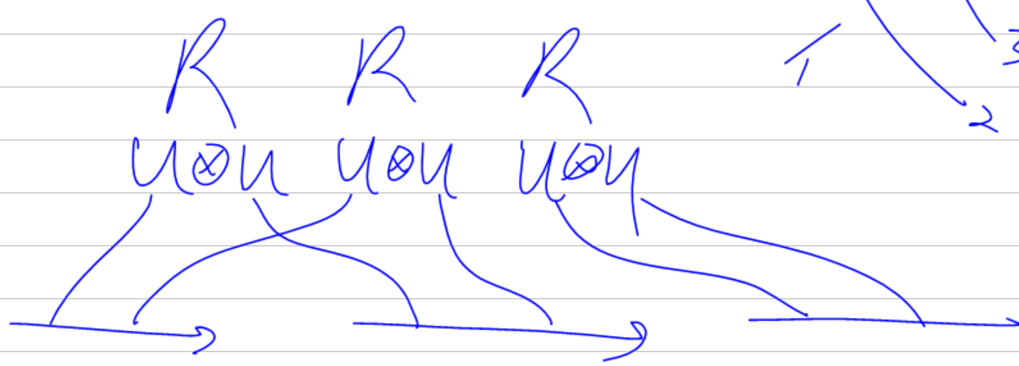
$$\{y, x\} = b + ca^b$$

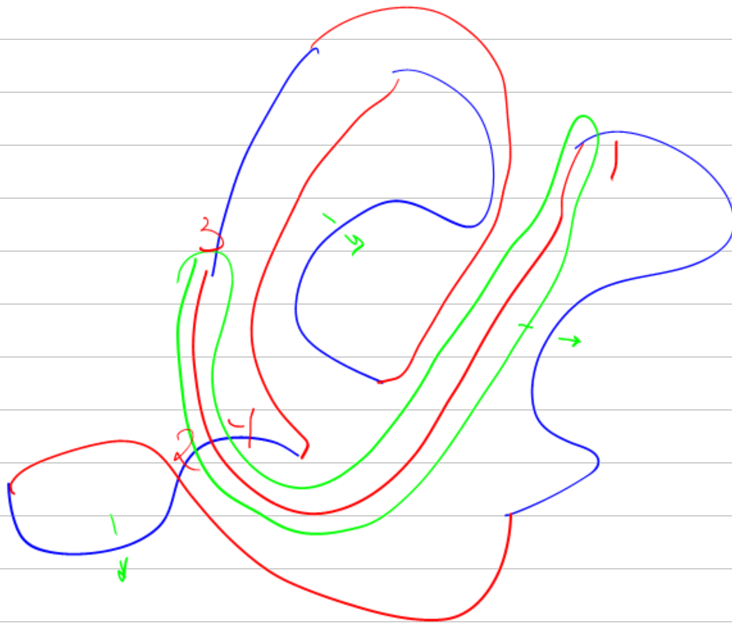
$$r = ba + yx \qquad R = e^r$$

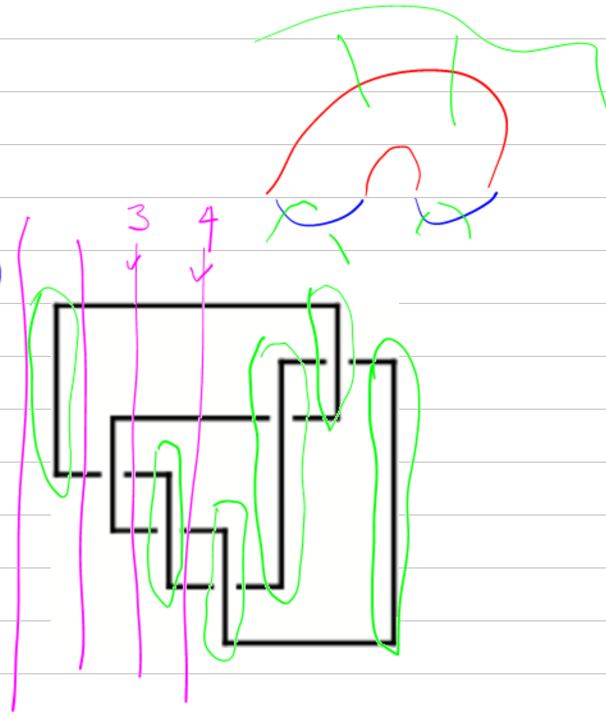
$$U = Q[a, b] \otimes \left\langle \begin{matrix} 1 & x \\ y & yx \end{matrix} \right\rangle$$

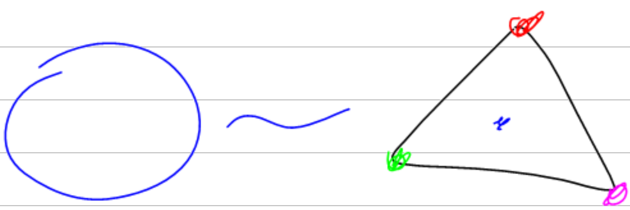
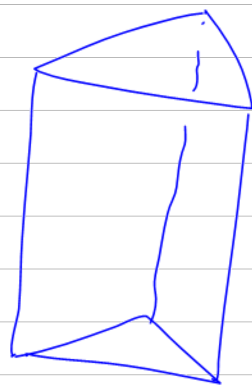
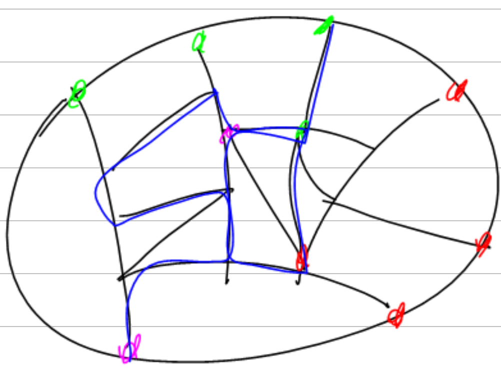




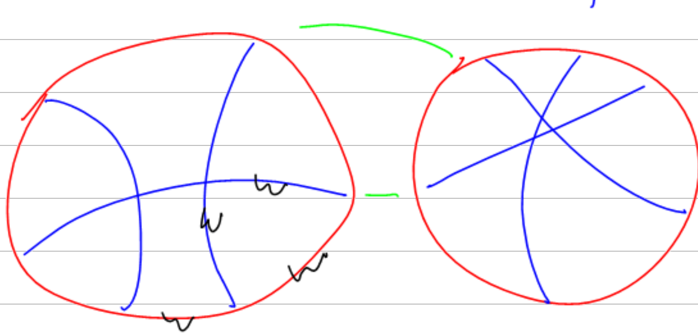


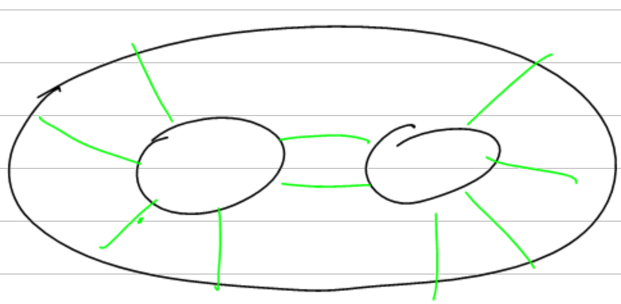




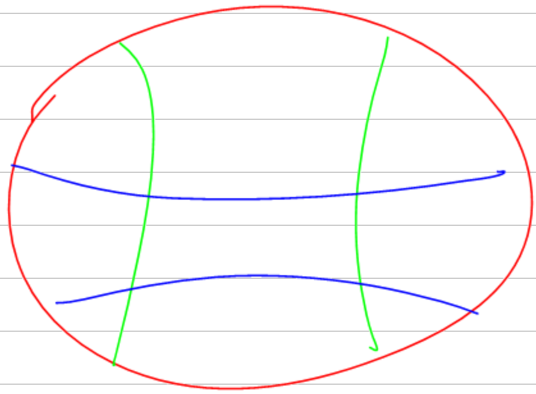
$2 \perp 2 = 4$





$A^{\otimes S}$

$A^{\otimes E} / A^{\otimes V}$



$R = e^{\sim 3}$

$$\mathbb{Q}[a] \xrightarrow[F]{e^{\alpha} x} \mathbb{Q}[x]$$

$a^n \longrightarrow 0$

$F(a^n) = x \not\to 0$

am

$A = U\left(\begin{matrix} x, a \\ [a, x] = x \end{matrix}\right)$

$a \cdots x$

$x a^n = \pm x + \dots$

$$\mathrm{Hom}_c(\mathbb{Q}[z_1] \to \mathbb{Q}[z_2]) \sim \mathbb{Q}[z_2][[\zeta_1]]$$

$$\mathrm{CHom}_{\mathbb{Q}[t]}\left(\mathbb{Q}[z][t] \circlearrowleft\right)$$

$$\simeq \mathbb{Q}[z][[\xi]][[t]]$$

$$am = \ldots\ldots e^{\alpha\xi}$$

$$\cancel{\mathbb{Q}[[a,x]]} \qquad e^{x} \qquad e^{tx}$$

$$\wr$$

$$U(a,x/[a,x]=x)[[t]]$$

$$e^{ab}$$

$$[x,y] = b + \underline{\underline{\epsilon}}a$$

$$\mathbb{Q}[[\alpha,\xi]] \checkmark$$

$$U_0 \subset U_1 \subset \ldots \subset U_k \subset \ldots$$

$$U_0^* \leftarrow U_1^* \leftarrow \ldots$$

$$\begin{array}{ccc} \mathbb{Q}[x] & \xrightarrow{\mathbb{Q}[y][[\xi]]} & \mathbb{Q}[y] \\ \wr & & \wr \\ U & & U \end{array}$$

$$\mathbb{D} = QU = \mathcal{D}(\mathbb{A}, \mathbb{B})$$

$$dm^{ij}_{k} \quad d\Delta \quad dS$$

$$dm_\lambda = \Delta\Delta // P_\lambda P_\lambda // mm$$

$$\downarrow \qquad \downarrow$$

$$dm$$

$$g(\Delta\Delta // PP // mm) = g(\Delta\Delta) // g(PP) // g(mm)$$

in $\mathbb{Q}[z, \hbar^{\pm}][[\zeta]]$

$$\cancel{x} \quad \cancel{b} = \cancel{x}^* \qquad B = e^{-\hbar}b$$

$$\mathbb{A} \otimes \mathbb{B} \longrightarrow \mathbb{Q}_\hbar$$

$$(a, b) = 1 \quad (x, y) = 1$$

$$\langle x^n, y^n \rangle = [n]!_\hbar$$

$$(y^{17}, \cdot)^{-1} (\hbar^{500} \ldots)$$

a^{18}, x^{18}

$$x^n \longrightarrow 0$$

$$|F(z)| \le \underbrace{|F|}_{}\,|z|$$

$$\begin{tikzcd}
K \arrow[r, squiggly] \arrow[r, bend right, "Z"'] & \tilde{A} \arrow[r, "g"] & U(\mathfrak{g})[[\hbar]] \arrow[loop below, "\cdot\hbar"]
\end{tikzcd}$$

$$\mathrm{Hom}_{\mathbb{Q}[\hbar]}(\mathbb{Q}[z][\hbar] \circlearrowleft) \overset{\wr}{\longleftrightarrow} \mathbb{Q}[z][\zeta][\hbar]$$

$$\mathrm{Hom}_{\underbrace{\mathbb{Q}[[\hbar]]}_{R}}(\mathbb{Q}[[\hbar]][z] \circlearrowleft) \longleftrightarrow \mathbb{Q}[[\hbar]][z][[\zeta]]$$

$$T(U) = R_{12}\, U_{34} \,//\, m_B^{13} m_A^{24} = I$$

$$T(U+\delta U) = T(U) + \underline{\underline{dT}}(\delta U) + o(\delta U)$$

$$\xrightarrow{a}\ \xrightarrow{b}\ \xrightarrow{c}$$

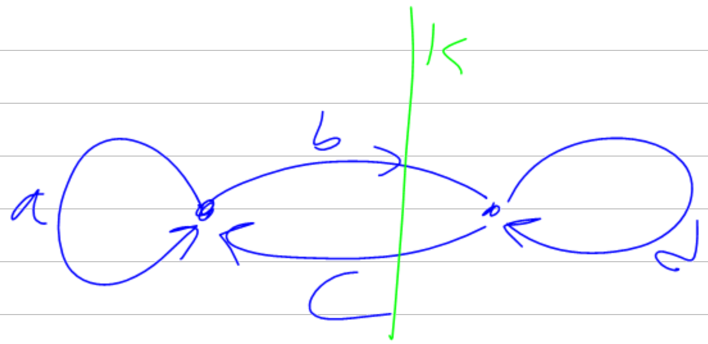
ab $\quad (ab)c \leftarrow$

bc $\quad a(bc) \leftarrow$

$$\underline{\mathrm{Hom}(V,W)} \cong \mathrm{Hom}(W^*,V^*)^*$$

$$(V^*\otimes W)[[h]]$$

$$S(p,q) \xrightarrow{\mathrm{Sym}} U([p,q]=1)$$



$$\rtimes \langle k\rangle / k^2=1$$

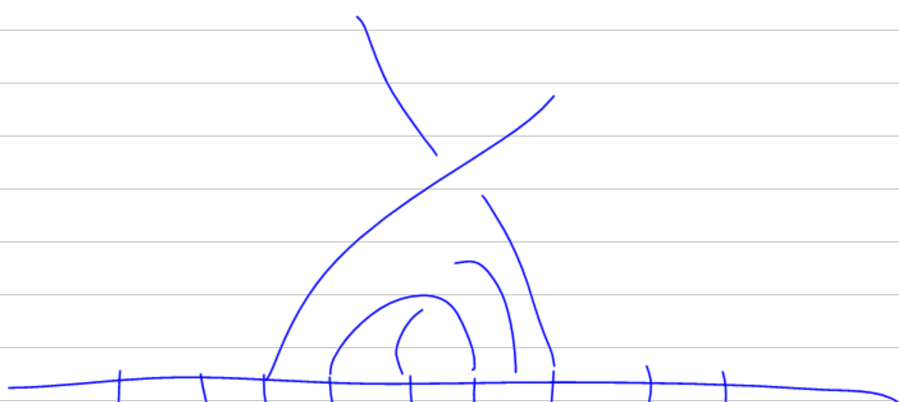
$$GL(2) \qquad \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

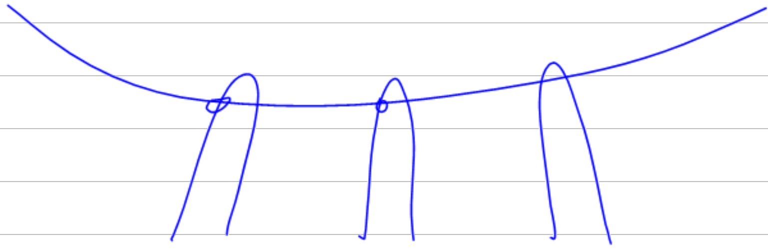
$$ka=ak \quad kd=dk \quad kb=-bk$$

$$\Delta a \neq a\otimes 1 + k\otimes a$$

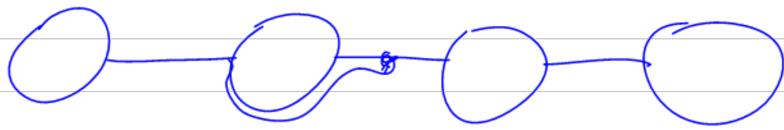
The Sweedler Algebra:

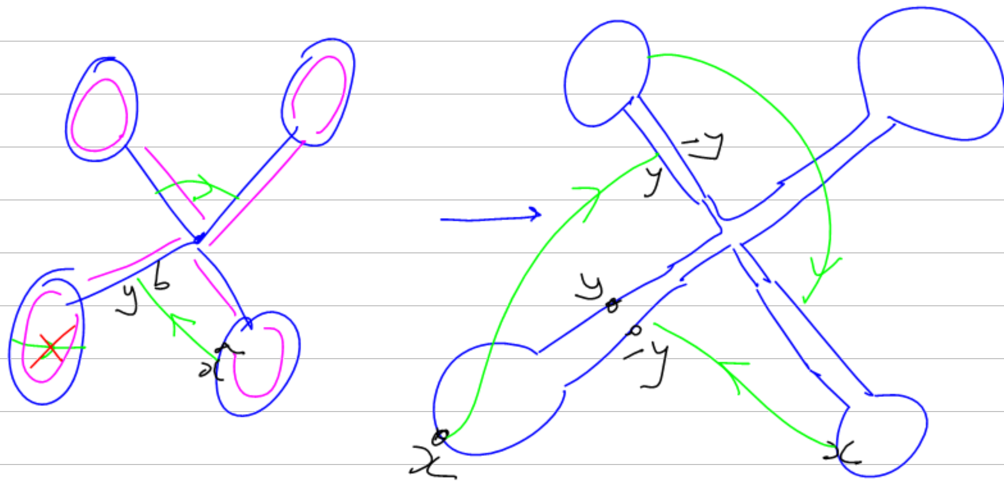
$$s^2=1 \quad w^2=0 \quad sw=-ws$$

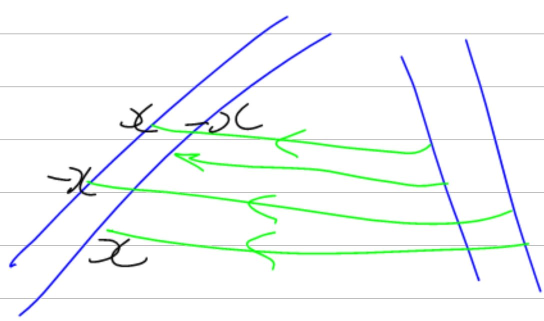




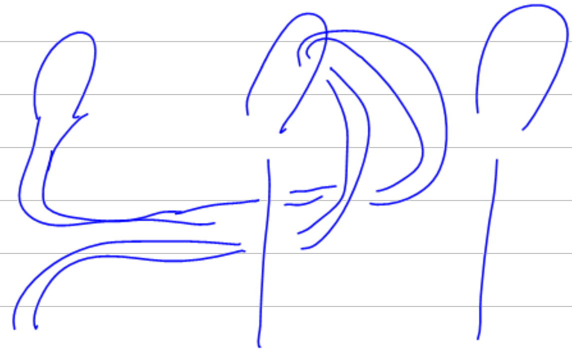
$$A_a(t) = f_1(t)\, f_2(t^{-1})$$







$$\begin{tikzcd}
 & \mathcal{A}(\uparrow_{2n}) \arrow[dl, "K"'] \arrow[dr, "\tau"] & \\
\mathcal{A}(\uparrow_n) & & \mathcal{A}(\uparrow_n)
\end{tikzcd}$$



$$x \qquad \left(= \mathbb{O}(\ldots) \right)$$

$$e(\lambda) = e_m^{\lambda x} = \mathbb{O}\left(e^{f(\lambda)}\right)$$

$$e(\lambda+\mu) = e(\lambda)e(\mu)$$

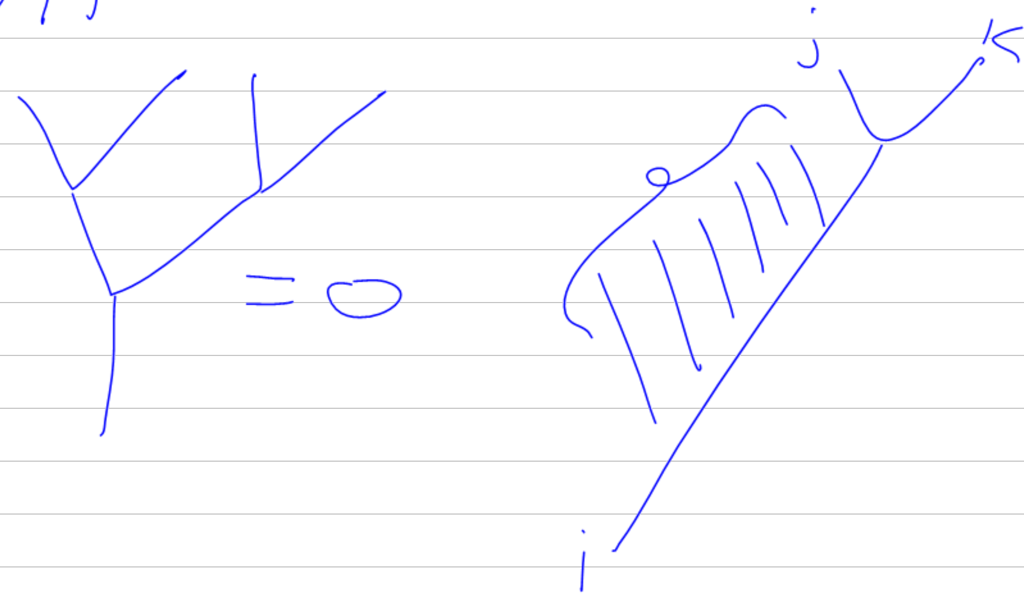
$$\mathbb{O}\left(e^{f(\lambda+\mu)}\right) = \mathbb{O}\left(e^{f(\lambda)}\right)\mathbb{O}\left(e^{f(\mu)}\right)$$

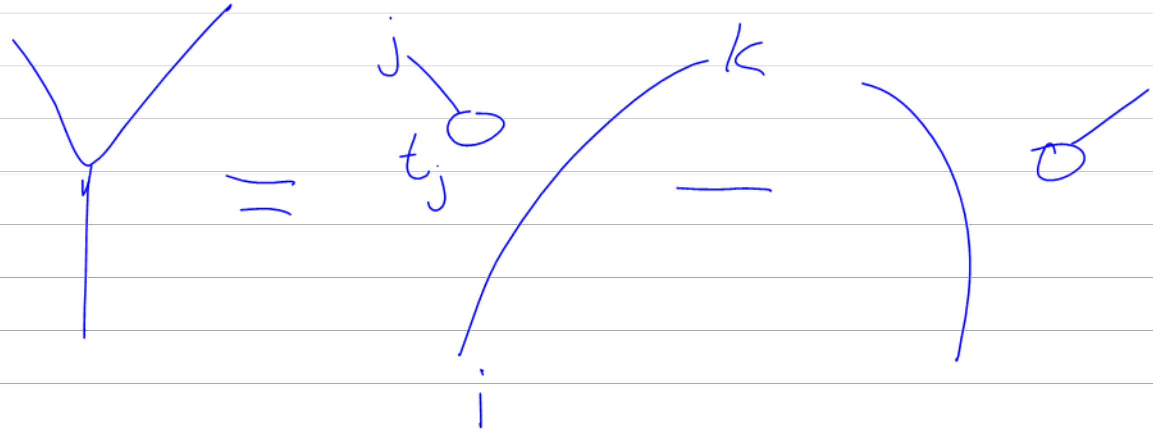
$$= \mathbb{O}\left(e^{h(\lambda,\mu)}\right)$$

$$\partial_\mu f(\lambda+\mu)\Big|_{\mu=0} = \partial_\mu h(\lambda,\mu)\Big|_{\mu=0}$$

$$e^{\vdash\!\dashv} = e^{\vdash\!\rightarrow\!\dashv \;+\; \vdash\!\leftarrow\!\dashv}$$

KBH





$a_{i\_} := i^2$

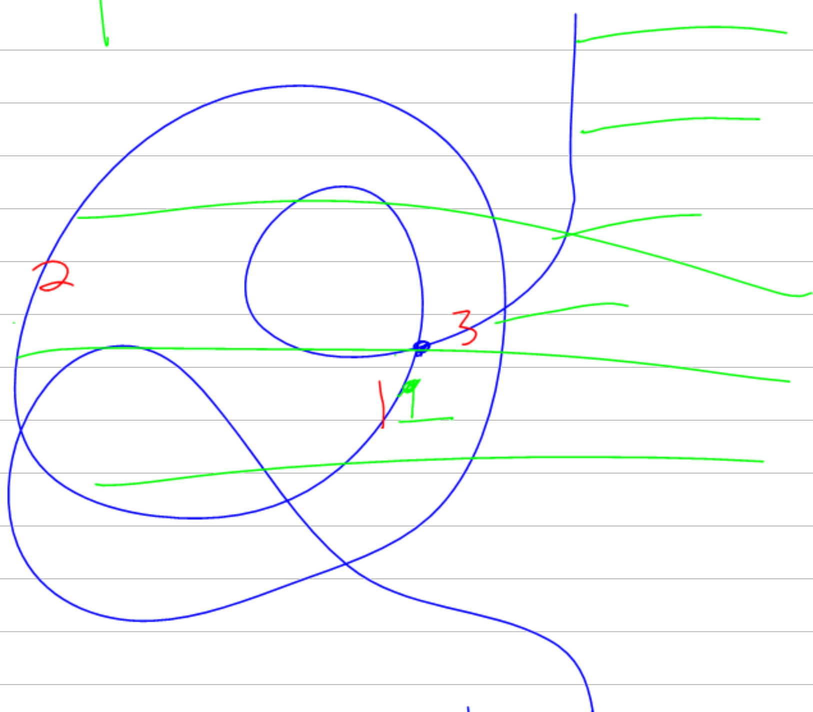
Subscript[a, i_] := i^2

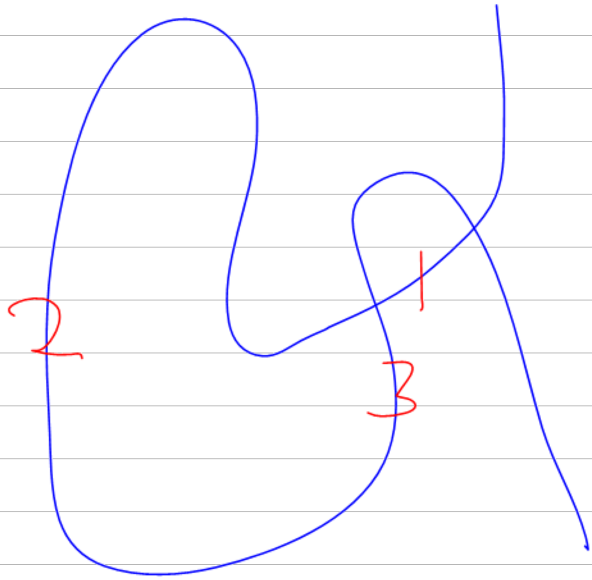
a /: $a_{i\_} := i^2$

S[k_] := S[n, k]

$$S[n\_, k\_] := S(n,k) = \_\ \_\ \_\ \_$$

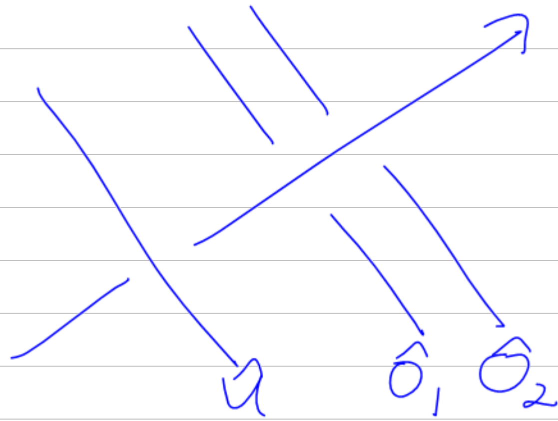
$$\mathcal{E}\_ // bm\_ := \mathcal{E} // bm\_ = \ldots$$



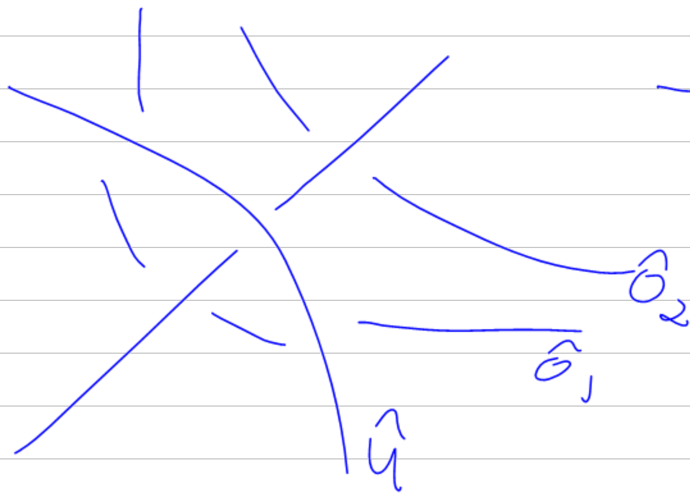


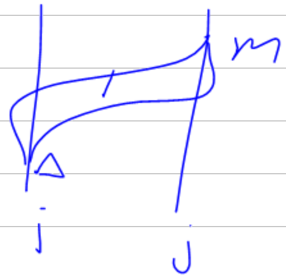
$\underline{\underline{W}}\, e^{Q+p}$

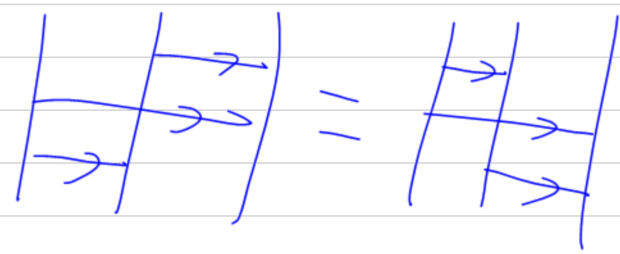
$\det Q \cdot (- \ - \ .$



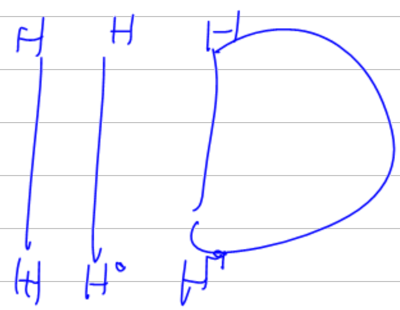
$u\ o_1\ o_2$



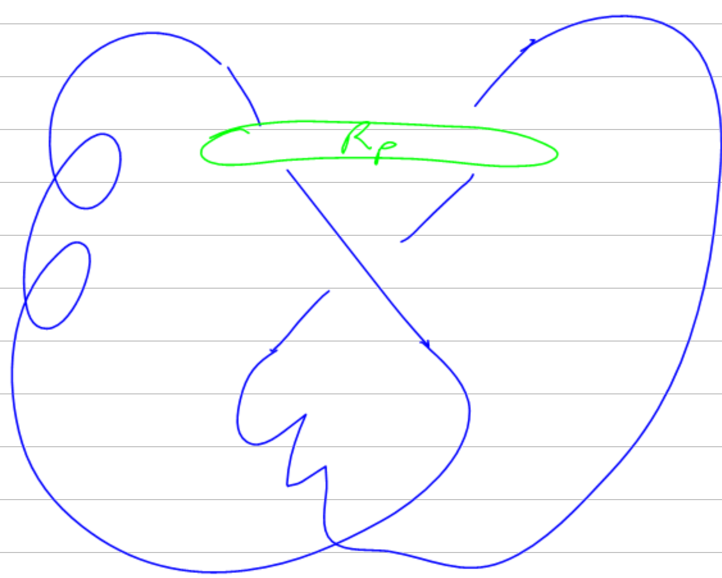
σ_{ij} 



$$H^{*}\otimes \dots \otimes H^{*} \otimes H \otimes \dots \otimes H$$



$$\begin{pmatrix} \alpha & \beta & \gamma \\ \delta & \epsilon & \eta \\ \lambda & \mu & \nu \end{pmatrix} \longrightarrow \beta\delta - \gamma\mu + \nu$$



$$\left\langle \sum \pi_i \rho_i + \xi_i x_i : MM \cdot \prod R \right\rangle$$

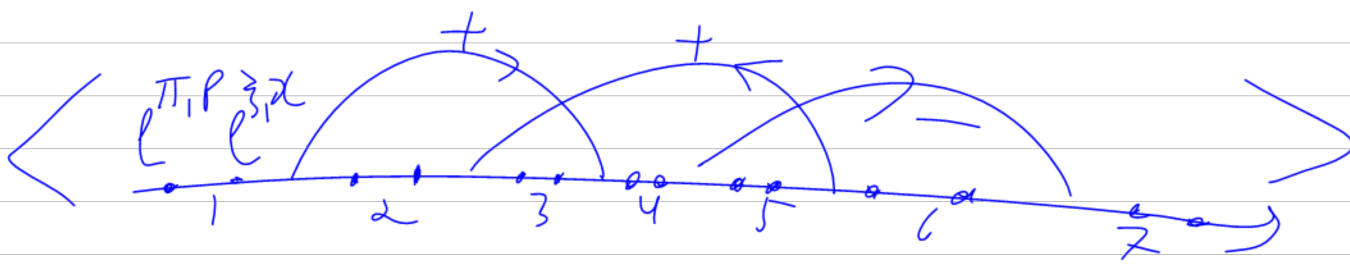
$$\langle F : G + P \rangle \qquad \rho_i \rho_j x_j^2$$

$$= \langle (\mathbb{I} + FG)^{-1} : P \rangle$$

$$R_{ij} = R^{\circ}_{ij}\left(1 + \rho_i \rho_j x_j^2\right)$$

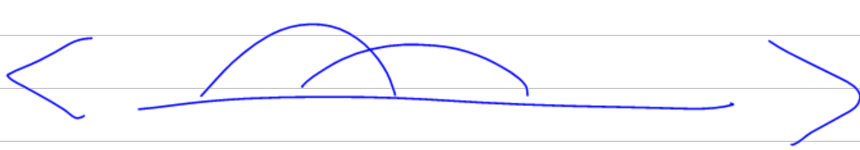
$$\Downarrow$$

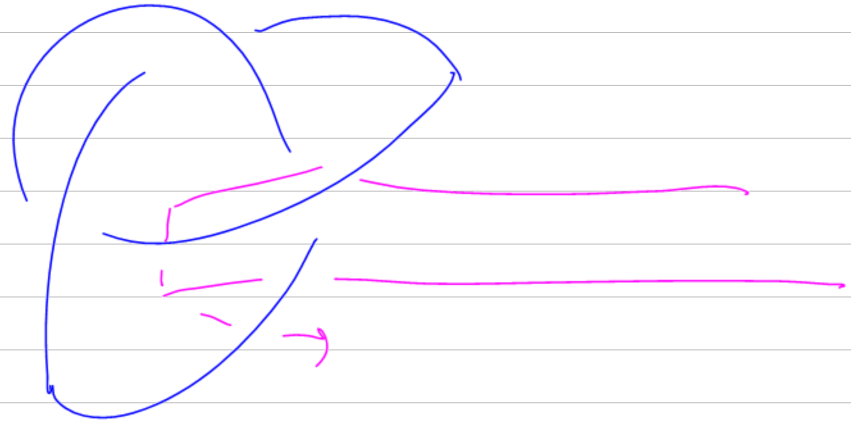
$$2\partial_{ij}\partial_{jj} \qquad \rho_i \rho_j x_j x_j$$

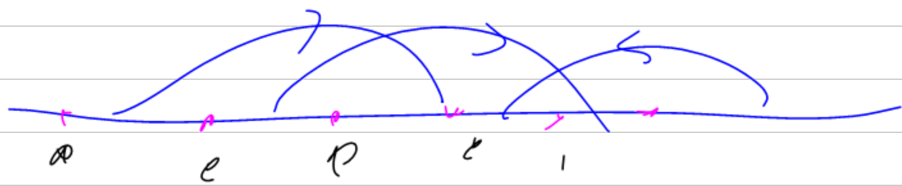


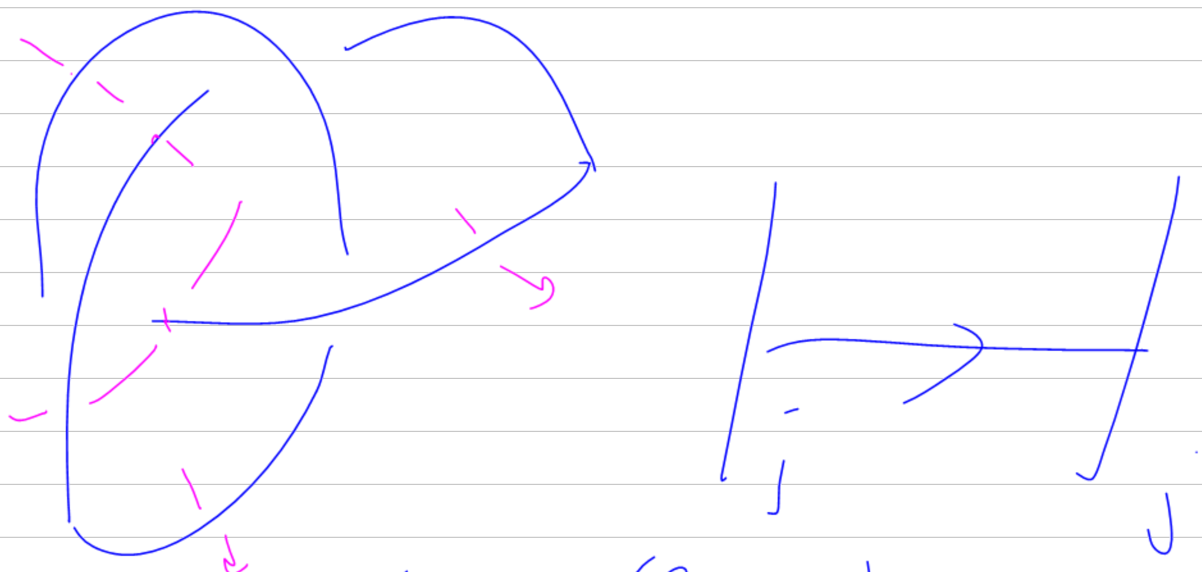
$$\|$$

$$\exp\left(2\partial_{ij}\pi_i \xi_j\right)$$

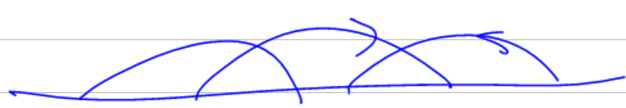








$$R_{ij} = e^{(T-1)(p_i - p_j)x_j}$$



$$\frac{df}{dp} = [x, f(p)x] = \cancel{x}$$

$$\left(\sum A^n\right)_{ij} = (1-A)^{-1}_{ij}$$

$\partial w = v$

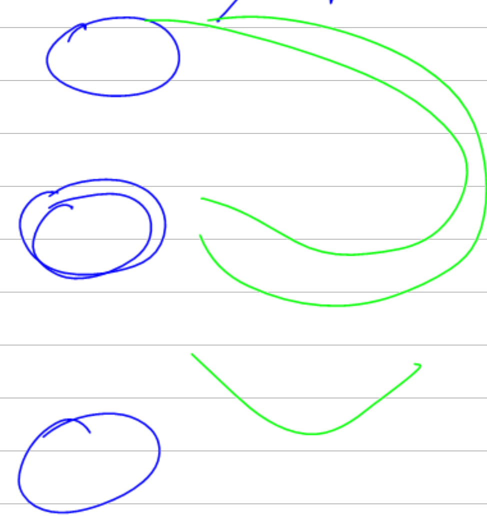
$x, y \in H_1(\tilde{C})$ $\quad lk(u,v) = int(u,w)$

torsion over $\mathbb{Z}[T^{\pm 1}]$

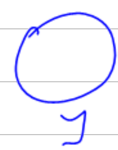
$\exists z$ s.t. $\partial z = py \quad p \in \mathbb{Z}[T^{\pm 1}]$

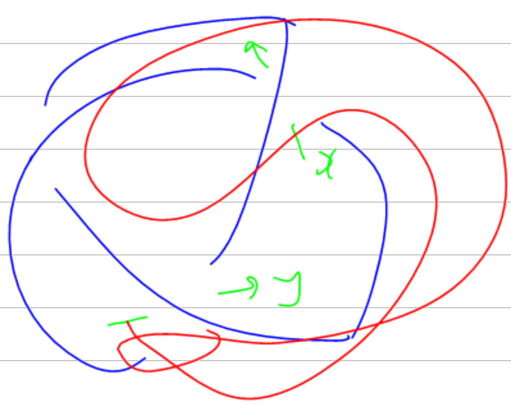
$$\langle x, y\rangle = \text{Equiv-int}(x, z)/p$$

$$= \sum_k T^k \, int(T^k x, z)/p$$

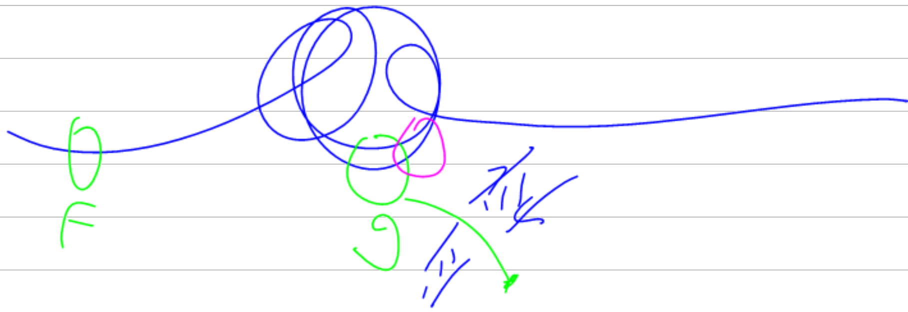


$(T+2+T^{-1})$





$$\boxed{G_{sl_2}} = \boxed{G}\,\boxed{G}\,\boxed{G}$$

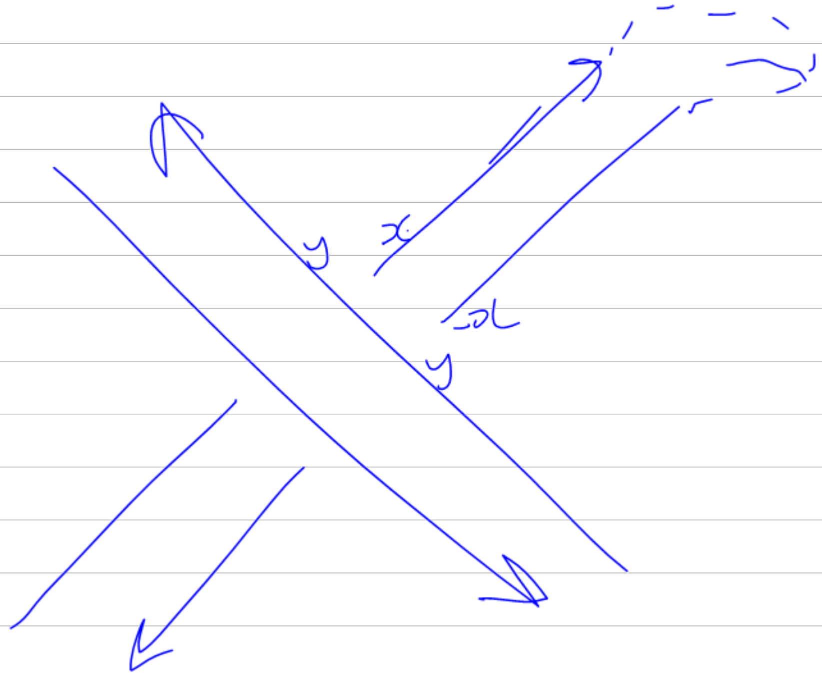


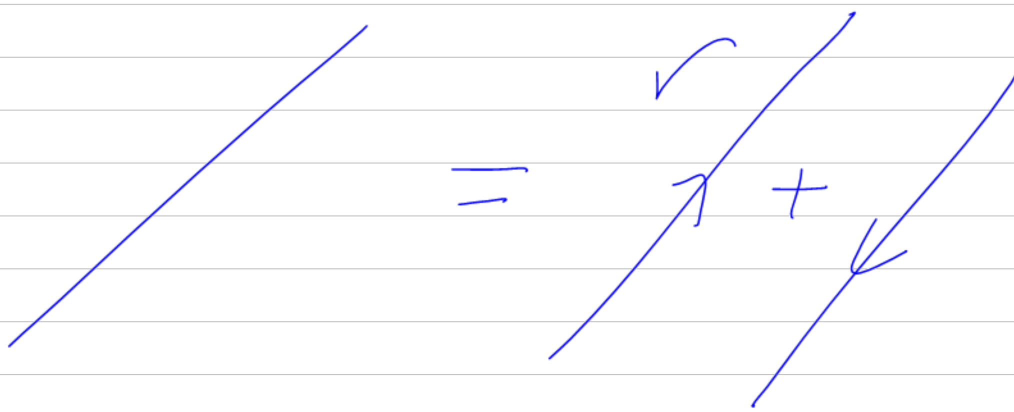
$$G^{ij}_{\alpha\beta}\left(- \quad - \quad . \right)$$

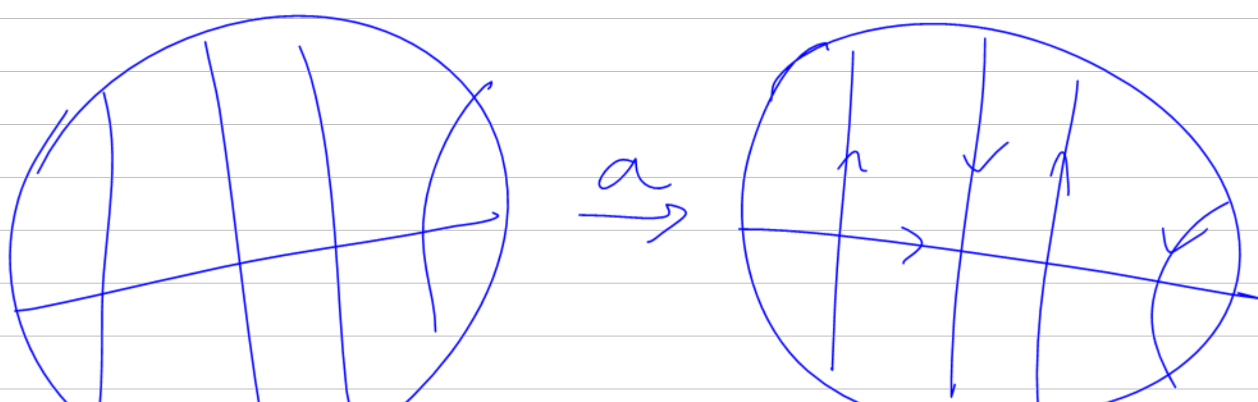
$$[e_{ik}, e_{jn}] \in \langle e_{in} \rangle$$

$\langle m^{ij}_{k}$ am aΔ

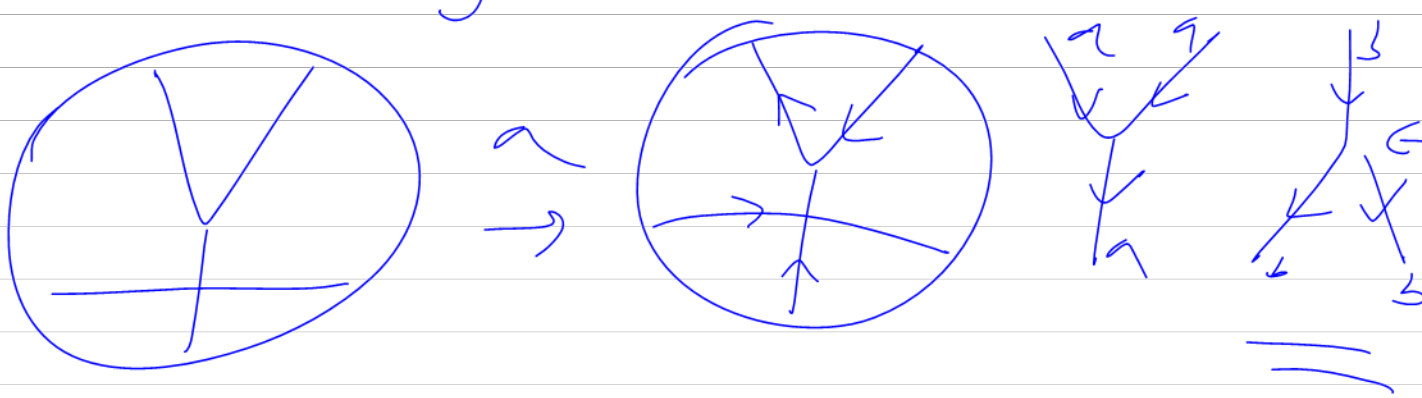
bm bΔ

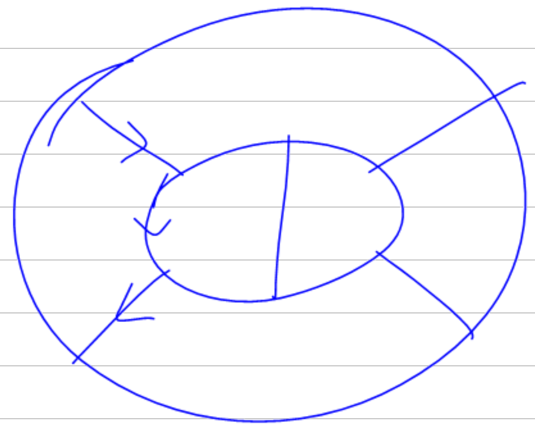






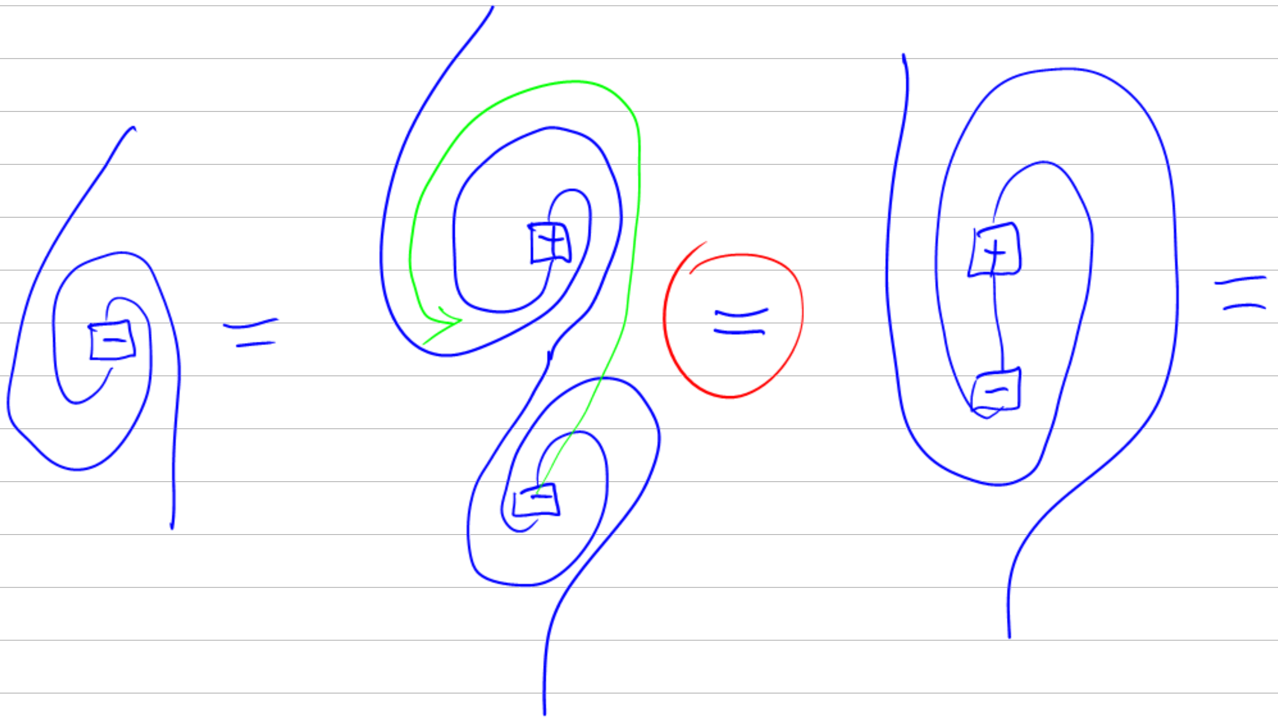
$g = a \oplus b$



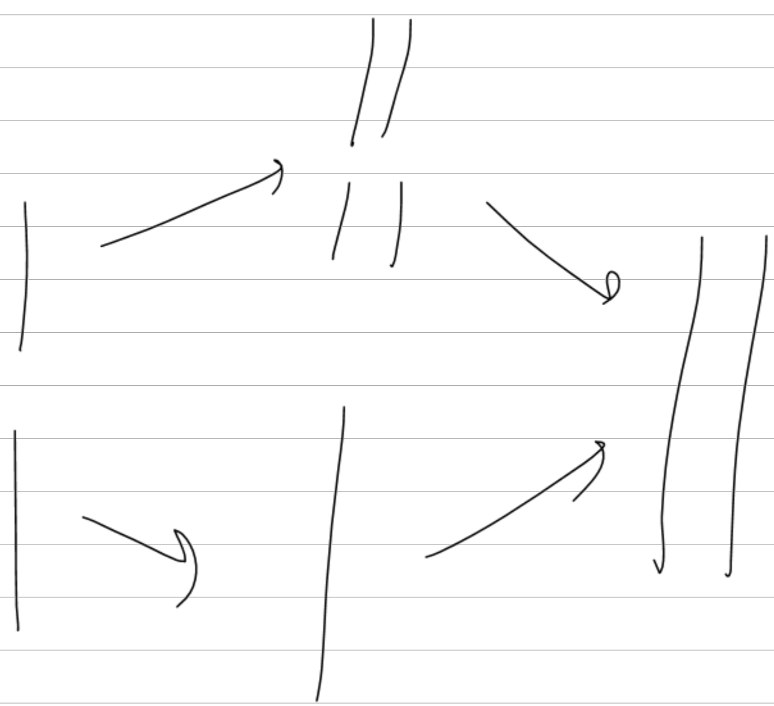


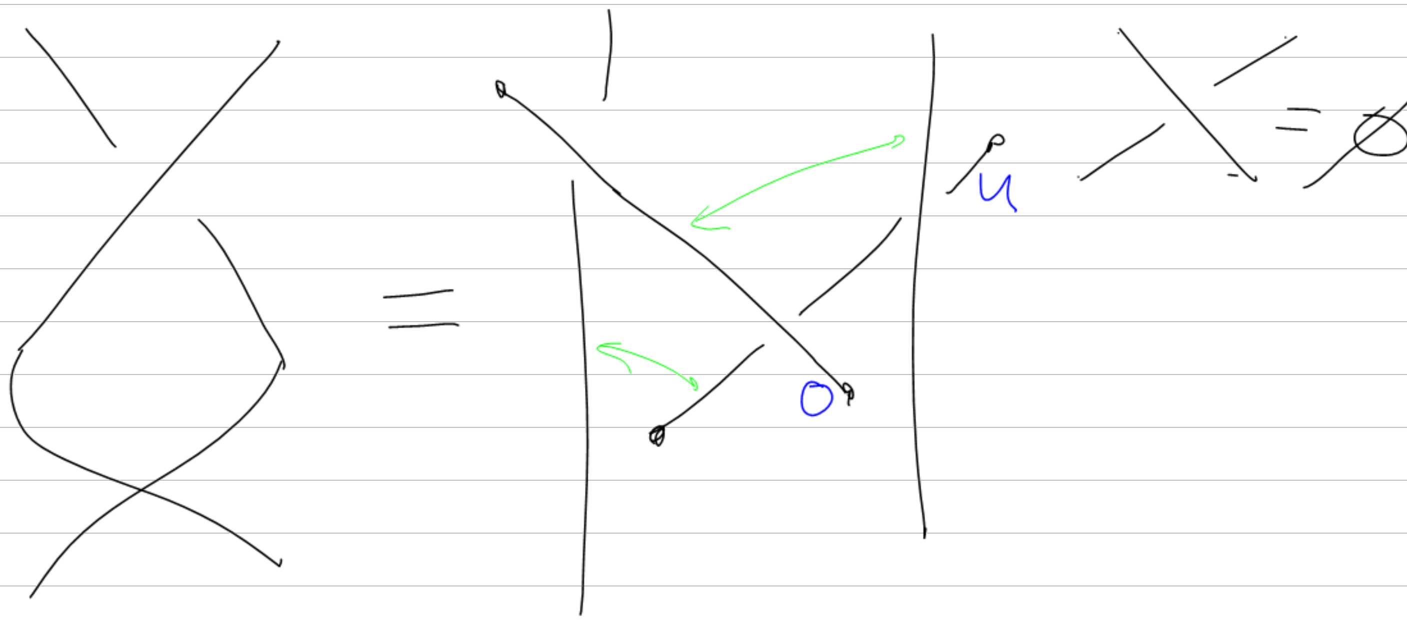
$\partial x_1 \quad y\partial_y$

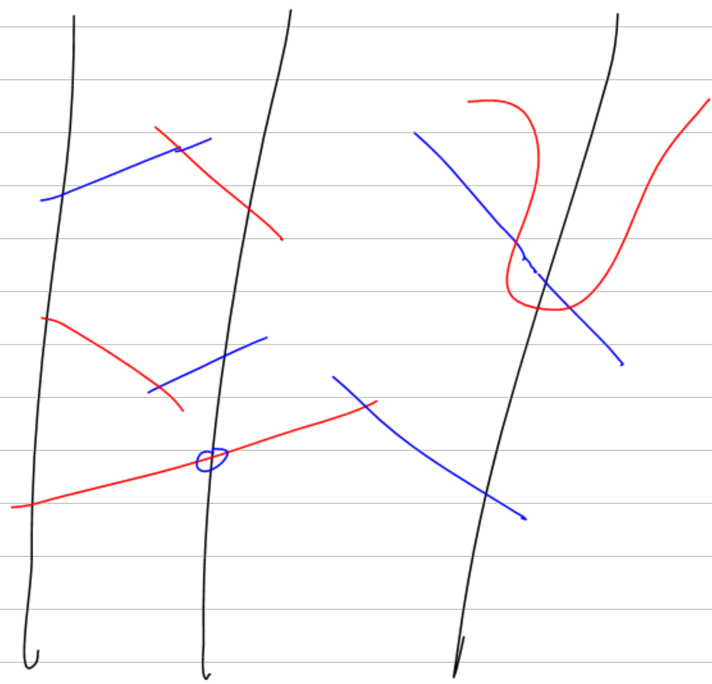
$$\begin{array}{c} 0 \\ \downarrow \\ K \\ \downarrow \\ \mathbb{Q}[y] \\ \downarrow \end{array}$$



$$0 = \int \partial_x \left(e^{Q} \rho \right) = \int e^{Q} \underbrace{\left(\rho\, \partial_x Q + \partial_x \rho \right)}$$







$$e^{\sum_c R^s_{ij}}\Big|_{\substack{p_i \to \partial/\partial\pi_i \\ x_i \to -\ \ -}} e^{g_{ij}\xi_i\pi_j}\Big|_0$$

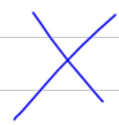
$$R^s_{ij} \in \mathbb{Q}[T^{\pm 1}, p_i, p_j, x_i, x_j, \epsilon]$$

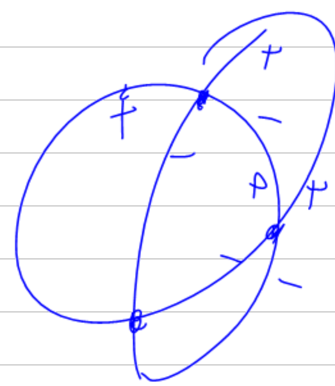
$$p_i p_j x_i x_j \longrightarrow \overset{7}{g_{ii}}\, g_{jj} \overset{3}{+} g_{ij}\, g_{ij}$$

$$\begin{array}{cc} \downarrow & \downarrow \\ x_i p_i & x_j p_j \end{array}$$

$$\langle g_{ik} g_{jl} - g_{jk} g_{il} \rangle \longrightarrow Q[g_{ij}] \longrightarrow Q[p_i, x_i]$$

$$\begin{array}{c} \cup \\ R \end{array}$$



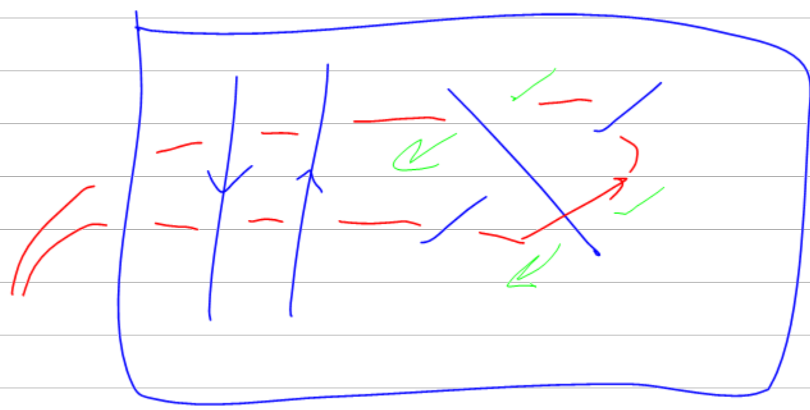


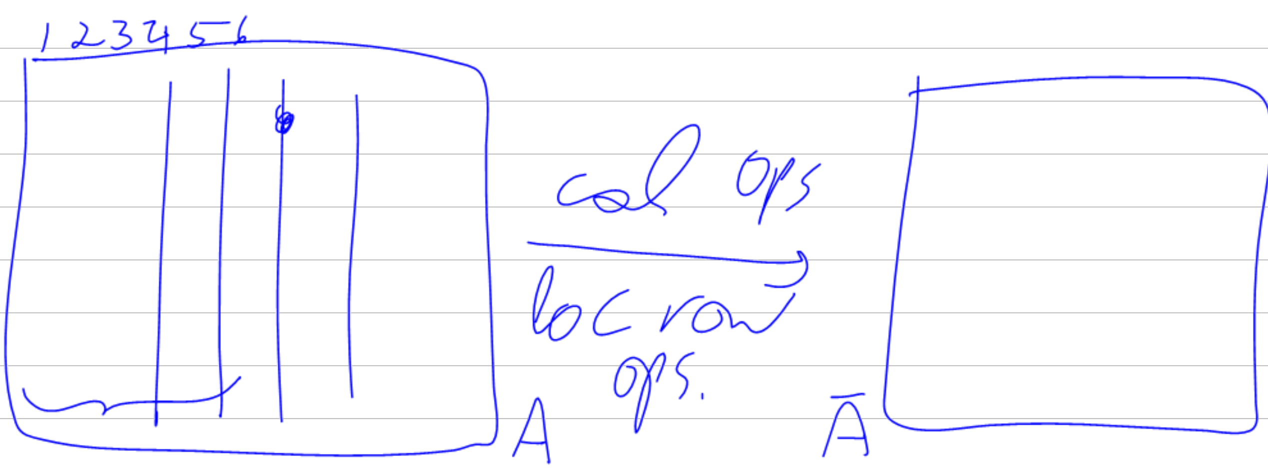
$R_1(g_{ij})$ $\quad$ $R_1(g_{i'j'}) \longleftarrow$ Gram

$R_1(x_i p_i)$ $\quad$ $R_1(x_{i'} p_{j'}) \longleftarrow$ Heis

$$G_{ij} = \begin{pmatrix} g_{ii} & g_{ij} \\ g_{ji} & g_{jj} \end{pmatrix}$$

$$\begin{tikzcd}
\mathbb{Q}[G_{ij}]^{\otimes 2} \arrow[r, "m"] \arrow[d, "\text{not } 1\text{-}1"] & \mathbb{Q}[G_{ijkl}] \\
\mathbb{Q}[p_{ij}, x_{ij}]^{\otimes 2} \arrow[r, "m"] & \mathbb{Q}[p_{ijkl}, x_{ijkl}] \arrow[u, "\text{Feynman}"']
\end{tikzcd}$$





$$\begin{pmatrix} \frac{1}{1-t} & & & 0 \\ & \ddots & & \\ & & \ddots & \\ & & & \frac{1}{1-t} \end{pmatrix}$$

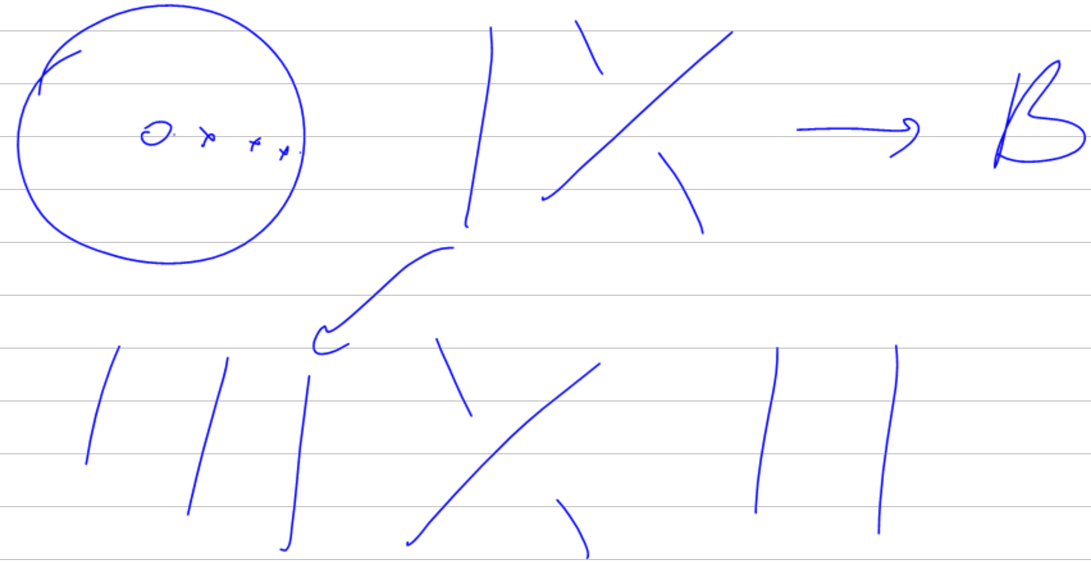
$$\langle a, b, c : a^b = c \rangle$$

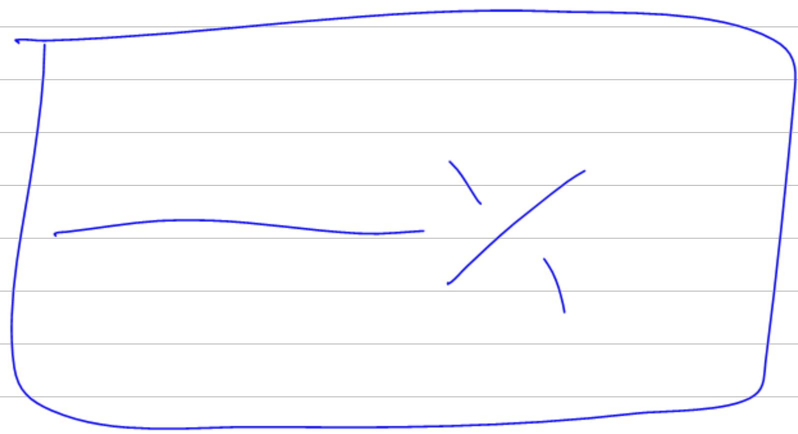
$$\downarrow$$

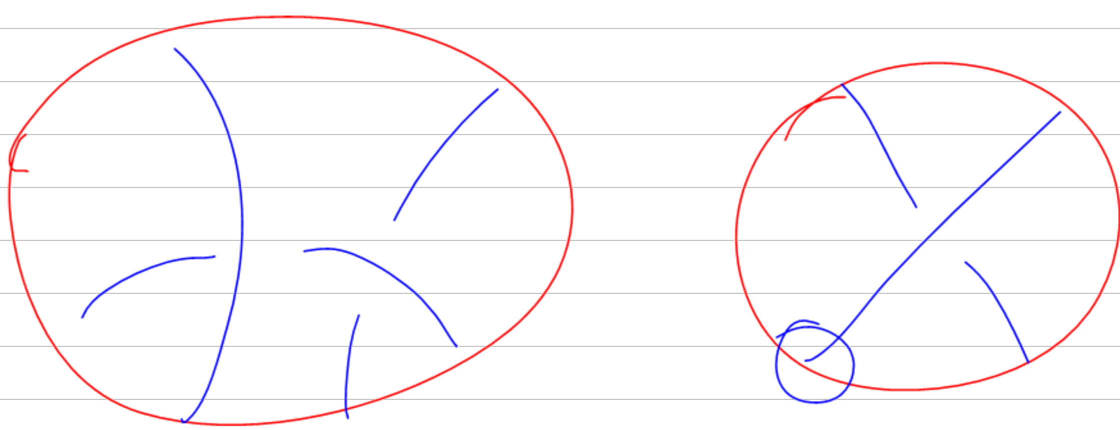
$$a^{(b^{-1})} = c$$

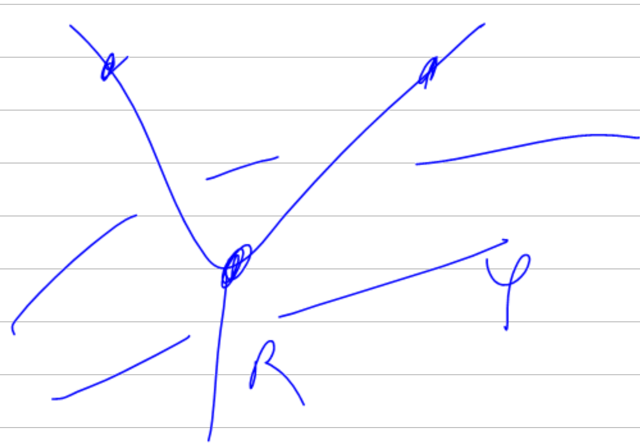
$$\begin{pmatrix} & & \\ & A & \\ & & \end{pmatrix} \longrightarrow \begin{pmatrix} & & \\ & \overline{A} & \\ & & \end{pmatrix}$$

$$G \qquad\qquad\qquad G$$

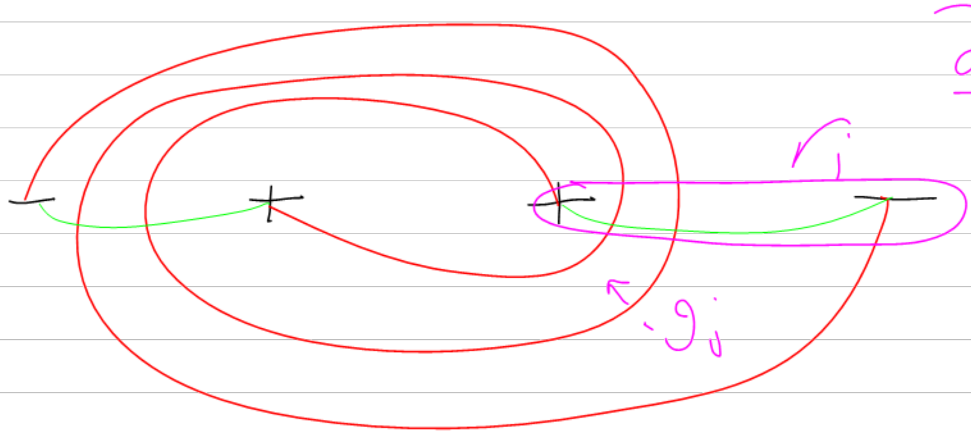




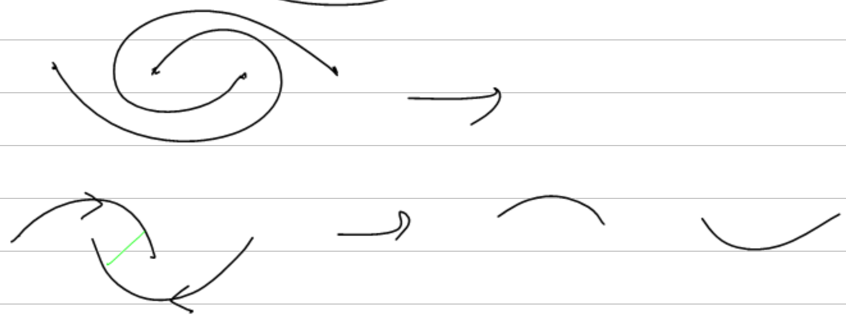


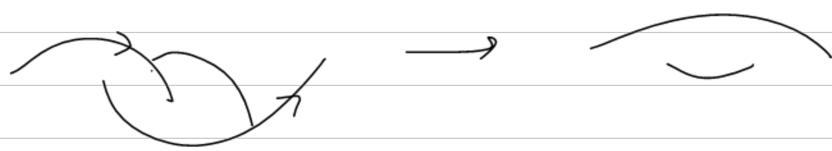


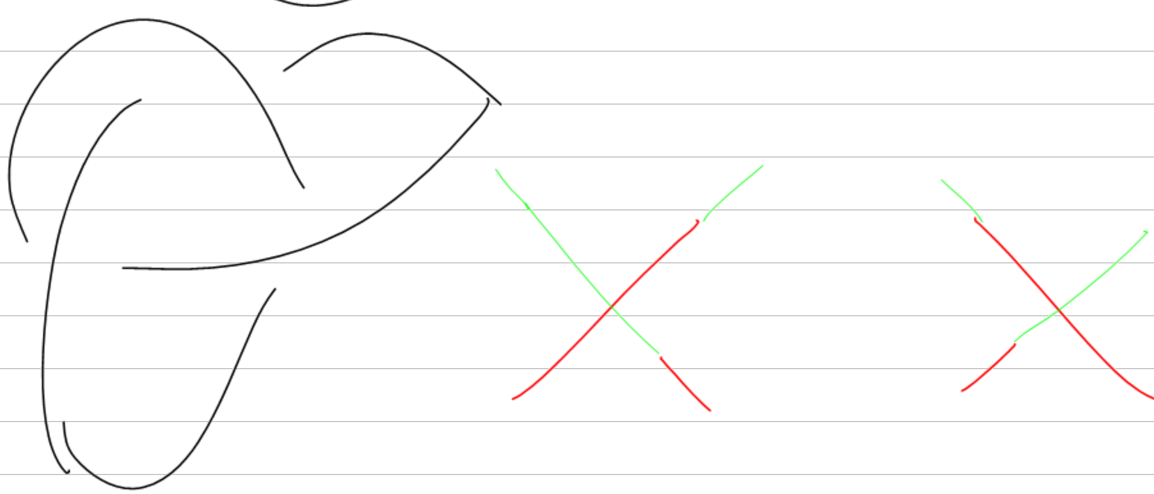
$$(\Delta \otimes 1)R = R^{13} R^{23}$$



$$\frac{\partial r_i}{\partial g_j}$$





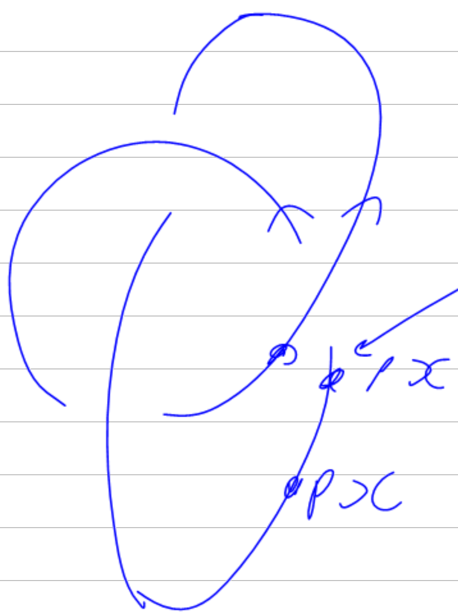


$B_n \circlearrowright V$ Burau.

$$\begin{tikzcd} \mathbb{Q}B_n \ltimes \overbrace{\mathcal{H}(V\oplus V^*)}^{A}[[\epsilon]]_{doc} \arrow[r, "\pi"'] & \mathbb{Q}B_n \arrow[l, bend right, "J"'] \end{tikzcd}$$

$$J : B_n \longrightarrow \mathcal{H}(p_i, x_i)$$

$$J(\beta_1 \cdot \beta_2) = \left(J(\beta_1) // \beta_2\right) \cdot J(\beta_2)$$

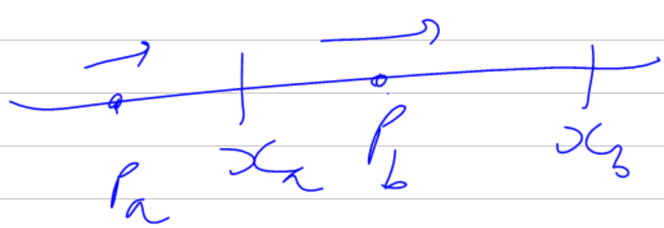


$e^{\pi p} e^{\xi x}$

$r_s(\varphi_i, \varphi_j, i, j)$

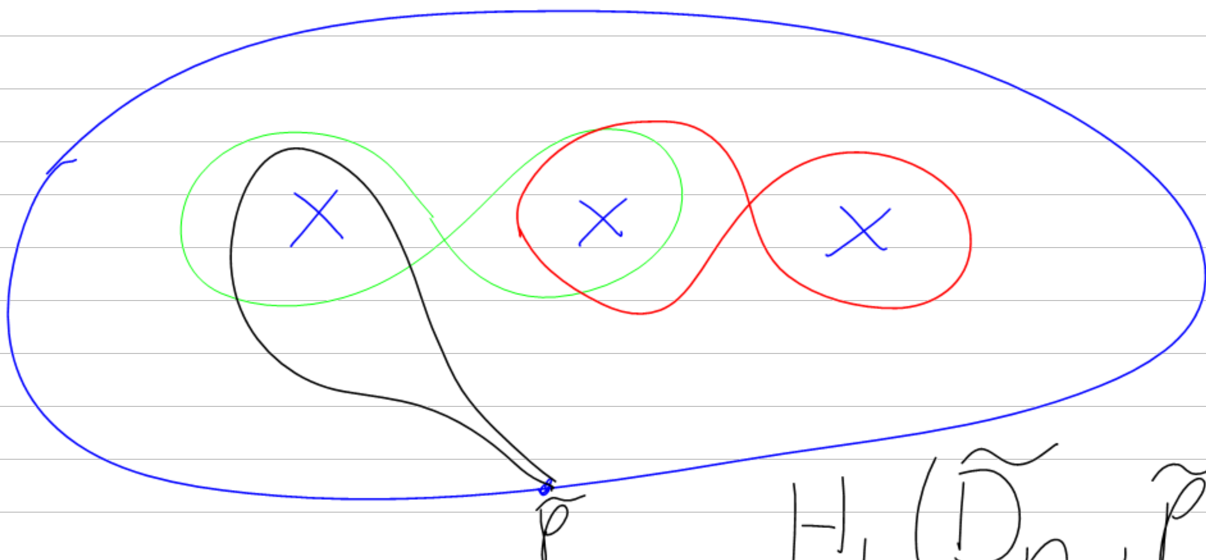
$r_s(0\ 0\ i\ j)$

γ

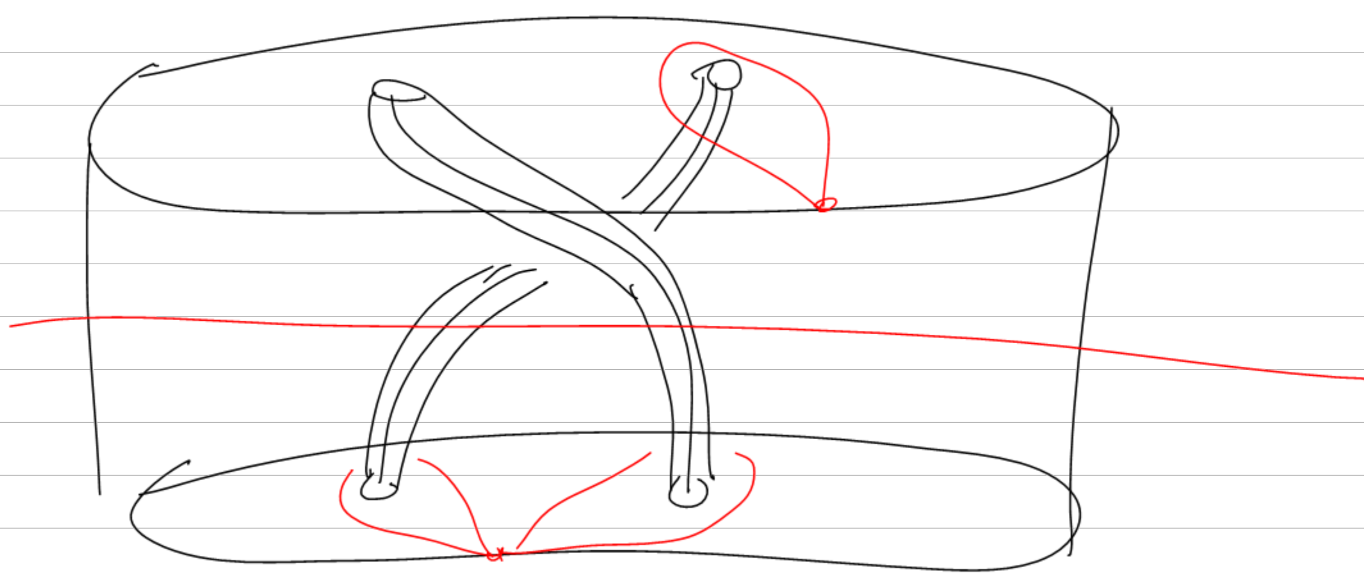


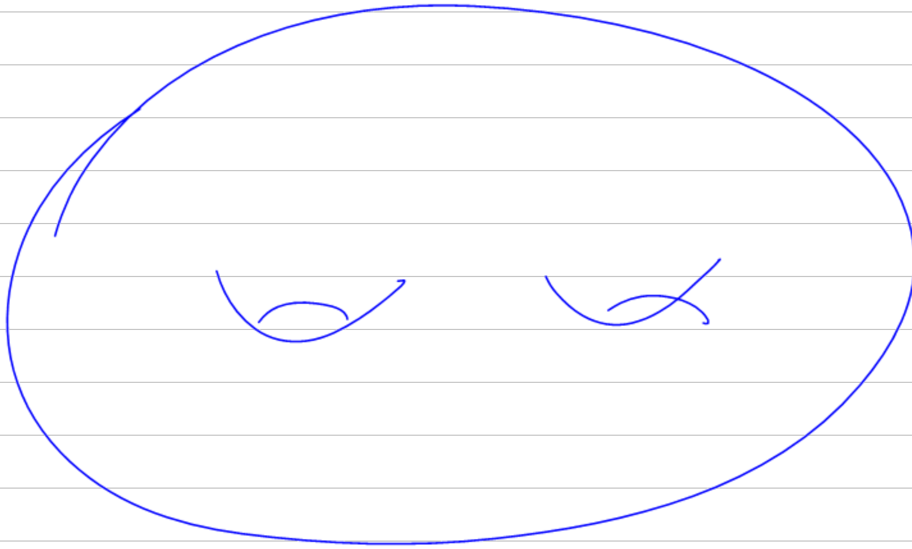
$\vartheta_{\alpha\beta}$ $\qquad p_\alpha \rightsquigarrow p^a_\alpha \; p^b_\alpha$

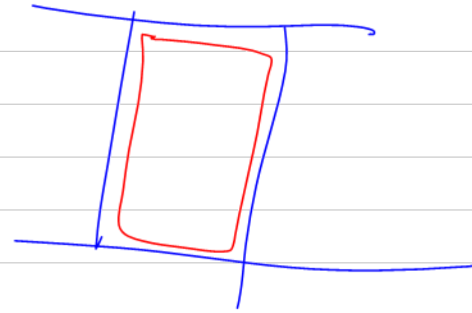
$$e^{\sum_{\alpha,\beta} \vartheta_{\alpha\beta}(p^a_\alpha + p^b_\alpha)\cdot(x^a_\beta + x^b_\beta) + -\sum_\alpha p^b_\alpha x^a_\alpha}$$

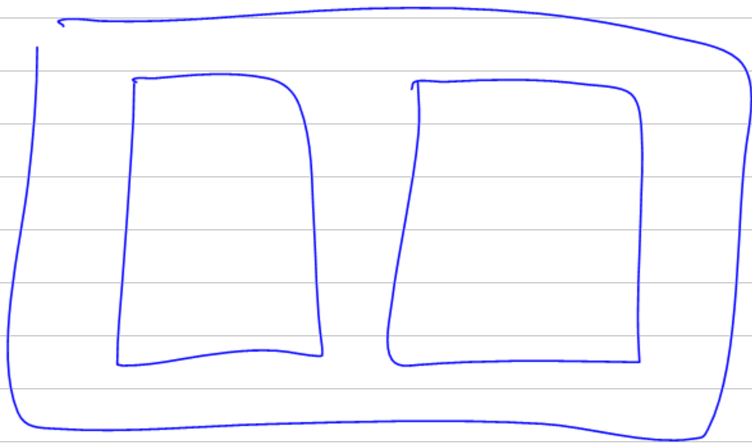


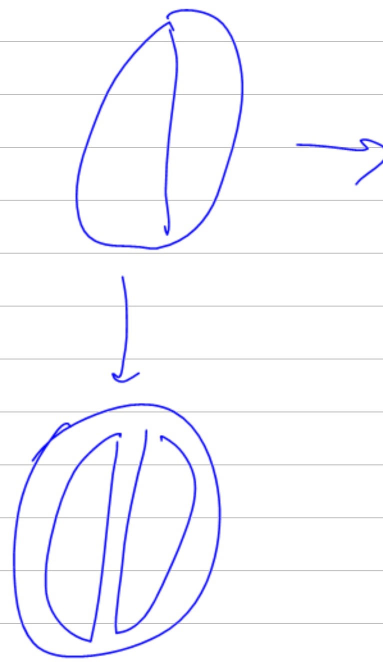
$H_1(\tilde{D}_n, \tilde{p})$

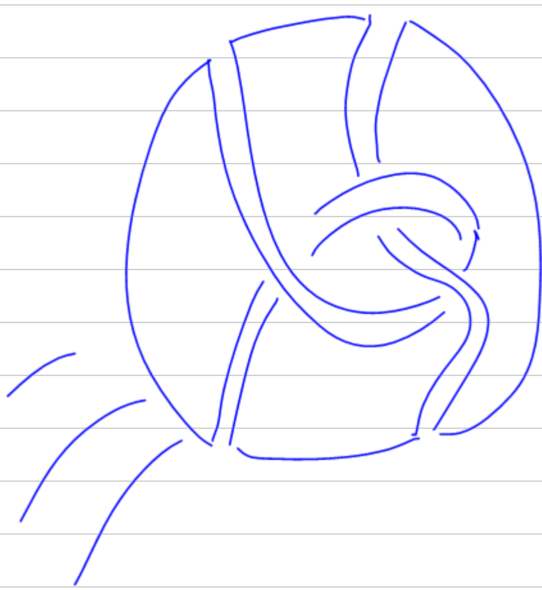


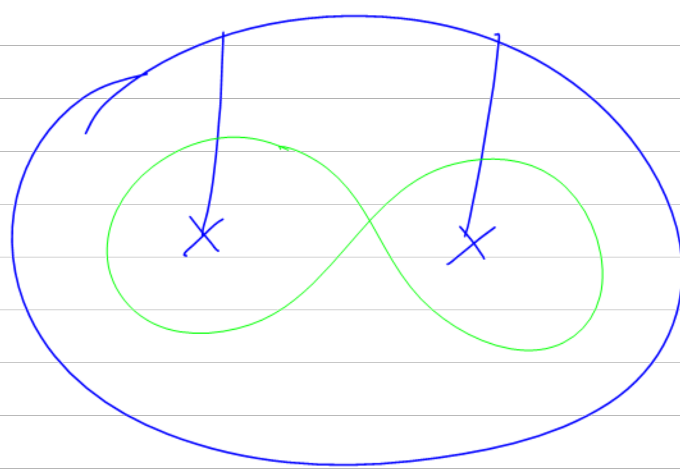


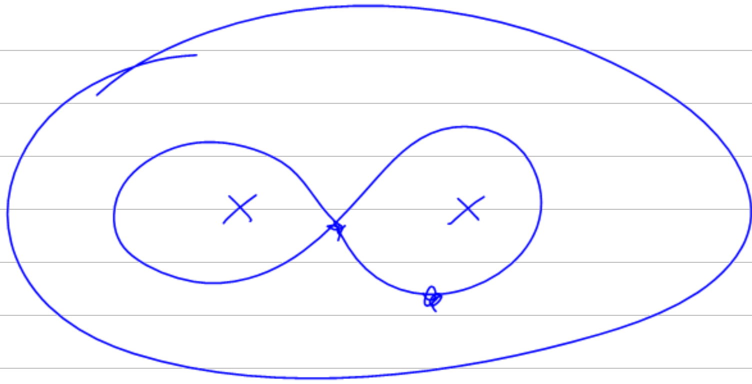


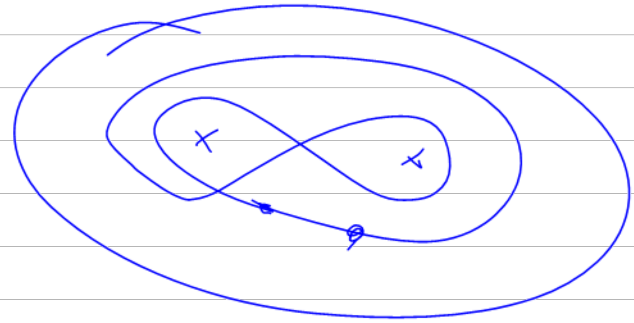


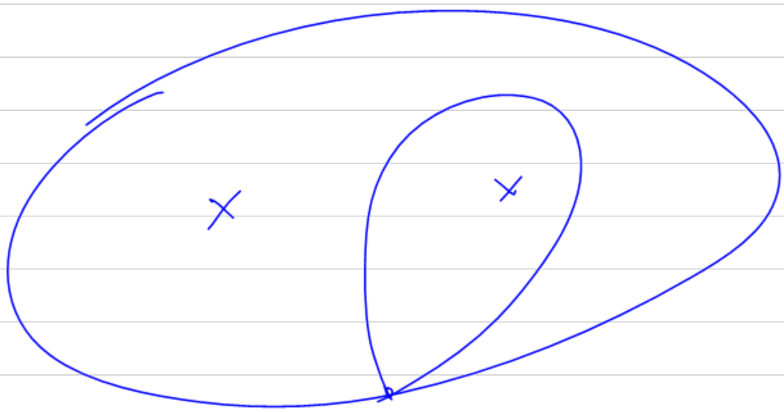


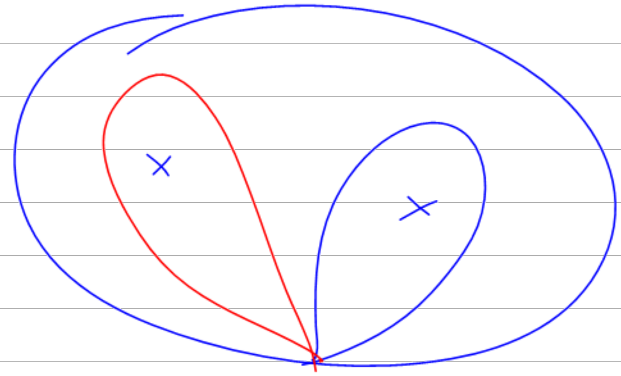


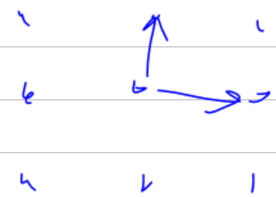




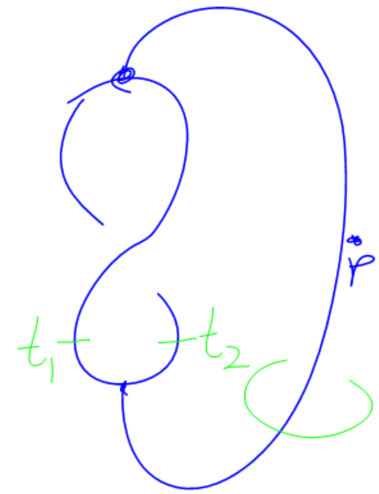








Let H be a genus g handlebody in $\mathbb{R}^3$, let $\Sigma = \partial H$, and let $p \in \Sigma$ be a basepoint (think "H is a tubular neighborhood of a pinched tangle T"). Let $C = (\operatorname{int} H)^c$ and let $\tau\colon \pi_1(C) \to H^1(H;\mathbb{Z}) \cong \mathbb{Z}^g$ be induced by Alexander (?) duality. Let $\tilde\Sigma \subset \tilde C$ be the τ-covers of $\Sigma \subset C$, respectively, with covering projections ϕ. Let $\tilde p = \phi^{-1}(p)$; it is a copy of $\mathbb{Z}^g$. Let $R \coloneqq \mathbb{Z}H^1(H;\mathbb{Z}) \cong \mathbb{Z}[T_i^{\pm 1}]_{i=1}^g$ be the group ring of $H^1(H;\mathbb{Z})$, the ring of Laurent polynomials in variables $T_1, \dots, T_g$, and note that $H_1(\tilde\Sigma, \tilde p)$ and $H_1(\tilde C, \tilde p)$ are R-modules.



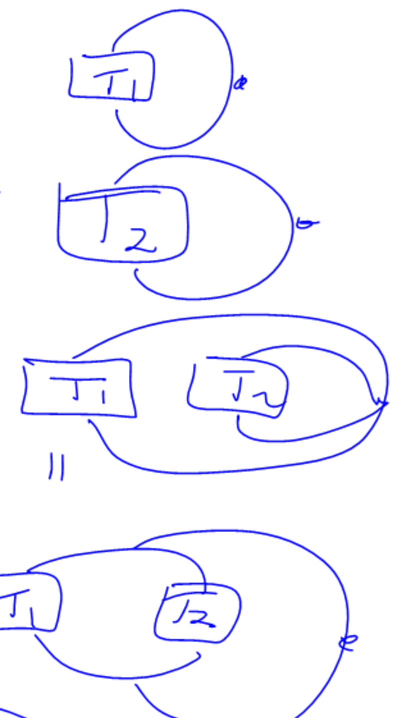
Definition 1. $A(H) \coloneqq \ker i_*\colon \underbrace{H_1(\tilde\Sigma, \tilde p)}_{2g} \to H_1(\tilde C, \tilde p)$, *the Alexander module of* H.

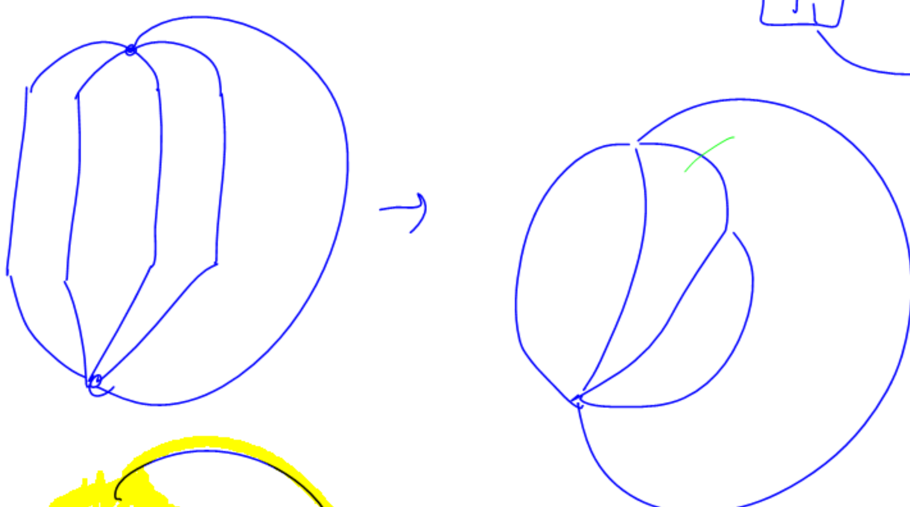
Def 2 < >

Thm 1 $A(H)$ is Lagrangian.

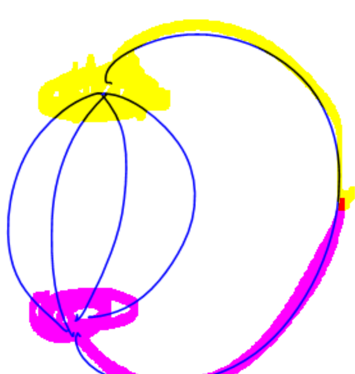
Thm 2 behavior under ⊗

Thm 3 Behavior under amputations.

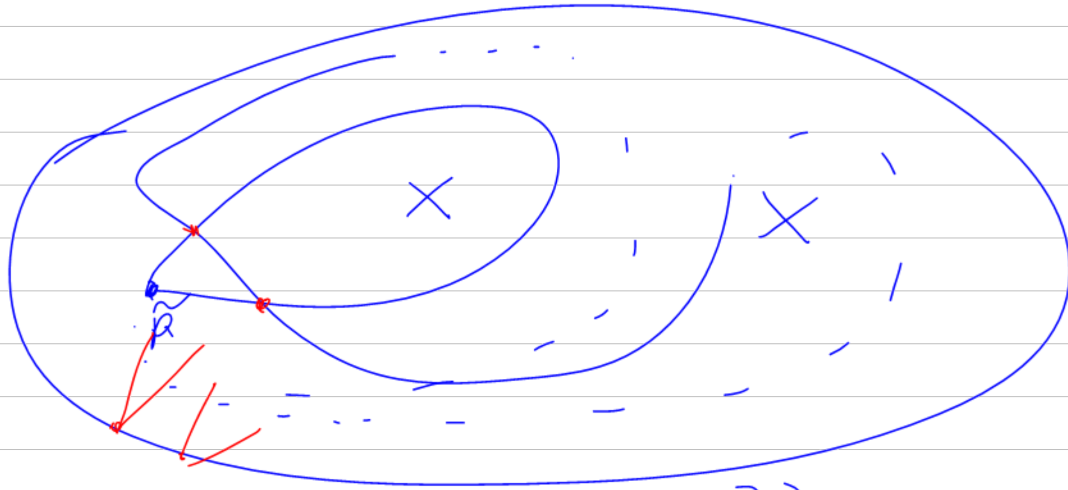




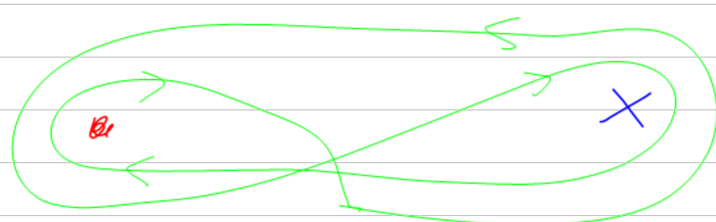
Def

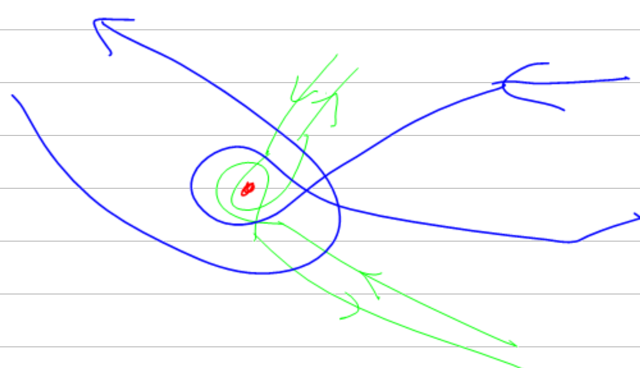


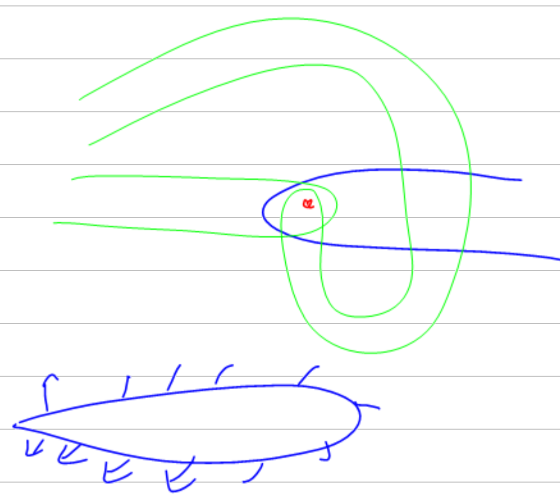
⟺ Gassner & unitarity

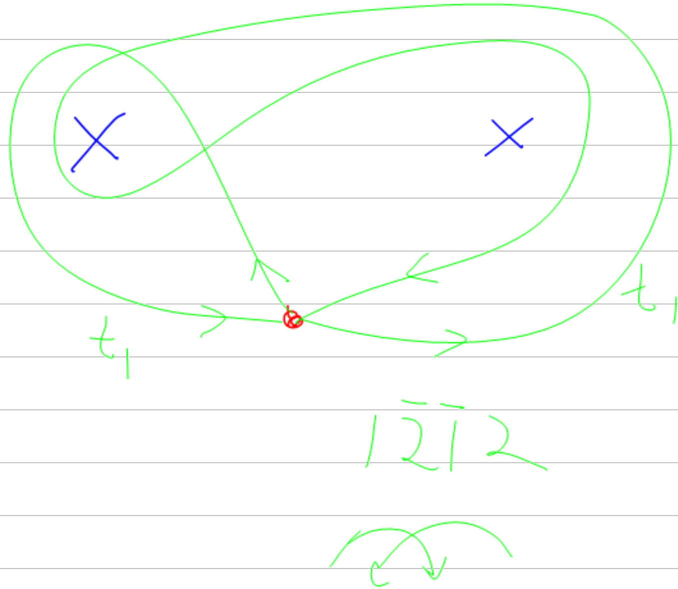


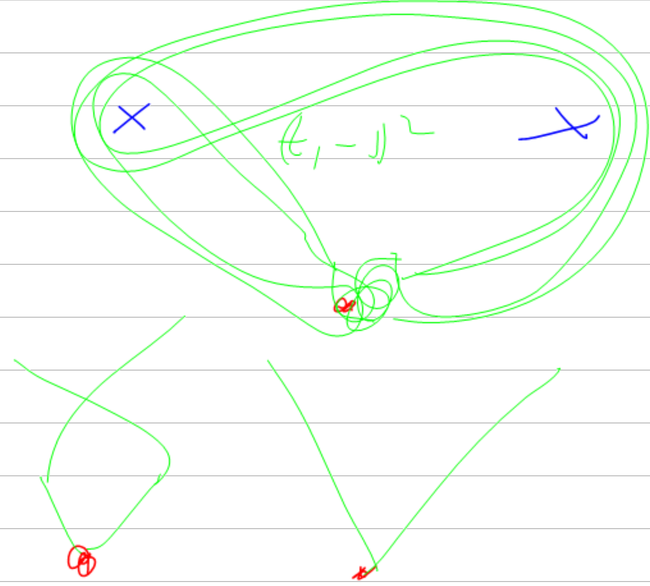
$$H_1(\tilde{D}_2, \tilde{p})^{\otimes 2} \longrightarrow R$$





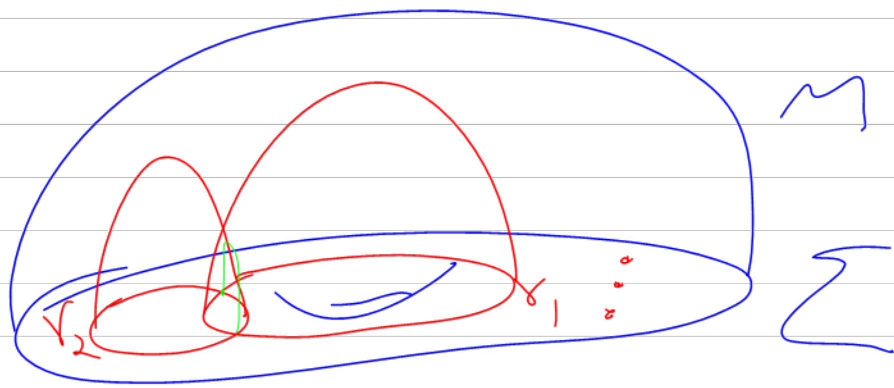


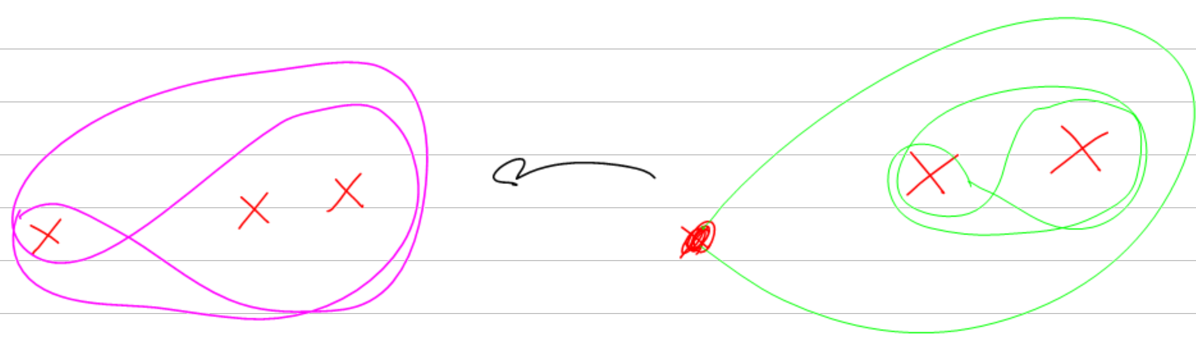


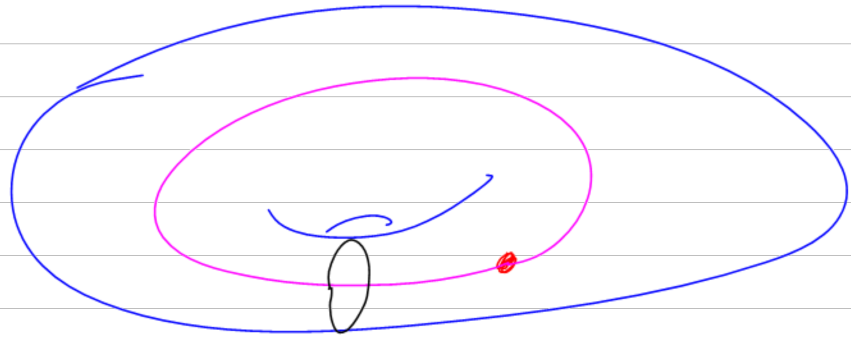


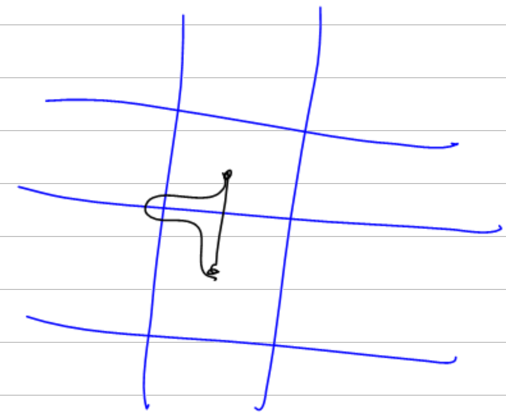
$$\Sigma = \partial M^3 \xrightarrow{i} M^3$$

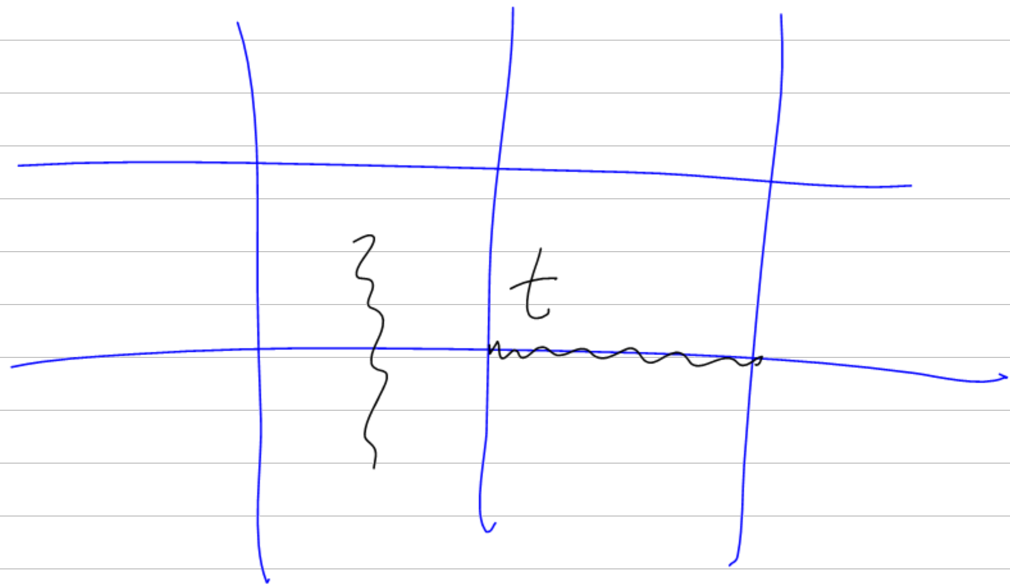
$\operatorname{Ker} i_* \subset H_1(\partial M)$ is isotropic rel. intersection of $\partial M^3 = \Sigma$

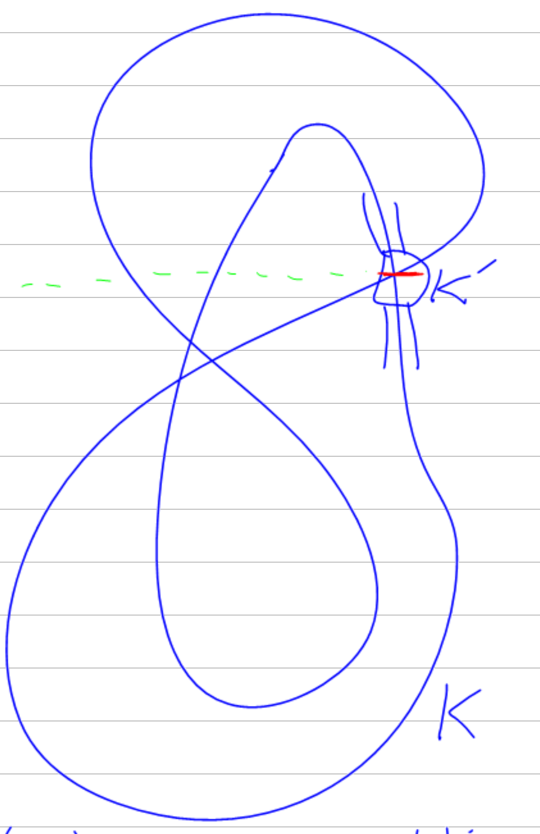


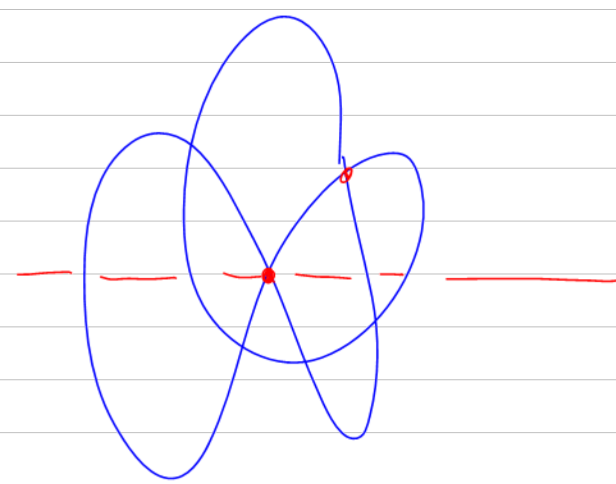


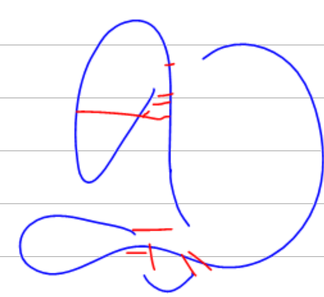












$$\varphi(K) = \sum_{\text{smoothings of } K}$$

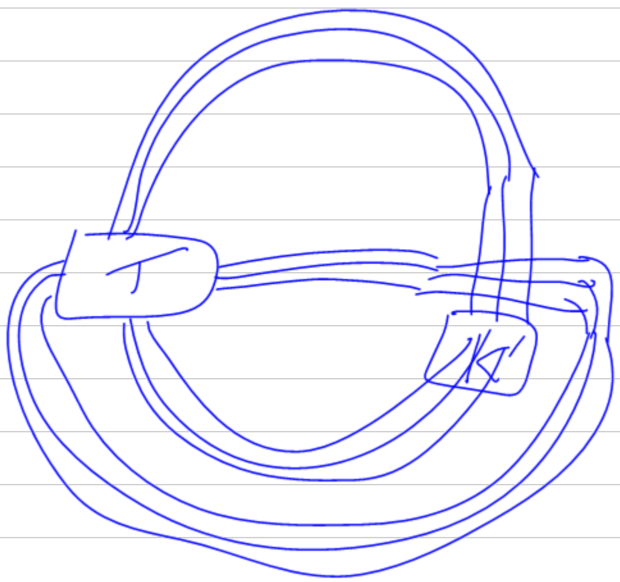
$\mathbb{Z}V \xrightarrow{[\ ]} V \qquad \mathbb{Z}\mathbb{Z}S \neq \mathbb{Z}S$

$D \in GD \qquad V = \mathbb{Z}(GD)$

$$Fi(D) = \sum_{a\in D} \sum_{D' \subset D\setminus a} D' \in \mathbb{Z}(\mathbb{Z}(GD))$$

V

$$\sum_\alpha a_\alpha x^{W(\sum_\beta b_{\alpha\beta} D_{\alpha\beta})} \mapsfrom \sum_\alpha a_\alpha \Big(\sum_\beta b_{\alpha\beta} D_{\alpha\beta}\Big)$$



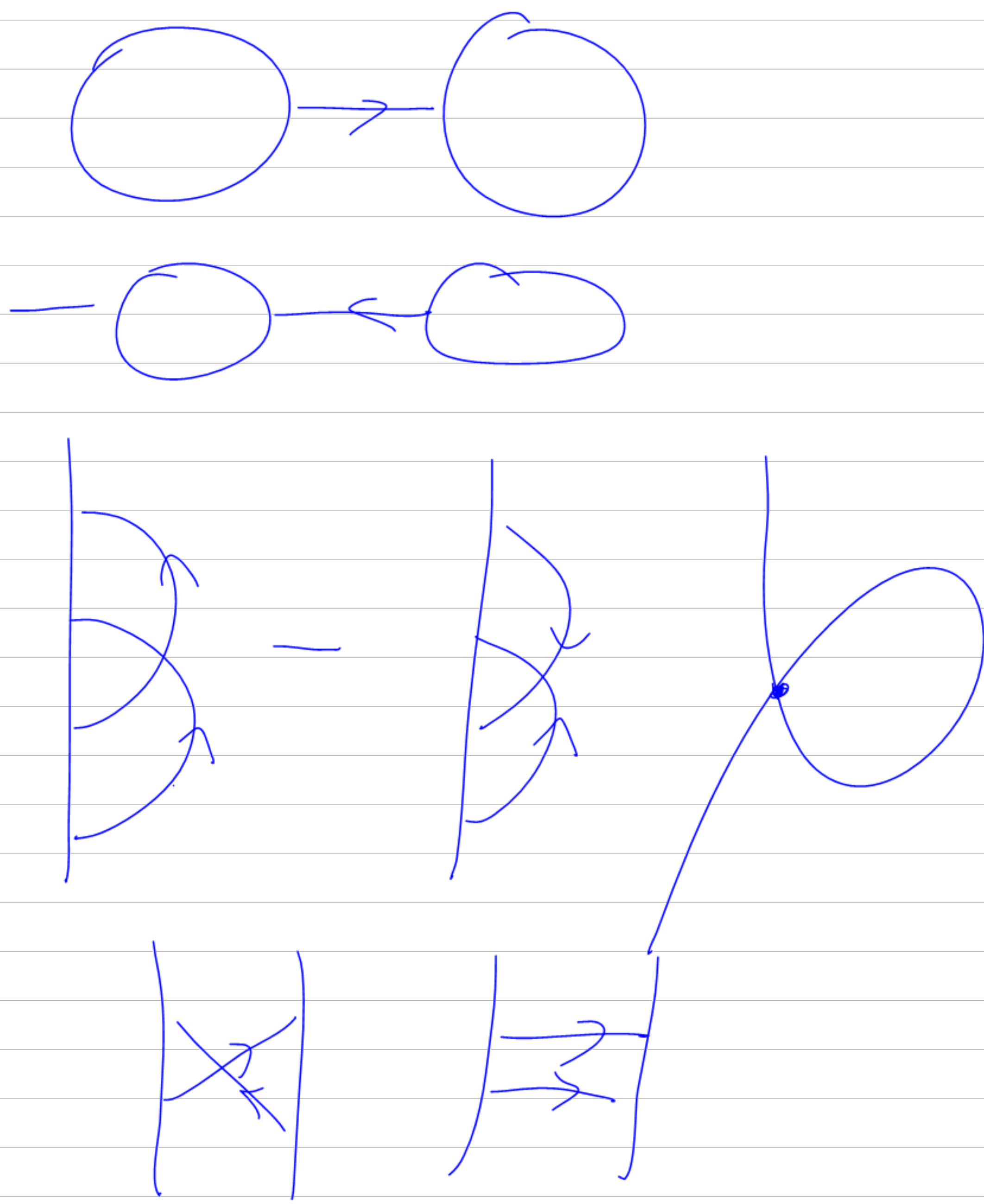
$$Fi_V(K) = \sum_{C\in K} (-1)^C x^{V(K\setminus C)}$$

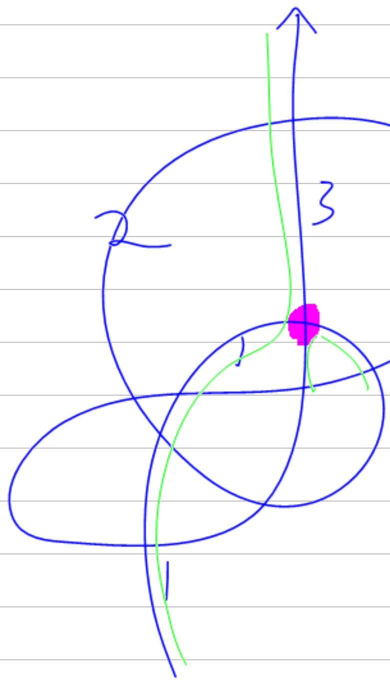
$$R = \mathbb{Z}[t^{\pm 1}]$$

$$\underset{0}{H_1(\tilde{A}) \oplus H_1(\tilde{B})} \to H_1(\widetilde{A \cup B}) \to H_0(\widetilde{A \cap B}) \to H_0(\tilde{A}) \oplus H_0(\tilde{B}) \to H_0(\widetilde{A \cup B})$$

$$\to R/(t^{rq}-1) \xrightarrow{(1,1)} R/(t^{r}-1) \oplus R/(t^{q}-1) \xrightarrow{(1,-1)} R/(t-1)$$

$$\Delta = \frac{(t^{rq}-1)(t-1)}{(t^{r}-1)(t^{q}-1)} = \frac{1+t^{q}+t^{2q}+\dots+t^{(r-1)q}}{1+t+\dots+t^{r-1}}$$

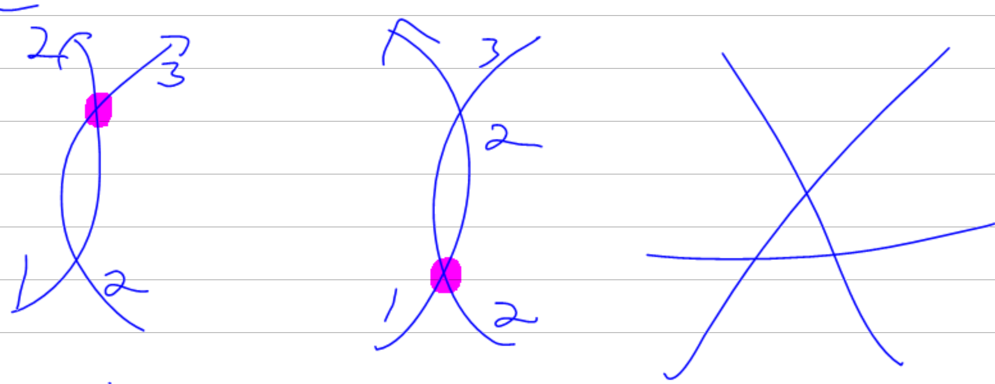




$$l_{12}+l_{32}=$$
$$l_{21}+l_{23}$$

l_{ij} # of times i goes over j

$$\sum_C (-1)^c x^{\sum a^t_{ij} l_{ij}}$$



$a_{12} = a_{23}$

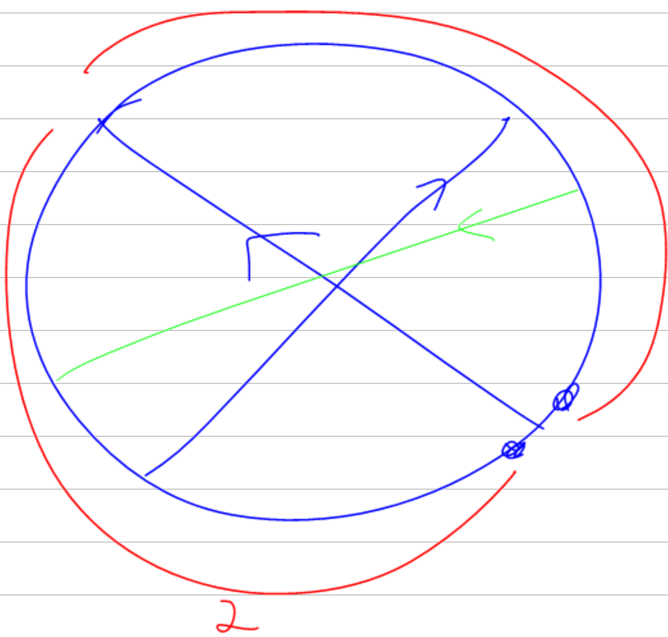
R3: u : 2 + (6, 8)
m : 1 −
l : 3 +

$R3[\sigma, S_{1...3}] =$

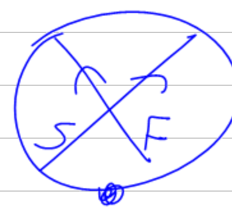
$A \otimes A \xrightarrow{m} A$

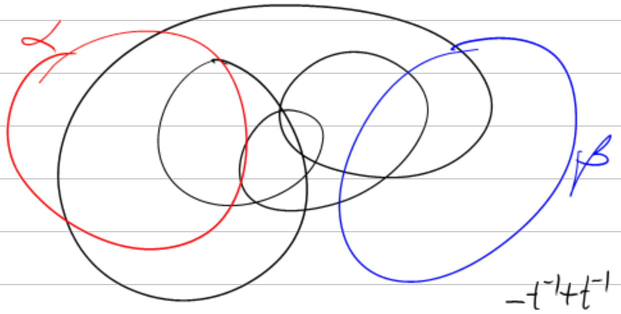
$A^{\otimes 3} \longrightarrow A^{\otimes 3}$

$$\begin{tikzcd}
A \otimes A \otimes A \arrow[rr, "\sigma"] \arrow[d, "m \otimes 1"] & & A \otimes A \otimes A \arrow[d, "1 \otimes m"] \\
A \otimes A \arrow[r, "m"] & A & A \otimes A \arrow[l, "m"']
\end{tikzcd}$$

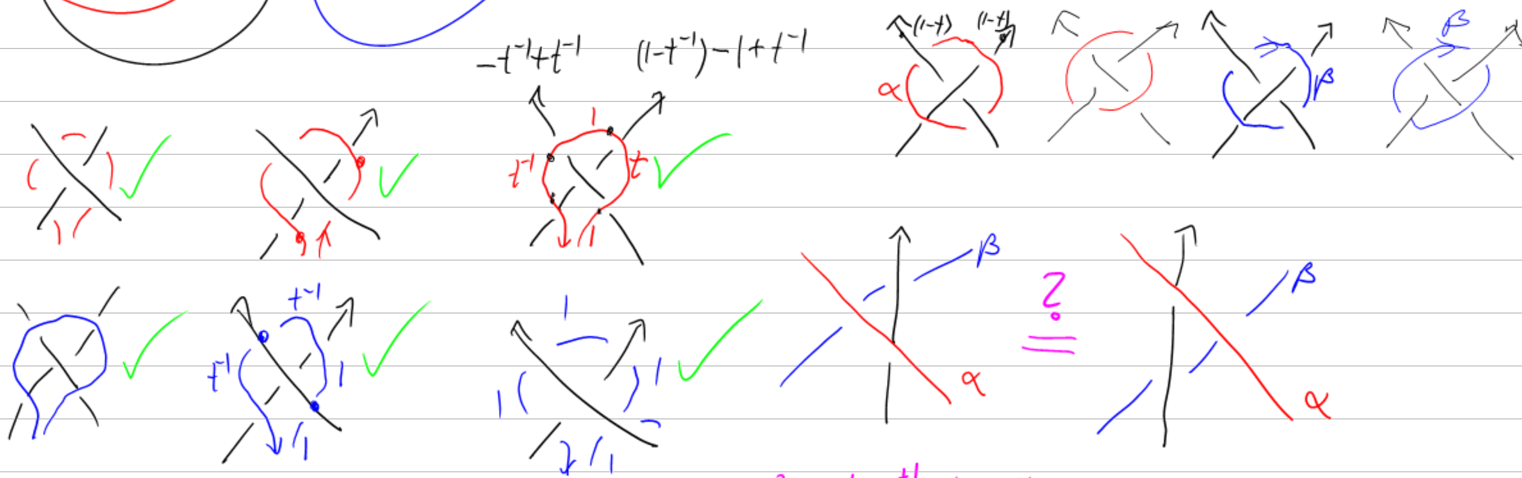


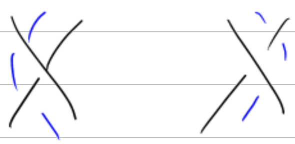
$\Delta = -\ell_{21} + \ell_{12}$



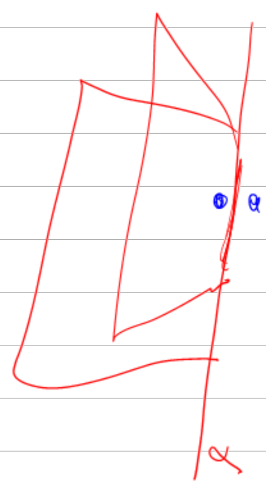


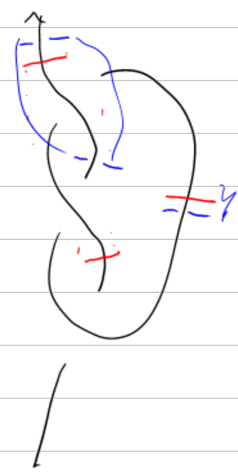
230228b Given a diagram D for a long K, the phase ϕ along a curve $\gamma \subset D^c$ multiplies by T^s whenever γ passes over D with sign s. **Conj.** $\mathrm{lk}_K(\alpha,\beta) = \langle \text{flow: generated by } \alpha, \text{ measured by } \beta \rangle + ?$. α generates $\pm\phi$ flow when it runs over D. β $\pm\phi^{-1}$-measures flow when it runs under D.





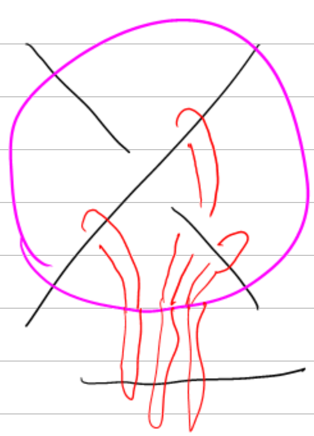
$? \overset{?}{=} \dfrac{\text{phase}^{\pm 1} \text{ xing \# of } \alpha \,\&\, \beta}{1-t}$





α_i: lower clasper

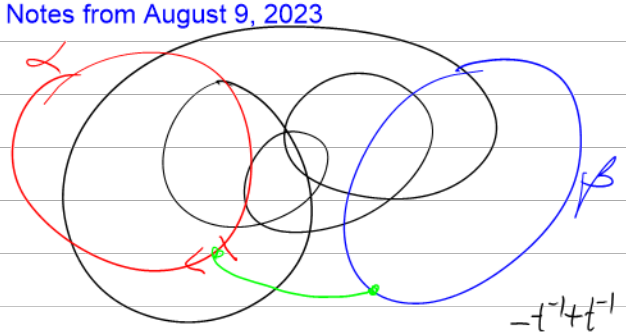
β_j: upper clasper



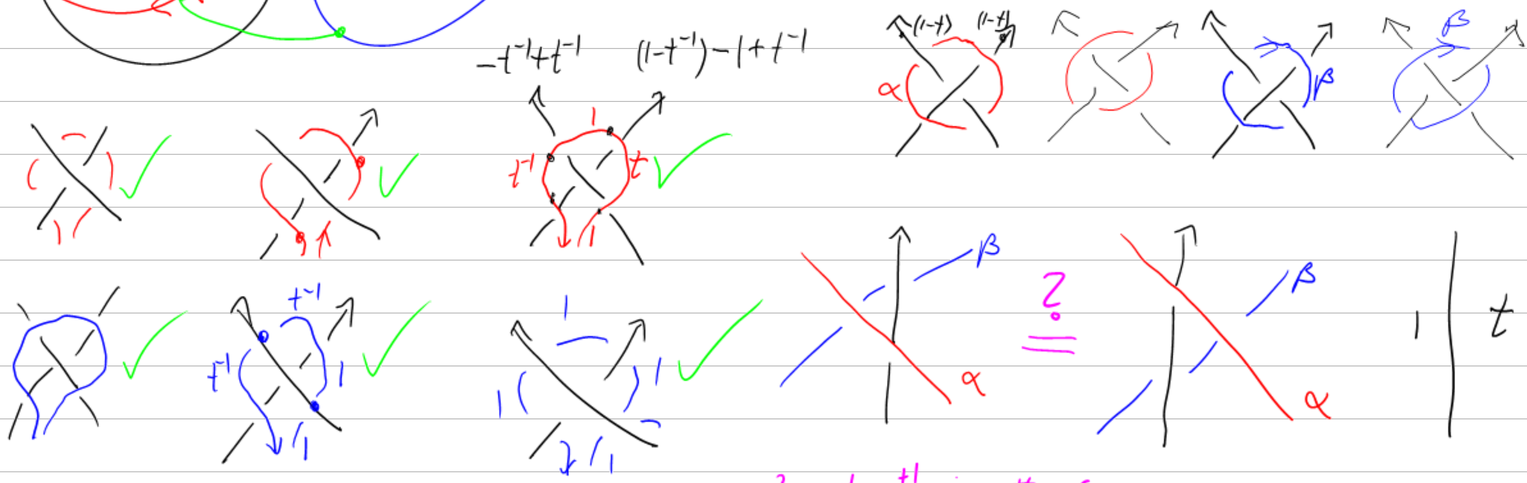
$\exists$ map $\left\{\text{3-comp classical}\right\} \longrightarrow \left\{\begin{matrix}\text{3-components}\\ \text{virtuals}\end{matrix}\right\} \Big/ \begin{matrix}\text{OC on red}\\ \text{UC on blue}\\ \text{homotopy on red \& blue}\end{matrix}$

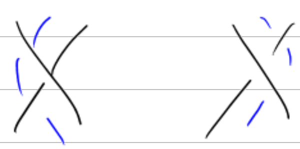
$(1-T) = (1-T) \qquad (1-T^{-1}) = \underset{\uparrow}{-}\,\underset{\phi}{T^{-1}}(1-T)$

Notes from August 9, 2023

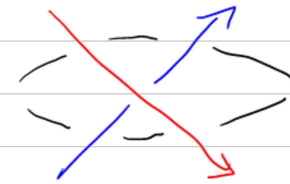


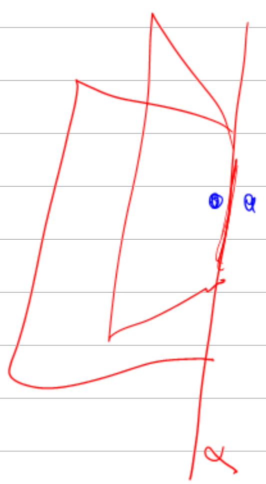
230228b In VanDerVeen_Journal: Given a diagram D for a long K, the phase ϕ along a curve $\gamma \subset D^c$ multiplies by T^s whenever γ passes over D with sign s. **Conj.** $\mathrm{lk}_K(\alpha,\beta) = \langle$flow: generated by α, measured by $\beta\rangle + \langle$phased α over β count$\rangle$ $/(1-t)$. α generates $\pm\phi$-flow when it runs over D. β measures $\pm\phi^{-1}$-flow when it runs under D.

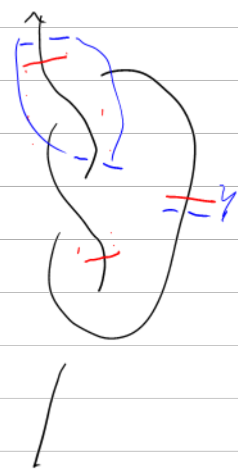




$?\overset{?}{=}$ phase$^{\pm1}$ xing # of α & β / $1-t$

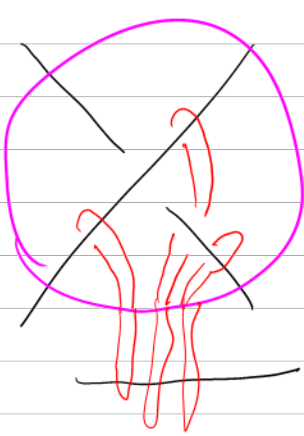


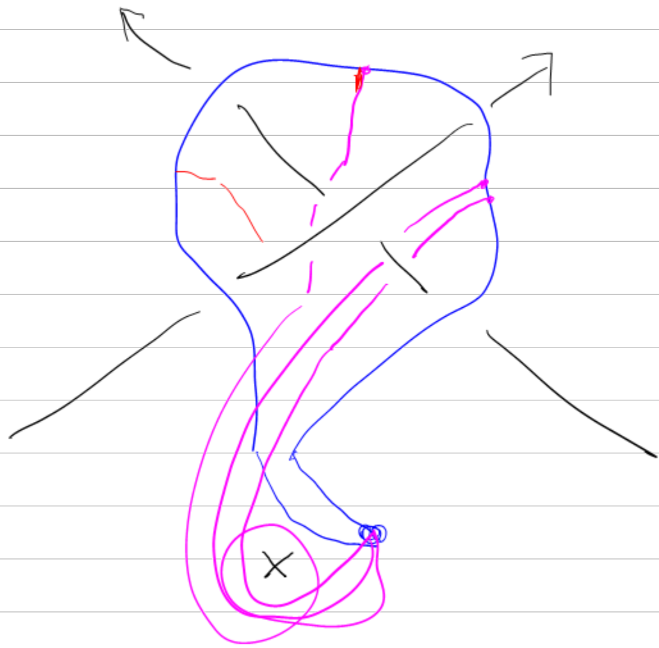


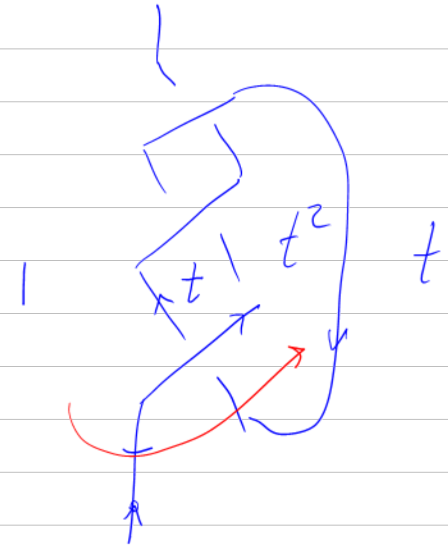


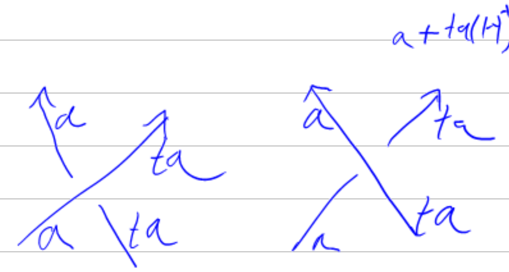
α_i: lower clasper

β_j: upper clasper

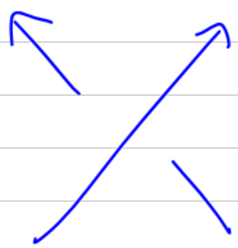


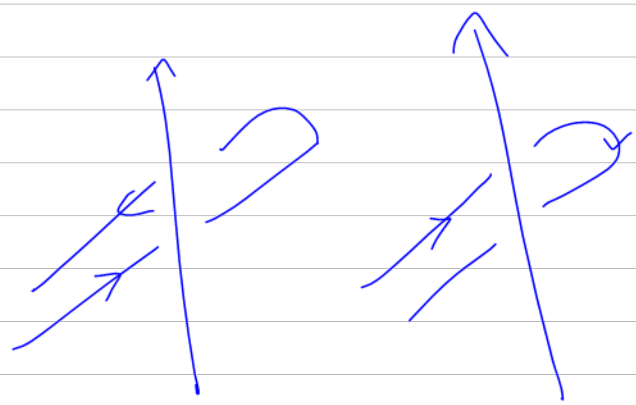






$$R = a \otimes b \qquad R^{-1} = a \otimes S(b)$$

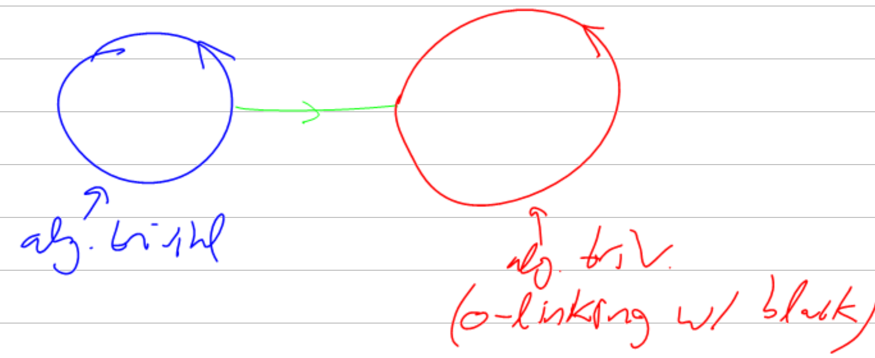




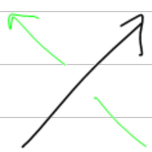
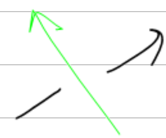
Expansion for virts/braid like $\overset{?}{\Longrightarrow}$ Expansion for rotational virtuals.

Properties of equivariant linking numbers in the background of a black knot:

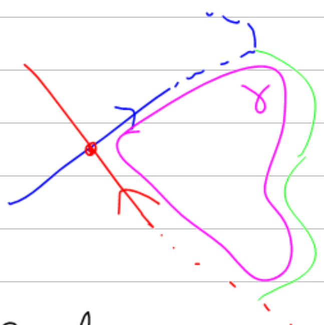
1. An invariant of



2. green is blind to red, blue & itself

3.  $= t$ 

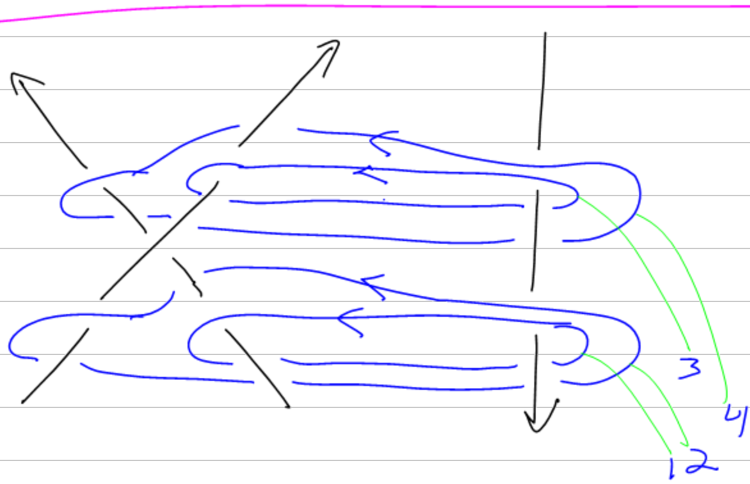
4. blue and red are blind to themselves.

5. 

$= t^{lk(\gamma, K)}$ 

6. "Bilinear":

7. Conjugation of blue/red multiplies by a power of t.



$$b_1 = b_4$$

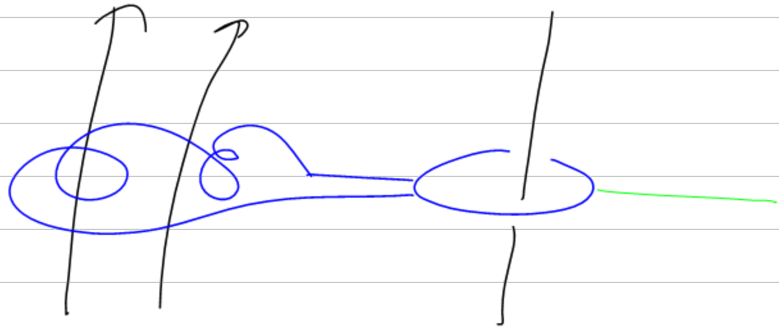
$$b_2 \overset{?}{=} t b_3 + (1-t) b_4$$

$$\Updownarrow$$

$$b_2 - t b_3 = b_1 - t b_4$$

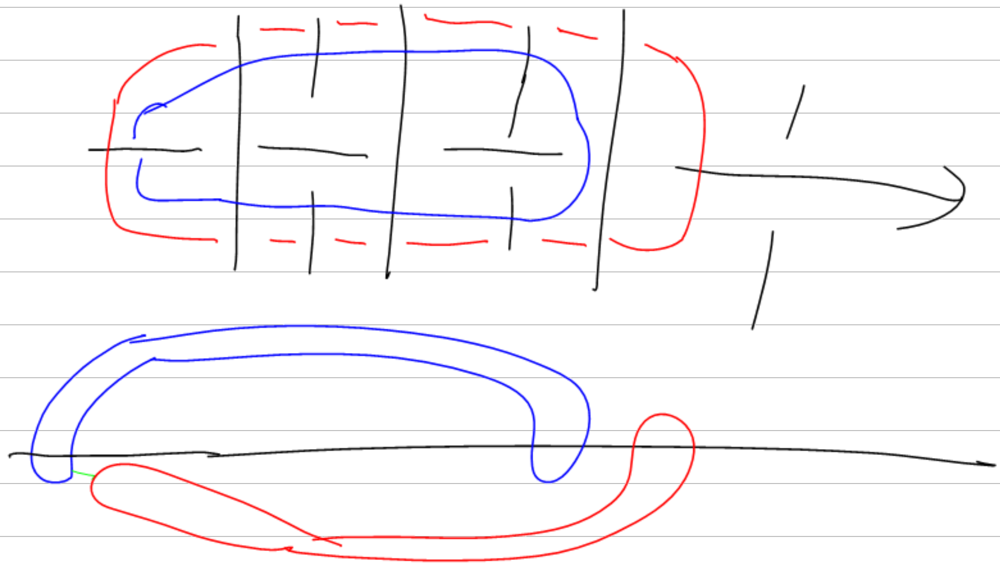
$$\Updownarrow$$

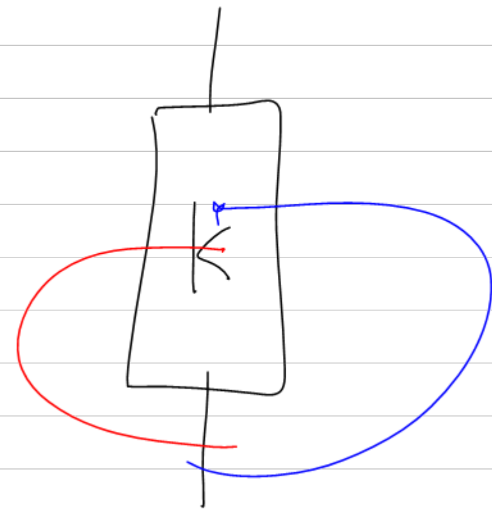
$$b_2 + t b_4 = b_1 + t b_3$$



$\gamma^a \alpha \qquad \gamma^b \beta$

$$\gamma^{a+b}\alpha\beta = \gamma^b \gamma^a \alpha \gamma^{-b} \gamma^b \beta$$
$$= (\gamma^a \alpha)^{\gamma^{-b}} \gamma^b \beta$$

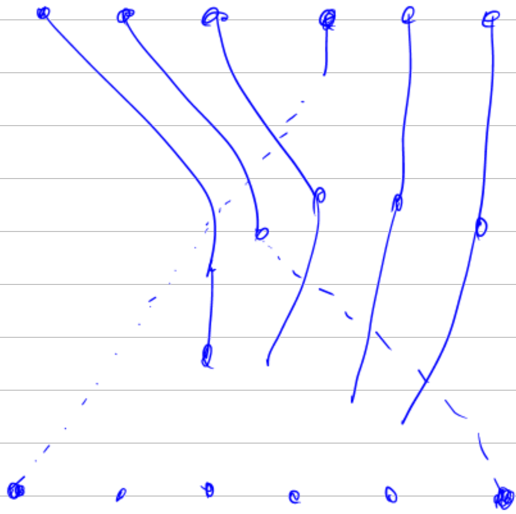


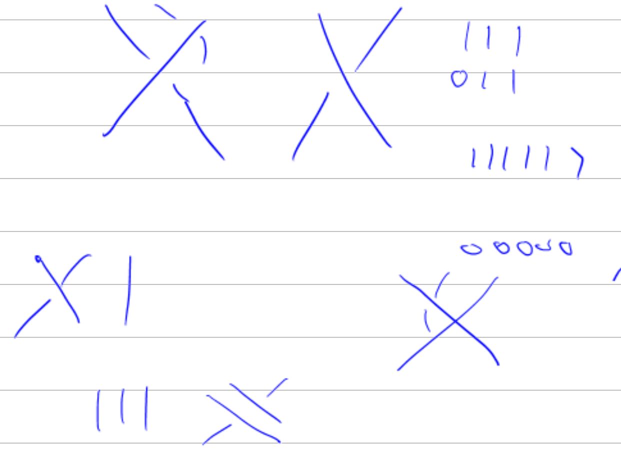


$GA = I \qquad LA = D$

$L = DG$

<u>conj</u> $A \otimes \bar{A} \cong \Lambda \otimes_{\Lambda_0} B$ where B is a Λ_0-module.



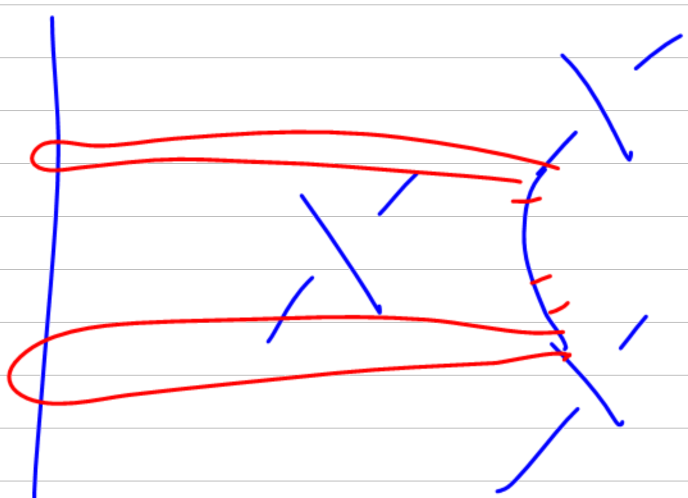


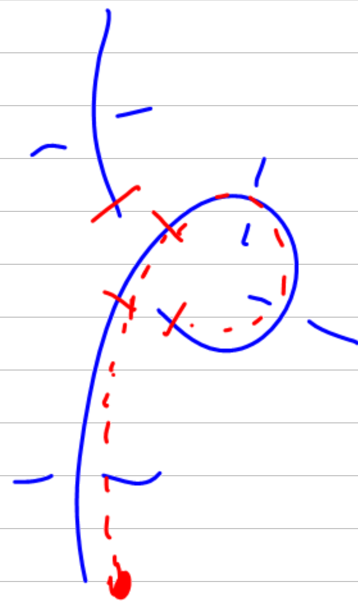
$1+3+3+2+2+2=13$

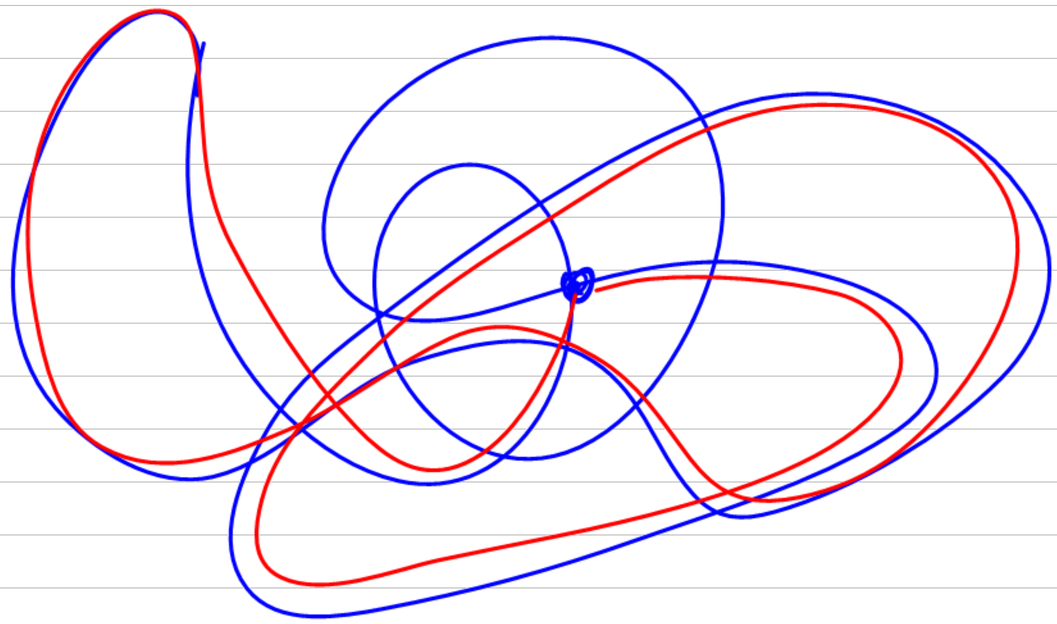
$$
\begin{tikzcd}
R_1 \arrow[r, "A_1"] & G_1 \arrow[dr] & \\
 & & A \\
R_2 \arrow[u, "Q"] \arrow[r, "A_2"] & G_2 \arrow[u, "P"] \arrow[ur] &
\end{tikzcd}
$$

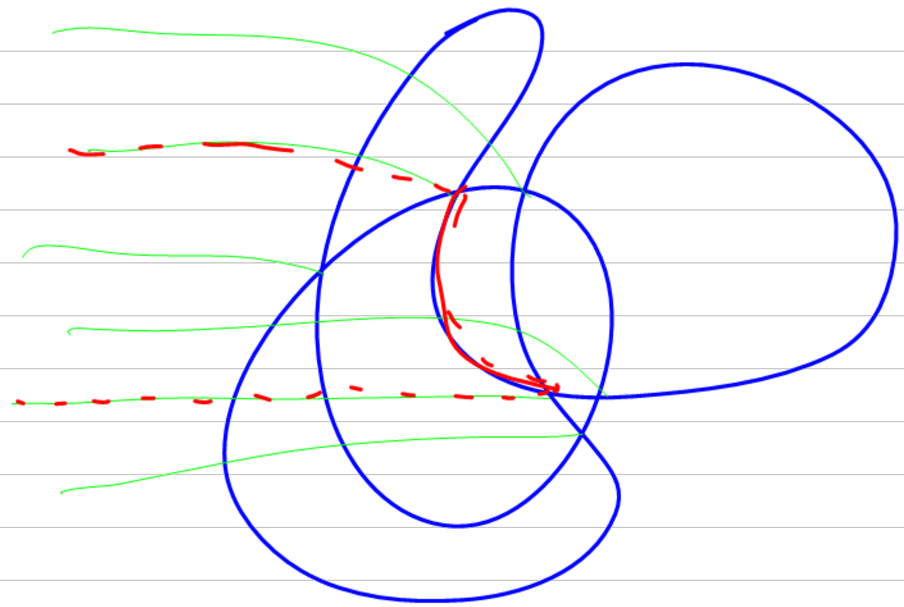
upper

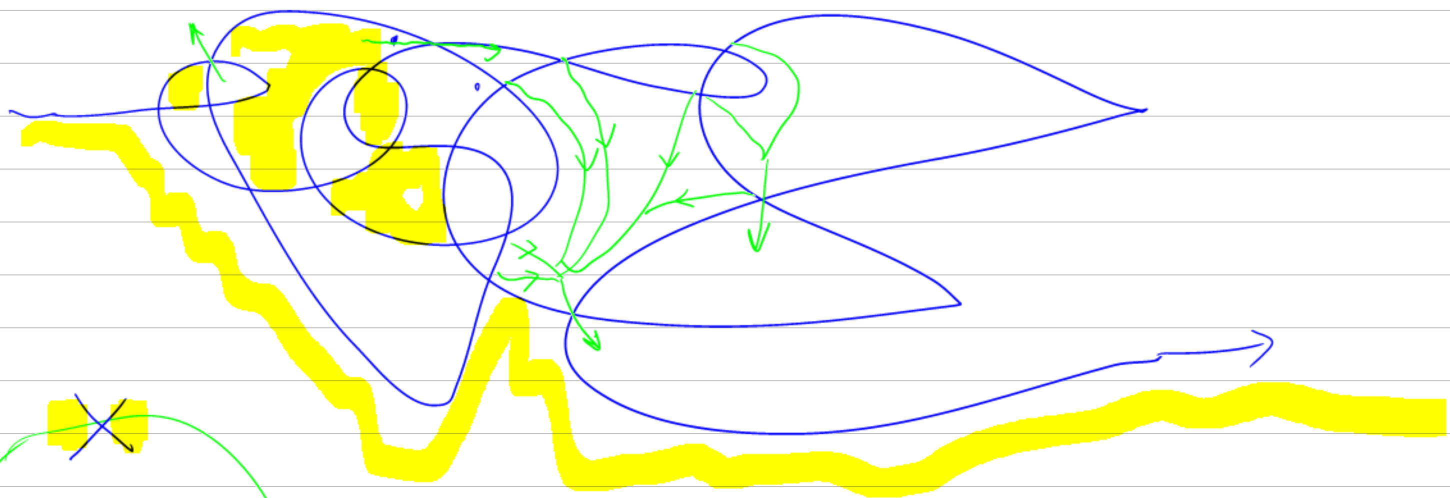
lower

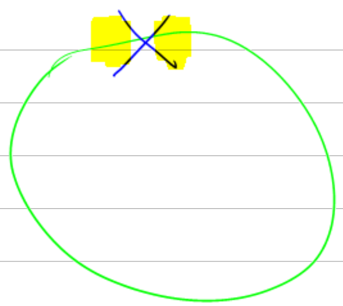




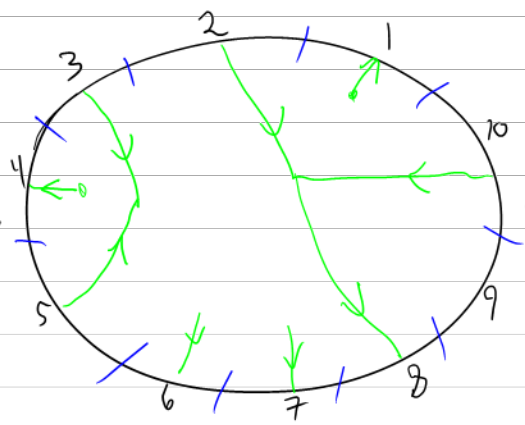






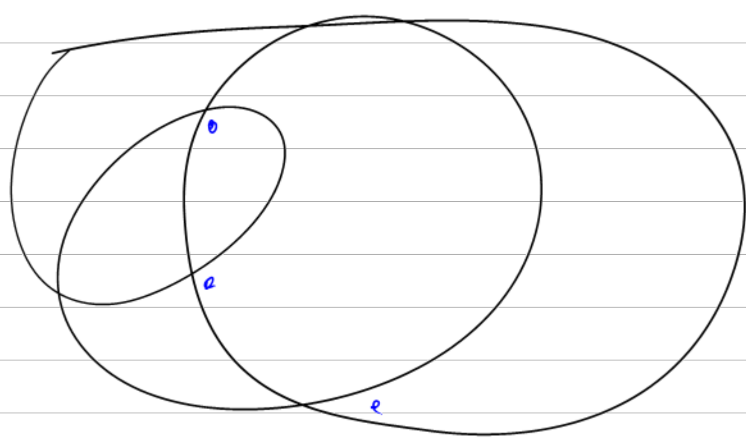


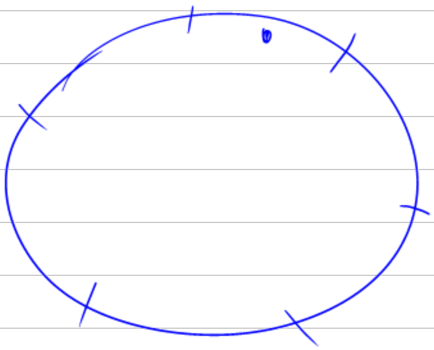
$\mathbb{Z}[T^{\pm 1}, m].$

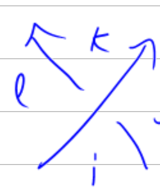


$\longleftrightarrow$ $Y_8[2,10]\,Y[3,5].$

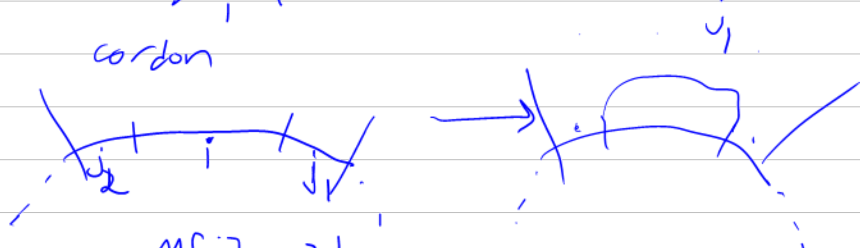
$Y_1[\,]\,Y_4[\,]\,Y_6[\,]\,Y_9[\,]$



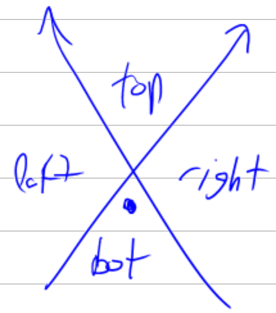


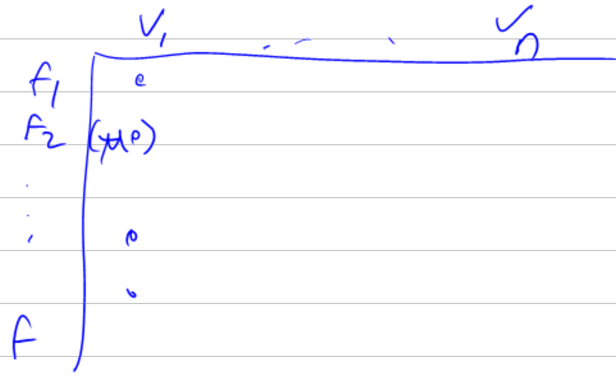


$M^T M[i] + M[j] + M[l] + M[k] \cdot T$



$M[i] \longrightarrow 1$


top
left
right
bot


v_1 ... v_n
f_1
f_2
f