Pensieve header: Modified from Roland's Dropbox/RhoRho/rhorho.nb

In[•]:=
```
Once[<< KnotTheory`];
```

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
```

In[•]:=
```
PD[epd_EPD] := PD @@ epd /. {X_i_,j_ :> X[j, i + 1, j + 1, i], Xbar_i_,j_ :> X[j, i, j + 1, i + 1]}
```

In[•]:=
```
Rot[pd_PD] := Module[{n, xs, x, rots, Xp, Xm, front = {1}, k},
   n = Length@pd; rots = Table[0, {2 n}];
   xs = Cases[pd, x_X :> {Xp[x[[4]], x[[1]]] PositiveQ@x
                          Xm[x[[2]], x[[1]]]      True   ];
   For[k = 1, k ≤ 2 n, ++k,
    If[FreeQ[front, -k],
     front = Flatten@Replace[front, k → (xs /. {
             Xp[k, l_] | Xm[l_, k] :> {l + 1, k + 1, -l},
             Xp[l_, k] | Xm[k, l_] :> (++rots[[l]]; {-l, k + 1, l + 1}),
             _Xp | _Xm :> {}
            }), {1}],
     Cases[front, k | -k] /. {k, -k} :> --rots[[k]];
    ]
   ];
   {xs /. {Xp[i_, j_] :> {+1, i, j}, Xm[i_, j_] :> {-1, i, j}}, rots}];
Rot[K_] := Rot[PD[K]];
```

In[•]:=
```
δ_i_,j_ := If[i === j, 1, 0];
gRules_s_,i_,j_ := {g_iβ_ :> δ_iβ + T^s g_i⁺,β + (1 - T^s) g_j⁺,β, g_jβ_ :> δ_jβ + g_j⁺,β,
  g_α_,i :> T^-s (g_α,i⁺ - δ_α,i⁺), g_α_,j :> g_α,j⁺ - (1 - T^s) g_αi - δ_α,j⁺}
fRules_s_,i_,j_ := {f_iβ_ :> δ_iβ + F^s f_i⁺,β + (1 - F^s) f_j⁺,β, f_jβ_ :> δ_jβ + f_j⁺,β,
  f_α_,i :> F^-s (f_α,i⁺ - δ_α,i⁺), f_α_,j :> f_α,j⁺ - (1 - F^s) f_αi - δ_α,j⁺}
(α_⁺)⁺ := α"++";
```

The ansatz for R, R^{-1} and C

In[ ]:=
```
R1[1, i_, j_] := Module[{gfs = Join[Flatten@Table[fa,b, {a, {i, j}}, {b, {i, j}}],
      Flatten@Table[ga,b, {a, {i, j}}, {b, {i, j}}]]},
  DeleteDuplicates[Times @@@ Tuples[
       gfs, 2]].Table[rk, {k, 36}] +
   gfs.Table[rk, {k, 36 + 1, 36 + 8}] + r36+9]
R1[-1, i_, j_] := Module[{gfs = Join[Flatten@Table[fa,b, {a, {i, j}}, {b, {i, j}}],
      Flatten@Table[ga,b, {a, {i, j}}, {b, {i, j}}]]},
  DeleteDuplicates[Times @@@ Tuples[
       gfs, 2]].Table[qk, {k, 36}] +
   gfs.Table[qk, {k, 36 + 1, 36 + 8}] + q36+9]
(* modified expression for the curl: *)
CC[s_, i_] := s (c1 fi,i + c2 gi,i + c3)
```

In[ ]:=
```
R1[1, i, j]
```

Out[ ]=

$$r_{45} + r_{37} f_{i,i} + r_1 f_{i,i}^2 + r_{38} f_{i,j} + r_2 f_{i,i} f_{i,j} + r_9 f_{i,j}^2 + r_{39} f_{j,i} + r_3 f_{i,i} f_{j,i} + r_{10} f_{i,j} f_{j,i} + r_{16} f_{j,i}^2 + r_{40} f_{j,j} + r_4 f_{i,i} f_{j,j} + r_{11} f_{i,j} f_{j,j} + r_{17} f_{j,i} f_{j,j} + r_{22} f_{j,j}^2 + r_{41} g_{i,i} + r_5 f_{i,i} g_{i,i} + r_{12} f_{i,j} g_{i,i} + r_{18} f_{j,i} g_{i,i} + r_{23} f_{j,j} g_{i,i} + r_{27} g_{i,i}^2 + r_{42} g_{i,j} + r_6 f_{i,i} g_{i,j} + r_{13} f_{i,j} g_{i,j} + r_{19} f_{j,i} g_{i,j} + r_{24} f_{j,j} g_{i,j} + r_{28} g_{i,i} g_{i,j} + r_{31} g_{i,j}^2 + r_{43} g_{j,i} + r_7 f_{i,i} g_{j,i} + r_{14} f_{i,j} g_{j,i} + r_{20} f_{j,i} g_{j,i} + r_{25} f_{j,j} g_{j,i} + r_{29} g_{i,i} g_{j,i} + r_{32} g_{i,j} g_{j,i} + r_{34} g_{j,i}^2 + r_{44} g_{j,j} + r_8 f_{i,i} g_{j,j} + r_{15} f_{i,j} g_{j,j} + r_{21} f_{j,i} g_{j,j} + r_{26} f_{j,j} g_{j,j} + r_{30} g_{i,i} g_{j,j} + r_{33} g_{i,j} g_{j,j} + r_{35} g_{j,i} g_{j,j} + r_{36} g_{j,j}^2$$

The Reidemeister moves as equations

In[ ]:=
```
lhs3 = R1[1, j, k] + R1[1, i, k+] + R1[1, i+, j+] //.
   gRules1,j,k ⋃ gRules1,i,k+ ⋃ gRules1,i+,j+ ⋃ fRules1,j,k ⋃ fRules1,i,k+ ⋃ fRules1,i+,j+;
rhs3 = R1[1, i, j] + R1[1, i+, k] + R1[1, j+, k+] //.
   gRules1,i,j ⋃ gRules1,i+,k ⋃ gRules1,j+,k+ ⋃ fRules1,i,j ⋃ fRules1,i+,k ⋃ fRules1,j+,k+;

lhs2c = R1[-1, i, j+] + R1[1, i+, j] + CC[1, j+] //.
   gRules-1,i,j+ ⋃ gRules1,i+,j ⋃ fRules-1,i,j+ ⋃ fRules1,i+,j;
rhs2c = CC[1, (j+)+];

lhs1l = R1[1, i+, i] + CC[1, i+] //. {gi+,β_ :> T^-1 δi+,β + gi"++",β, gi,β_ :> δi,β + gi+,β,
    gα_,i+ :> T^-1 (gα,i"++" - δα,i+), gα_,i :> T gα,i+ - δα,i+,
    fi+,β_ :> F^-1 δi+,β + fi"++",β, fi,β_ :> δi,β + fi+,β,
    fα_,i+ :> F^-1 (fα,i"++" - δα,i+), fα_,i :> T fα,i+ - δα,i+};
lhs1r = R1[1, i, i+] + CC[-1, i+] //. {
    giβ_ :> δiβ + T gi+,β + (1 - T) gi++,β, gi+,β_ :> δi+,β + gi++,β,
    gα_,i :> T^-1 (gα,i+ - δα,i+), gα_,i+ :> T gα,i++ - (1 - T) δα,i+ - T δα,i++,
    fiβ_ :> δiβ + F fi+,β + (1 - F) fi++,β, fi+,β_ :> δi+,β + fi++,β,
    fα_,i :> F^-1 (fα,i+ - δα,i+), fα_,i+ :> T fα,i++ - (1 - F) δα,i+ - F δα,i++};
rhs1l = 0;
rhs1r = 0;
lhssw =
  R1[1, i, j] + CC[-1, i] + CC[-1, j] + CC[1, i+] + CC[1, j+] //. gRules1,i,j ⋃ fRules1,i,j;
rhssw = R1[1, i, j] //. gRules1,i,j ⋃ fRules1,i,j;
```

Solve for the coefficients in the Reidemeister equations

In[ ]:=
```
V = Join[Flatten@Table[ga,b, {a, {i"++", j"++", k"++"}}, {b, {i"++", j"++", k"++"}}],
    Flatten@Table[fa,b, {a, {i"++", j"++", k"++"}}, {b, {i"++", j"++", k"++"}}]];
eq3 = Thread[Last /@ CoefficientRules[lhs3 - rhs3, V] == 0];
eq2c = Thread[Last /@ CoefficientRules[lhs2c - rhs2c, V] == 0];
eq1l = Thread[Last /@ CoefficientRules[lhs1l - rhs1l, V] == 0];
eq1r = Thread[Last /@ CoefficientRules[lhs1r - rhs1r, V] == 0];
eqsw = Thread[Last /@ CoefficientRules[lhssw - rhssw, V] == 0];
```

In[ ]:=
```
Soln = First@Solve[Join[eq3, eq2c, eq1l, eq1r, eqsw],
    Join[Table[q_i, {i, 1, 45}], Table[r_i, {i, 1, 45}], Table[c_i, {i, 1, 2}]]] // Simplify
```

Solve: Equations may not give solutions for all "solve" variables.

Out[ ]=

$$\left\{q_1 \to 0,\ q_2 \to 0,\ q_3 \to \frac{(-1+F)\ r_{17}}{2\,F},\ q_4 \to \frac{r_{17}}{2},\ q_5 \to 0,\ q_6 \to 0,\ q_7 \to \frac{(-1+T)\ r_{21}}{(-1+F)\ T^2},\ q_8 \to -\frac{(-1+T)\ r_{21}}{(-1+F)\ T},\right.$$
$$q_9 \to 0,\ q_{10} \to \frac{r_{17}}{2},\ q_{11} \to 0,\ q_{12} \to 0,\ q_{13} \to 0,\ q_{14} \to 0,\ q_{15} \to 0,\ q_{16} \to -\frac{(-1+F)\ r_{17}}{2\,F},$$
$$q_{17} \to -r_{17},\ q_{18} \to \frac{r_{21}}{F},\ q_{19} \to 0,\ q_{20} \to -\frac{\left(-1+T^2\right)\ r_{21}}{F\,T^2},\ q_{21} \to -\frac{r_{21}}{F\,T},\ q_{22} \to 0,\ q_{23} \to \frac{(-1+T)\ r_{21}}{(-1+F)\ T},$$
$$q_{24} \to 0,\ q_{25} \to -\frac{(-1+T)\ r_{21}}{(-1+F)\ T},\ q_{26} \to 0,\ q_{27} \to 0,\ q_{28} \to 0,\ q_{29} \to \frac{(-3+T)\ r_{29} + 2\ (-1+T)\ r_{35}}{T\ (1+T)},$$
$$q_{30} \to \frac{r_{29}+r_{35}}{1+T},\ q_{31} \to 0,\ q_{32} \to \frac{r_{29}+r_{35}}{1+T},\ q_{33} \to 0,\ q_{34} \to -\frac{(-1+T)\ \left((-2+T)\ r_{29} + (-1+2\,T)\ r_{35}\right)}{T^2\ (1+T)},$$
$$q_{35} \to \frac{-2\ (-1+T)\ r_{29} + (1-3\,T)\ r_{35}}{T\ (1+T)},\ q_{36} \to 0,\ q_{37} \to -r_{37},\ q_{38} \to 0,\ q_{39} \to r_{37},\ q_{40} \to 0,$$
$$q_{41} \to -r_{41},\ q_{42} \to 0,\ q_{43} \to \frac{2\,c_3 - r_{37} + (-1+T)\ r_{41}}{T},\ q_{44} \to -2\,c_3 + r_{37} + r_{41},\ q_{45} \to c_3,\ r_1 \to 0,$$
$$r_2 \to 0,\ r_3 \to \frac{1}{2}\ (-1+F)\ r_{17},\ r_4 \to -\frac{r_{17}}{2},\ r_5 \to 0,\ r_6 \to 0,\ r_7 \to -\frac{(-1+T)\ r_{21}}{-1+F},\ r_8 \to \frac{(-1+T)\ r_{21}}{(-1+F)\ T},$$
$$r_9 \to 0,\ r_{10} \to -\frac{r_{17}}{2},\ r_{11} \to 0,\ r_{12} \to 0,\ r_{13} \to 0,\ r_{14} \to 0,\ r_{15} \to 0,\ r_{16} \to -\frac{1}{2}\ (-1+F)\ r_{17},$$
$$r_{18} \to -\frac{r_{21}}{T},\ r_{19} \to 0,\ r_{20} \to -\frac{\left(-1+T^2\right)\ r_{21}}{T},\ r_{22} \to 0,\ r_{23} \to -\frac{(-1+T)\ r_{21}}{(-1+F)\ T},\ r_{24} \to 0,$$
$$r_{25} \to \frac{(-1+T)\ r_{21}}{(-1+F)\ T},\ r_{26} \to 0,\ r_{27} \to 0,\ r_{28} \to 0,\ r_{30} \to -\frac{r_{29}+r_{35}}{1+T},\ r_{31} \to 0,\ r_{32} \to -\frac{r_{29}+r_{35}}{1+T},$$
$$r_{33} \to 0,\ r_{34} \to \frac{(-1+T)\ (r_{29} - T\,r_{35})}{1+T},\ r_{36} \to 0,\ r_{38} \to 0,\ r_{39} \to -r_{37},\ r_{40} \to 0,\ r_{42} \to 0,$$
$$\left.r_{43} \to -2\,T\,c_3 + T\,r_{37} + (-1+T)\ r_{41},\ r_{44} \to 2\,c_3 - r_{37} - r_{41},\ r_{45} \to -c_3,\ c_1 \to -\frac{r_{17}}{2},\ c_2 \to -\frac{r_{29}+r_{35}}{1+T}\right\}$$

In[ ]:=
```
CCC[s_, i_] := s (CC[1, i] /. Soln)
```

In[ ]:=
```
CC[1, i] /. Soln
R1[1, i, j] /. Soln
R1[-1, i, j] /. Soln
```

Out[ ]=

$$c_3 - \frac{1}{2} r_{17} f_{i,i} - \frac{(r_{29} + r_{35})\, g_{i,i}}{1+T}$$

Out[ ]=

$$-c_3 + r_{37} f_{i,i} - r_{37} f_{j,i} + \frac{1}{2}(-1+F) r_{17} f_{i,i} f_{j,i} - \frac{1}{2} r_{17} f_{i,j} f_{j,i} - \frac{1}{2}(-1+F) r_{17} f_{j,i}^2 - \frac{1}{2} r_{17} f_{i,i} f_{j,j} + r_{17} f_{j,i} f_{j,j} + r_{41} g_{i,i} - \frac{r_{21} f_{j,i} g_{i,i}}{T} - \frac{(-1+T) r_{21} f_{j,j} g_{i,i}}{(-1+F) T} + (-2 T c_3 + T r_{37} + (-1+T) r_{41}) g_{j,i} - \frac{(-1+T) r_{21} f_{i,i} g_{j,i}}{-1+F} - \frac{\left(-1+T^2\right) r_{21} f_{j,i} g_{j,i}}{T} + \frac{(-1+T) r_{21} f_{j,j} g_{j,i}}{(-1+F) T} + r_{29} g_{i,i} g_{j,i} - \frac{(r_{29} + r_{35}) g_{i,j} g_{j,i}}{1+T} + \frac{(-1+T)(r_{29} - T r_{35}) g_{j,i}^2}{1+T} + (2 c_3 - r_{37} - r_{41}) g_{j,j} + \frac{(-1+T) r_{21} f_{i,i} g_{j,j}}{(-1+F) T} + r_{21} f_{j,i} g_{j,j} - \frac{(r_{29} + r_{35}) g_{i,i} g_{j,j}}{1+T} + r_{35} g_{j,i} g_{j,j}$$

Out[ ]=

$$c_3 - r_{37} f_{i,i} + r_{37} f_{j,i} + \frac{(-1+F) r_{17} f_{i,i} f_{j,i}}{2F} + \frac{1}{2} r_{17} f_{i,j} f_{j,i} - \frac{(-1+F) r_{17} f_{j,i}^2}{2F} + \frac{1}{2} r_{17} f_{i,i} f_{j,j} - r_{17} f_{j,i} f_{j,j} - r_{41} g_{i,i} + \frac{r_{21} f_{j,i} g_{i,i}}{F} + \frac{(-1+T) r_{21} f_{j,j} g_{i,i}}{(-1+F) T} + \frac{(2 c_3 - r_{37} + (-1+T) r_{41}) g_{j,i}}{T} + \frac{(-1+T) r_{21} f_{i,i} g_{j,i}}{(-1+F) T^2} - \frac{\left(-1+T^2\right) r_{21} f_{j,i} g_{j,i}}{F T^2} - \frac{(-1+T) r_{21} f_{j,j} g_{j,i}}{(-1+F) T} + \frac{((-3+T) r_{29} + 2(-1+T) r_{35}) g_{i,i} g_{j,i}}{T(1+T)} + \frac{(r_{29} + r_{35}) g_{i,j} g_{j,i}}{1+T} - \frac{(-1+T)((-2+T) r_{29} + (-1+2T) r_{35}) g_{j,i}^2}{T^2(1+T)} + (-2 c_3 + r_{37} + r_{41}) g_{j,j} - \frac{(-1+T) r_{21} f_{i,i} g_{j,j}}{(-1+F) T} - \frac{r_{21} f_{j,i} g_{j,j}}{FT} + \frac{(r_{29} + r_{35}) g_{i,i} g_{j,j}}{1+T} + \frac{(-2(-1+T) r_{29} + (1-3T) r_{35}) g_{j,i} g_{j,j}}{T(1+T)}$$

In[ ]:=
```
CCC[s_, i_] := s (CC[1, i] /. Soln)
RR1[1, i_, j_] := R1[1, i, j] /. Soln
RR1[-1, i_, j_] := R1[-1, i, j] /. Soln
ρρ[K_] := Module[{Cs, φ, n, A, s, i, j, k, Δ, G, FF, ρ1},
  {Cs, φ} = Rot[K]; n = Length[Cs];
  A = IdentityMatrix[2 n + 1];
  Cases[Cs, {s_, i_, j_} :> (A[[{i, j}, {i + 1, j + 1}]] += ({{-T^s, T^s - 1}, {0, -1}}))];
  Δ = T^((-Total[φ] - Total[Cs[[All, 1]]])/2) Det[A];
  G = Inverse[A]; FF = G /. T → F;
  ρ1 = Sum_{k=1}^{n} RR1 @@ Cs[[k]] + Sum_{k=1}^{2 n} φ[[k]] CCC[1, k];
  Factor@{Δ, (Δ^2 /. T → F) Δ^2 ρ1 /. α_^+ :> α + 1 /. {g_{α_,β_} :> G[[α, β]], f_{α_,β_} :> FF[[α, β]]}}];
Rold1[s_, i_, j_] := s (g_{ji} (g_{j^+,j} + g_{j,j^+} - g_{ij}) - g_{ii} (g_{j,j^+} - 1) - 1/2);
Cold[s_, i_] := s (-g_{ii} + 1/2)
ρ[K_] := Module[{Cs, φ, n, A, s, i, j, k, Δ, G, FF, ρ1},
  {Cs, φ} = Rot[K]; n = Length[Cs];
  A = IdentityMatrix[2 n + 1];
  Cases[Cs, {s_, i_, j_} :> (A[[{i, j}, {i + 1, j + 1}]] += ({{-T^s, T^s - 1}, {0, -1}}))];
  Δ = T^((-Total[φ] - Total[Cs[[All, 1]]])/2) Det[A];
  G = Inverse[A];
  ρ1 = Sum_{k=1}^{n} ((Rold1 @@ Cs[[k]])) + Sum_{k=1}^{2 n} φ[[k]] Cold[1, k];
  Factor@{Δ, Δ^2 ρ1 /. α_^+ :> α + 1 /. {g_{α_,β_} :> G[[α, β]]}}];
```

In[ ]:=
```
ρ[Knot[3, 1]]
Expand[F^2 ρρ[Knot[3, 1]]] /. F → 0
```

KnotTheory: Loading precomputed data in PD4Knots`.

Out[ ]=

$$\left\{\frac{1-T+T^2}{T},\ \frac{(-1+T)^2\left(1+T^2\right)}{T^2}\right\}$$

Out[ ]=

$$\left\{0,\ -\frac{2\,c_3}{1+T}+\frac{2\,c_3}{T^2\,(1+T)}+\frac{2\,T\,c_3}{1+T}-\frac{2\,T^3\,c_3}{1+T}+\frac{2\,r_{29}}{1+T}-\frac{r_{29}}{T\,(1+T)}-\frac{3\,T\,r_{29}}{1+T}+\right.$$
$$\left.\frac{2\,T^2\,r_{29}}{1+T}+\frac{2\,r_{35}}{1+T}-\frac{r_{35}}{T\,(1+T)}-\frac{3\,T\,r_{35}}{1+T}+\frac{2\,T^2\,r_{35}}{1+T}+\frac{2\,r_{37}}{1+T}-\frac{r_{37}}{T\,(1+T)}-\frac{T^2\,r_{37}}{1+T}+\frac{2\,T^3\,r_{37}}{1+T}\right\}$$

In[•]:=
```
(*ρρ does not distinguish the famous mutants*)
ρρ[Knot[11, NonAlternating, 34]] ==
 ρρ[Knot[11, NonAlternating, 42]]
```

KnotTheory: Loading precomputed data in DTCode4KnotsTo11`.

KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

Out[•]=
```
True
```

```
(*Yet ρρ does outperform our usual ρ=
 ρ1 a little on all 11 crossing non-alternating knots:*)
```

In[•]:=
```
ρ11na = ρ /@ Table[PD@Knot[11, NonAlternating, k], {k, 1, 185}];
```

In[•]:=
```
ρρ11na = ρρ /@ Table[PD@Knot[11, NonAlternating, k], {k, 1, 185}];
```

In[•]:=
```
Tally[ρ11na] // Length
Tally[Factor[ρρ11na]] // Length
```

Out[•]=
```
177
```

Out[•]=
```
177
```

In[•]:=
```
Thread[ρρ11na[[3]] == Factor[ρρ11na[[3]]]]
```

Out[•]=

$$\left\{\text{True},\ \frac{1}{2\,F^4\,T^4\,(1+T)}\left(648\,c_3-4752\,F\,c_3+15\,192\,F^2\,c_3-28\,512\,F^3\,c_3+34\,920\,F^4\,c_3-28\,512\,F^5\,c_3+15\,192\,F^6\,c_3-4752\,F^7\,c_3+648\,F^8\,c_3-2916\,T\,c_3+21\,384\,F\,T\,c_3-68\,364\,F^2\,T\,c_3+128\,304\,F^3\,T\,c_3-157\,140\,F^4\,T\,c_3+128\,304\,F^5\,T\,c_3-68\,364\,F^6\,T\,c_3+21\,384\,F^7\,T\,c_3-2916\,F^8\,T\,c_3+4032\,T^2\,c_3-29\,568\,F\,T^2\,c_3+94\,528\,F^2\,T^2\,c_3-177\,408\,F^3\,T^2\,c_3+217\,280\,F^4\,T^2\,c_3-177\,408\,F^5\,T^2\,c_3+94\,528\,F^6\,T^2\,c_3-29\,568\,F^7\,T^2\,c_3+4032\,F^8\,T^2\,c_3+468\,T^3\,c_3-3432\,F\,T^3\,c_3+10\,972\,F^2\,T^3\,c_3-20\,592\,F^3\,T^3\,c_3+25\,220\,F^4\,T^3\,c_3-20\,592\,F^5\,T^3\,c_3+10\,972\,F^6\,T^3\,c_3-3432\,F^7\,T^3\,c_3+468\,F^8\,T^3\,c_3-7128\,T^4\,c_3+52\,272\,F\,T^4\,c_3-167\,112\,F^2\,T^4\,c_3+313\,632\,F^3\,T^4\,c_3-384\,120\,F^4\,T^4\,c_3+313\,632\,F^5\,T^4\,c_3-167\,112\,F^6\,T^4\,c_3+52\,272\,F^7\,T^4\,c_3-7128\,F^8\,T^4\,c_3+7128\,T^5\,c_3-52\,272\,F\,T^5\,c_3+167\,112\,F^2\,T^5\,c_3-313\,632\,F^3\,T^5\,c_3+384\,120\,F^4\,T^5\,c_3-313\,632\,F^5\,T^5\,c_3+167\,112\,F^6\,T^5\,c_3-52\,272\,F^7\,T^5\,c_3+7128\,F^8\,T^5\,c_3-468\,T^6\,c_3+3432\,F\,T^6\,c_3-10\,972\,F^2\,T^6\,c_3+20\,592\,F^3\,T^6\,c_3-25\,220\,F^4\,T^6\,c_3+20\,592\,F^5\,T^6\,c_3-10\,972\,F^6\,T^6\,c_3+3432\,F^7\,T^6\,c_3-468\,F^8\,T^6\,c_3-4032\,T^7\,c_3+29\,568\,F\,T^7\,c_3-94\,528\,F^2\,T^7\,c_3+177\,408\,F^3\,T^7\,c_3-217\,280\,F^4\,T^7\,c_3+177\,408\,F^5\,T^7\,c_3-94\,528\,F^6\,T^7\,c_3+29\,568\,F^7\,T^7\,c_3-4032\,F^8\,T^7\,c_3+2916\,T^8\,c_3-21\,384\,F\,T^8\,c_3+68\,364\,F^2\,T^8\,c_3-128\,304\,F^3\,T^8\,c_3+157\,140\,F^4\,T^8\,c_3-128\,304\,F^5\,T^8\,c_3+68\,364\,F^6\,T^8\,c_3-21\,384\,F^7\,T^8\,c_3+2916\,F^8\,T^8\,c_3-648\,T^9\,c_3+4752\,F\,T^9\,c_3-15\,192\,F^2\,T^9\,c_3+28\,512\,F^3\,T^9\,c_3-34\,920\,F^4\,T^9\,c_3+28\,512\,F^5\,T^9\,c_3-15\,192\,F^6\,T^9\,c_3+4752\,F^7\,T^9\,c_3-648\,F^8\,T^9\,c_3-90\,r_{17}+441\,F\,r_{17}-702\,F^2\,r_{17}-90\,F^3\,r_{17}+2106\,F^4\,r_{17}-3654\,F^5\,r_{17}+3096\,F^6\,r_{17}-1341\,F^7\,r_{17}+234\,F^8\,r_{17}+570\,T\,r_{17}-2793\,F\,T\,r_{17}+4446\,F^2\,T\,r_{17}+570\,F^3\,T\,r_{17}-13\,338\,F^4\,T\,r_{17}+23\,142\,F^5\,T\,r_{17}-19\,608\,F^6\,T\,r_{17}+8493\,F^7\,T\,r_{17}-1482\,F^8\,T\,r_{17}-1450\,T^2\,r_{17}+7105\,F\,T^2\,r_{17}-11\,310\,F^2\,T^2\,r_{17}-1450\,F^3\,T^2\,r_{17}+33\,930\,F^4\,T^2\,r_{17}-58\,870\,F^5\,T^2\,r_{17}+49\,880\,F^6\,T^2\,r_{17}-21\,605\,F^7\,T^2\,r_{17}+3770\,F^8\,T^2\,r_{17}+1850\,T^3\,r_{17}-9065\,F\,T^3\,r_{17}+14\,430\,F^2\,T^3\,r_{17}+1850\,F^3\,T^3\,r_{17}-\right.$$

$$43\,290\,F^4\,T^3\,r_{17} + 75\,110\,F^5\,T^3\,r_{17} - 63\,640\,F^6\,T^3\,r_{17} + 27\,565\,F^7\,T^3\,r_{17} - 4810\,F^8\,T^3\,r_{17} - 890\,T^4\,r_{17} + 4361\,F\,T^4\,r_{17} - 6942\,F^2\,T^4\,r_{17} - 890\,F^3\,T^4\,r_{17} + 20\,826\,F^4\,T^4\,r_{17} - 36\,134\,F^5\,T^4\,r_{17} + 30\,616\,F^6\,T^4\,r_{17} - 13\,261\,F^7\,T^4\,r_{17} + 2314\,F^8\,T^4\,r_{17} - 890\,T^5\,r_{17} + 4361\,F\,T^5\,r_{17} - 6942\,F^2\,T^5\,r_{17} - 890\,F^3\,T^5\,r_{17} + 20\,826\,F^4\,T^5\,r_{17} - 36\,134\,F^5\,T^5\,r_{17} + 30\,616\,F^6\,T^5\,r_{17} - 13\,261\,F^7\,T^5\,r_{17} + 2314\,F^8\,T^5\,r_{17} + 1850\,T^6\,r_{17} - 9065\,F\,T^6\,r_{17} + 14\,430\,F^2\,T^6\,r_{17} + 1850\,F^3\,T^6\,r_{17} - 43\,290\,F^4\,T^6\,r_{17} + 75\,110\,F^5\,T^6\,r_{17} - 63\,640\,F^6\,T^6\,r_{17} + 27\,565\,F^7\,T^6\,r_{17} - 4810\,F^8\,T^6\,r_{17} - 1450\,T^7\,r_{17} + 7105\,F\,T^7\,r_{17} - 11\,310\,F^2\,T^7\,r_{17} - 1450\,F^3\,T^7\,r_{17} + 33\,930\,F^4\,T^7\,r_{17} - 58\,870\,F^5\,T^7\,r_{17} + 49\,880\,F^6\,T^7\,r_{17} - 21\,605\,F^7\,T^7\,r_{17} + 3770\,F^8\,T^7\,r_{17} + 570\,T^8\,r_{17} - 2793\,F\,T^8\,r_{17} + 4446\,F^2\,T^8\,r_{17} + 570\,F^3\,T^8\,r_{17} - 13\,338\,F^4\,T^8\,r_{17} + 23\,142\,F^5\,T^8\,r_{17} - 19\,608\,F^6\,T^8\,r_{17} + 8493\,F^7\,T^8\,r_{17} - 1482\,F^8\,T^8\,r_{17} - 90\,T^9\,r_{17} + 441\,F\,T^9\,r_{17} - 702\,F^2\,T^9\,r_{17} - 90\,F^3\,T^9\,r_{17} + 2106\,F^4\,T^9\,r_{17} - 3654\,F^5\,T^9\,r_{17} + 3096\,F^6\,T^9\,r_{17} - 1341\,F^7\,T^9\,r_{17} + 234\,F^8\,T^9\,r_{17} - 180\,r_{29} + 1320\,F\,r_{29} - 4220\,F^2\,r_{29} + 7920\,F^3\,r_{29} - 9700\,F^4\,r_{29} + 7920\,F^5\,r_{29} - 4220\,F^6\,r_{29} + 1320\,F^7\,r_{29} - 180\,F^8\,r_{29} + 882\,T\,r_{29} - 6468\,F\,T\,r_{29} + 20\,678\,F^2\,T\,r_{29} - 38\,808\,F^3\,T\,r_{29} + 47\,530\,F^4\,T\,r_{29} - 38\,808\,F^5\,T\,r_{29} + 20\,678\,F^6\,T\,r_{29} - 6468\,F^7\,T\,r_{29} + 882\,F^8\,T\,r_{29} - 1404\,T^2\,r_{29} + 10\,296\,F\,T^2\,r_{29} - 32\,916\,F^2\,T^2\,r_{29} + 61\,776\,F^3\,T^2\,r_{29} - 75\,660\,F^4\,T^2\,r_{29} + 61\,776\,F^5\,T^2\,r_{29} - 32\,916\,F^6\,T^2\,r_{29} + 10\,296\,F^7\,T^2\,r_{29} - 1404\,F^8\,T^2\,r_{29} - 180\,T^3\,r_{29} + 1320\,F\,T^3\,r_{29} - 4220\,F^2\,T^3\,r_{29} + 7920\,F^3\,T^3\,r_{29} - 9700\,F^4\,T^3\,r_{29} + 7920\,F^5\,T^3\,r_{29} - 4220\,F^6\,T^3\,r_{29} + 1320\,F^7\,T^3\,r_{29} - 180\,F^8\,T^3\,r_{29} + 4212\,T^4\,r_{29} - 30\,888\,F\,T^4\,r_{29} + 98\,748\,F^2\,T^4\,r_{29} - 185\,328\,F^3\,T^4\,r_{29} + 226\,980\,F^4\,T^4\,r_{29} - 185\,328\,F^5\,T^4\,r_{29} + 98\,748\,F^6\,T^4\,r_{29} - 30\,888\,F^7\,T^4\,r_{29} + 4212\,F^8\,T^4\,r_{29} - 7308\,T^5\,r_{29} + 53\,592\,F\,T^5\,r_{29} - 171\,332\,F^2\,T^5\,r_{29} + 321\,552\,F^3\,T^5\,r_{29} - 393\,820\,F^4\,T^5\,r_{29} + 321\,552\,F^5\,T^5\,r_{29} - 171\,332\,F^6\,T^5\,r_{29} + 53\,592\,F^7\,T^5\,r_{29} - 7308\,F^8\,T^5\,r_{29} + 6192\,T^6\,r_{29} - 45\,408\,F\,T^6\,r_{29} + 145\,168\,F^2\,T^6\,r_{29} - 272\,448\,F^3\,T^6\,r_{29} + 333\,680\,F^4\,T^6\,r_{29} - 272\,448\,F^5\,T^6\,r_{29} + 145\,168\,F^6\,T^6\,r_{29} - 45\,408\,F^7\,T^6\,r_{29} + 6192\,F^8\,T^6\,r_{29} - 2682\,T^7\,r_{29} + 19\,668\,F\,T^7\,r_{29} - 62\,878\,F^2\,T^7\,r_{29} + 118\,008\,F^3\,T^7\,r_{29} - 144\,530\,F^4\,T^7\,r_{29} + 118\,008\,F^5\,T^7\,r_{29} - 62\,878\,F^6\,T^7\,r_{29} + 19\,668\,F^7\,T^7\,r_{29} - 2682\,F^8\,T^7\,r_{29} + 468\,T^8\,r_{29} - 3432\,F\,T^8\,r_{29} + 10\,972\,F^2\,T^8\,r_{29} - 20\,592\,F^3\,T^8\,r_{29} + 25\,220\,F^4\,T^8\,r_{29} - 20\,592\,F^5\,T^8\,r_{29} + 10\,972\,F^6\,T^8\,r_{29} - 3432\,F^7\,T^8\,r_{29} + 468\,F^8\,T^8\,r_{29} - 180\,r_{35} + 1320\,F\,r_{35} - 4220\,F^2\,r_{35} + 7920\,F^3\,r_{35} - 9700\,F^4\,r_{35} + 7920\,F^5\,r_{35} - 4220\,F^6\,r_{35} + 1320\,F^7\,r_{35} - 180\,F^8\,r_{35} + 882\,T\,r_{35} - 6468\,F\,T\,r_{35} + 20\,678\,F^2\,T\,r_{35} - 38\,808\,F^3\,T\,r_{35} + 47\,530\,F^4\,T\,r_{35} - 38\,808\,F^5\,T\,r_{35} + 20\,678\,F^6\,T\,r_{35} - 6468\,F^7\,T\,r_{35} + 882\,F^8\,T\,r_{35} - 1404\,T^2\,r_{35} + 10\,296\,F\,T^2\,r_{35} - 32\,916\,F^2\,T^2\,r_{35} + 61\,776\,F^3\,T^2\,r_{35} - 75\,660\,F^4\,T^2\,r_{35} + 61\,776\,F^5\,T^2\,r_{35} - 32\,916\,F^6\,T^2\,r_{35} + 10\,296\,F^7\,T^2\,r_{35} - 1404\,F^8\,T^2\,r_{35} - 180\,T^3\,r_{35} + 1320\,F\,T^3\,r_{35} - 4220\,F^2\,T^3\,r_{35} + 7920\,F^3\,T^3\,r_{35} - 9700\,F^4\,T^3\,r_{35} + 7920\,F^5\,T^3\,r_{35} - 4220\,F^6\,T^3\,r_{35} + 1320\,F^7\,T^3\,r_{35} - 180\,F^8\,T^3\,r_{35} + 4212\,T^4\,r_{35} - 30\,888\,F\,T^4\,r_{35} + 98\,748\,F^2\,T^4\,r_{35} - 185\,328\,F^3\,T^4\,r_{35} + 226\,980\,F^4\,T^4\,r_{35} - 185\,328\,F^5\,T^4\,r_{35} + 98\,748\,F^6\,T^4\,r_{35} - 30\,888\,F^7\,T^4\,r_{35} + 4212\,F^8\,T^4\,r_{35} - 7308\,T^5\,r_{35} + 53\,592\,F\,T^5\,r_{35} - 171\,332\,F^2\,T^5\,r_{35} + 321\,552\,F^3\,T^5\,r_{35} - 393\,820\,F^4\,T^5\,r_{35} + 321\,552\,F^5\,T^5\,r_{35} - 171\,332\,F^6\,T^5\,r_{35} + 53\,592\,F^7\,T^5\,r_{35} - 7308\,F^8\,T^5\,r_{35} + 6192\,T^6\,r_{35} - 45\,408\,F\,T^6\,r_{35} + 145\,168\,F^2\,T^6\,r_{35} - 272\,448\,F^3\,T^6\,r_{35} + 333\,680\,F^4\,T^6\,r_{35} - 272\,448\,F^5\,T^6\,r_{35} + 145\,168\,F^6\,T^6\,r_{35} - 45\,408\,F^7\,T^6\,r_{35} + 6192\,F^8\,T^6\,r_{35} - 2682\,T^7\,r_{35} + 19\,668\,F\,T^7\,r_{35} - 62\,878\,F^2\,T^7\,r_{35} + 118\,008\,F^3\,T^7\,r_{35} - 144\,530\,F^4\,T^7\,r_{35} + 118\,008\,F^5\,T^7\,r_{35} - 62\,878\,F^6\,T^7\,r_{35} + 19\,668\,F^7\,T^7\,r_{35} - 2682\,F^8\,T^7\,r_{35} + 468\,T^8\,r_{35} - 3432\,F\,T^8\,r_{35} + 10\,972\,F^2\,T^8\,r_{35} - 20\,592\,F^3\,T^8\,r_{35} + 25\,220\,F^4\,T^8\,r_{35} - 20\,592\,F^5\,T^8\,r_{35} + 10\,972\,F^6\,T^8\,r_{35} - 3432\,F^7\,T^8\,r_{35} + 468\,F^8\,T^8\,r_{35} + 594\,F\,r_{37} - 3798\,F^2\,r_{37} + 10\,692\,F^3\,r_{37} - 17\,460\,F^4\,r_{37} + 17\,820\,F^5\,r_{37} - 11\,394\,F^6\,r_{37} + 4158\,F^7\,r_{37} - 648\,F^8\,r_{37} - 594\,T\,r_{37} + 594\,F\,T\,r_{37} + 10\,128\,F^2\,T\,r_{37} - 41\,580\,F^3\,T\,r_{37} + 78\,570\,F^4\,T\,r_{37} - 86\,724\,F^5\,T\,r_{37} + 58\,236\,F^6\,T\,r_{37} - 21\,978\,F^7\,T\,r_{37} + 3510\,F^8\,T\,r_{37} + 3204\,T^2\,r_{37} - 13\,926\,F\,T^2\,r_{37} + 13\,926\,F^2\,T^2\,r_{37} + 31\,284\,F^3\,T^2\,r_{37} - 108\,640\,F^4\,T^2\,r_{37} + 146\,124\,F^5\,T^2\,r_{37} - 108\,454\,F^6\,T^2\,r_{37} + 43\,494\,F^7\,T^2\,r_{37} - 7236\,F^8\,T^2\,r_{37} - 6894\,T^3\,r_{37} + 38\,346\,F\,T^3\,r_{37} - 83\,556\,F^2\,T^3\,r_{37} + 83\,556\,F^3\,T^3\,r_{37} - 12\,610\,F^4\,T^3\,r_{37} - 62\,964\,F^5\,T^3\,r_{37} + 72\,584\,F^6\,T^3\,r_{37} - 34\,914\,F^7\,T^3\,r_{37} + 6426\,F^8\,T^3\,r_{37} + 6768\,T^4\,r_{37} - 43\,758\,F\,T^4\,r_{37} + 121\,114\,F^2\,T^4\,r_{37} - 192\,060\,F^3\,T^4\,r_{37} + 192\,060\,F^4\,T^4\,r_{37} - 121\,572\,F^5\,T^4\,r_{37} + 45\,998\,F^6\,T^4\,r_{37} - 8514\,F^7\,T^4\,r_{37} +$$

$$360\,F^8\,T^4\,r_{37} - 360\,T^5\,r_{37} + 8514\,F\,T^5\,r_{37} - 45\,998\,F^2\,T^5\,r_{37} + 121\,572\,F^3\,T^5\,r_{37} - 192\,060\,F^4\,T^5\,r_{37} + 192\,060\,F^5\,T^5\,r_{37} - 121\,114\,F^6\,T^5\,r_{37} + 43\,758\,F^7\,T^5\,r_{37} - 6768\,F^8\,T^5\,r_{37} - 6426\,T^6\,r_{37} + 34\,914\,F\,T^6\,r_{37} - 72\,584\,F^2\,T^6\,r_{37} + 62\,964\,F^3\,T^6\,r_{37} + 12\,610\,F^4\,T^6\,r_{37} - 83\,556\,F^5\,T^6\,r_{37} + 83\,556\,F^6\,T^6\,r_{37} - 38\,346\,F^7\,T^6\,r_{37} + 6894\,F^8\,T^6\,r_{37} + 7236\,T^7\,r_{37} - 43\,494\,F\,T^7\,r_{37} + 108\,454\,F^2\,T^7\,r_{37} - 146\,124\,F^3\,T^7\,r_{37} + 108\,640\,F^4\,T^7\,r_{37} - 31\,284\,F^5\,T^7\,r_{37} - 13\,926\,F^6\,T^7\,r_{37} + 13\,926\,F^7\,T^7\,r_{37} - 3204\,F^8\,T^7\,r_{37} - 3510\,T^8\,r_{37} + 21\,978\,F\,T^8\,r_{37} - 58\,236\,F^2\,T^8\,r_{37} + 86\,724\,F^3\,T^8\,r_{37} - 78\,570\,F^4\,T^8\,r_{37} + 41\,580\,F^5\,T^8\,r_{37} - 10\,128\,F^6\,T^8\,r_{37} - 594\,F^7\,T^8\,r_{37} + 594\,F^8\,T^8\,r_{37} + 648\,T^9\,r_{37} - 4158\,F\,T^9\,r_{37} + 11\,394\,F^2\,T^9\,r_{37} - 17\,820\,F^3\,T^9\,r_{37} + 17\,460\,F^4\,T^9\,r_{37} - 10\,692\,F^5\,T^9\,r_{37} + 3798\,F^6\,T^9\,r_{37} - 594\,F^7\,T^9\,r_{37}\Big) ==$$

$$-\frac{1}{2\,F^4\,T^4\,(1+T)}\Big(-648\,c_3 + 4752\,F\,c_3 - 15\,192\,F^2\,c_3 + 28\,512\,F^3\,c_3 - 34\,920\,F^4\,c_3 + 28\,512\,F^5\,c_3 - 15\,192\,F^6\,c_3 + 4752\,F^7\,c_3 - 648\,F^8\,c_3 + 2916\,T\,c_3 - 21\,384\,F\,T\,c_3 + 68\,364\,F^2\,T\,c_3 - 128\,304\,F^3\,T\,c_3 + 157\,140\,F^4\,T\,c_3 - 128\,304\,F^5\,T\,c_3 + 68\,364\,F^6\,T\,c_3 - 21\,384\,F^7\,T\,c_3 + 2916\,F^8\,T\,c_3 - 4032\,T^2\,c_3 + 29\,568\,F\,T^2\,c_3 - 94\,528\,F^2\,T^2\,c_3 + 177\,408\,F^3\,T^2\,c_3 - 217\,280\,F^4\,T^2\,c_3 + 177\,408\,F^5\,T^2\,c_3 - 94\,528\,F^6\,T^2\,c_3 + 29\,568\,F^7\,T^2\,c_3 - 4032\,F^8\,T^2\,c_3 - 468\,T^3\,c_3 + 3432\,F\,T^3\,c_3 - 10\,972\,F^2\,T^3\,c_3 + 20\,592\,F^3\,T^3\,c_3 - 25\,220\,F^4\,T^3\,c_3 + 20\,592\,F^5\,T^3\,c_3 - 10\,972\,F^6\,T^3\,c_3 + 3432\,F^7\,T^3\,c_3 - 468\,F^8\,T^3\,c_3 + 7128\,T^4\,c_3 - 52\,272\,F\,T^4\,c_3 + 167\,112\,F^2\,T^4\,c_3 - 313\,632\,F^3\,T^4\,c_3 + 384\,120\,F^4\,T^4\,c_3 - 313\,632\,F^5\,T^4\,c_3 + 167\,112\,F^6\,T^4\,c_3 - 52\,272\,F^7\,T^4\,c_3 + 7128\,F^8\,T^4\,c_3 - 7128\,T^5\,c_3 + 52\,272\,F\,T^5\,c_3 - 167\,112\,F^2\,T^5\,c_3 + 313\,632\,F^3\,T^5\,c_3 - 384\,120\,F^4\,T^5\,c_3 + 313\,632\,F^5\,T^5\,c_3 - 167\,112\,F^6\,T^5\,c_3 + 52\,272\,F^7\,T^5\,c_3 - 7128\,F^8\,T^5\,c_3 + 468\,T^6\,c_3 - 3432\,F\,T^6\,c_3 + 10\,972\,F^2\,T^6\,c_3 - 20\,592\,F^3\,T^6\,c_3 + 25\,220\,F^4\,T^6\,c_3 - 20\,592\,F^5\,T^6\,c_3 + 10\,972\,F^6\,T^6\,c_3 - 3432\,F^7\,T^6\,c_3 + 468\,F^8\,T^6\,c_3 + 4032\,T^7\,c_3 - 29\,568\,F\,T^7\,c_3 + 94\,528\,F^2\,T^7\,c_3 - 177\,408\,F^3\,T^7\,c_3 + 217\,280\,F^4\,T^7\,c_3 - 177\,408\,F^5\,T^7\,c_3 + 94\,528\,F^6\,T^7\,c_3 - 29\,568\,F^7\,T^7\,c_3 + 4032\,F^8\,T^7\,c_3 - 2916\,T^8\,c_3 + 21\,384\,F\,T^8\,c_3 - 68\,364\,F^2\,T^8\,c_3 + 128\,304\,F^3\,T^8\,c_3 - 157\,140\,F^4\,T^8\,c_3 + 128\,304\,F^5\,T^8\,c_3 - 68\,364\,F^6\,T^8\,c_3 + 21\,384\,F^7\,T^8\,c_3 - 2916\,F^8\,T^8\,c_3 + 648\,T^9\,c_3 - 4752\,F\,T^9\,c_3 + 15\,192\,F^2\,T^9\,c_3 - 28\,512\,F^3\,T^9\,c_3 + 34\,920\,F^4\,T^9\,c_3 - 28\,512\,F^5\,T^9\,c_3 + 15\,192\,F^6\,T^9\,c_3 - 4752\,F^7\,T^9\,c_3 + 648\,F^8\,T^9\,c_3 + 90\,r_{17} - 441\,F\,r_{17} + 702\,F^2\,r_{17} + 90\,F^3\,r_{17} - 2106\,F^4\,r_{17} + 3654\,F^5\,r_{17} - 3096\,F^6\,r_{17} + 1341\,F^7\,r_{17} - 234\,F^8\,r_{17} - 570\,T\,r_{17} + 2793\,F\,T\,r_{17} - 4446\,F^2\,T\,r_{17} - 570\,F^3\,T\,r_{17} + 13\,338\,F^4\,T\,r_{17} - 23\,142\,F^5\,T\,r_{17} + 19\,608\,F^6\,T\,r_{17} - 8493\,F^7\,T\,r_{17} + 1482\,F^8\,T\,r_{17} + 1450\,T^2\,r_{17} - 7105\,F\,T^2\,r_{17} + 11\,310\,F^2\,T^2\,r_{17} + 1450\,F^3\,T^2\,r_{17} - 33\,930\,F^4\,T^2\,r_{17} + 58\,870\,F^5\,T^2\,r_{17} - 49\,880\,F^6\,T^2\,r_{17} + 21\,605\,F^7\,T^2\,r_{17} - 3770\,F^8\,T^2\,r_{17} - 1850\,T^3\,r_{17} + 9065\,F\,T^3\,r_{17} - 14\,430\,F^2\,T^3\,r_{17} - 1850\,F^3\,T^3\,r_{17} + 43\,290\,F^4\,T^3\,r_{17} - 75\,110\,F^5\,T^3\,r_{17} + 63\,640\,F^6\,T^3\,r_{17} - 27\,565\,F^7\,T^3\,r_{17} + 4810\,F^8\,T^3\,r_{17} + 890\,T^4\,r_{17} - 4361\,F\,T^4\,r_{17} + 6942\,F^2\,T^4\,r_{17} + 890\,F^3\,T^4\,r_{17} - 20\,826\,F^4\,T^4\,r_{17} + 36\,134\,F^5\,T^4\,r_{17} - 30\,616\,F^6\,T^4\,r_{17} + 13\,261\,F^7\,T^4\,r_{17} - 2314\,F^8\,T^4\,r_{17} + 890\,T^5\,r_{17} - 4361\,F\,T^5\,r_{17} + 6942\,F^2\,T^5\,r_{17} + 890\,F^3\,T^5\,r_{17} - 20\,826\,F^4\,T^5\,r_{17} + 36\,134\,F^5\,T^5\,r_{17} - 30\,616\,F^6\,T^5\,r_{17} + 13\,261\,F^7\,T^5\,r_{17} - 2314\,F^8\,T^5\,r_{17} - 1850\,T^6\,r_{17} + 9065\,F\,T^6\,r_{17} - 14\,430\,F^2\,T^6\,r_{17} - 1850\,F^3\,T^6\,r_{17} + 43\,290\,F^4\,T^6\,r_{17} - 75\,110\,F^5\,T^6\,r_{17} + 63\,640\,F^6\,T^6\,r_{17} - 27\,565\,F^7\,T^6\,r_{17} + 4810\,F^8\,T^6\,r_{17} + 1450\,T^7\,r_{17} - 7105\,F\,T^7\,r_{17} + 11\,310\,F^2\,T^7\,r_{17} + 1450\,F^3\,T^7\,r_{17} - 33\,930\,F^4\,T^7\,r_{17} + 58\,870\,F^5\,T^7\,r_{17} - 49\,880\,F^6\,T^7\,r_{17} + 21\,605\,F^7\,T^7\,r_{17} - 3770\,F^8\,T^7\,r_{17} - 570\,T^8\,r_{17} + 2793\,F\,T^8\,r_{17} - 4446\,F^2\,T^8\,r_{17} - 570\,F^3\,T^8\,r_{17} + 13\,338\,F^4\,T^8\,r_{17} - 23\,142\,F^5\,T^8\,r_{17} + 19\,608\,F^6\,T^8\,r_{17} - 8493\,F^7\,T^8\,r_{17} + 1482\,F^8\,T^8\,r_{17} + 90\,T^9\,r_{17} - 441\,F\,T^9\,r_{17} + 702\,F^2\,T^9\,r_{17} + 90\,F^3\,T^9\,r_{17} - 2106\,F^4\,T^9\,r_{17} + 3654\,F^5\,T^9\,r_{17} - 3096\,F^6\,T^9\,r_{17} + 1341\,F^7\,T^9\,r_{17} - 234\,F^8\,T^9\,r_{17} + 180\,r_{29} - 1320\,F\,r_{29} + 4220\,F^2\,r_{29} - 7920\,F^3\,r_{29} + 9700\,F^4\,r_{29} - 7920\,F^5\,r_{29} + 4220\,F^6\,r_{29} - 1320\,F^7\,r_{29} + 180\,F^8\,r_{29} - 882\,T\,r_{29} + 6468\,F\,T\,r_{29} - 20\,678\,F^2\,T\,r_{29} + 38\,808\,F^3\,T\,r_{29} - 47\,530\,F^4\,T\,r_{29} + 38\,808\,F^5\,T\,r_{29} - 20\,678\,F^6\,T\,r_{29} + 6468\,F^7\,T\,r_{29} - 882\,F^8\,T\,r_{29} + 1404\,T^2\,r_{29} - 10\,296\,F\,T^2\,r_{29} + 32\,916\,F^2\,T^2\,r_{29} - 61\,776\,F^3\,T^2\,r_{29} + 75\,660\,F^4\,T^2\,r_{29} - 61\,776\,F^5\,T^2\,r_{29} + 32\,916\,F^6\,T^2\,r_{29} - 10\,296\,F^7\,T^2\,r_{29} + 1404\,F^8\,T^2\,r_{29} + 180\,T^3\,r_{29} -$$

$$1320\,F\,T^3\,r_{29}+4220\,F^2\,T^3\,r_{29}-7920\,F^3\,T^3\,r_{29}+9700\,F^4\,T^3\,r_{29}-7920\,F^5\,T^3\,r_{29}+4220\,F^6\,T^3\,r_{29}-1320\,F^7\,T^3\,r_{29}+180\,F^8\,T^3\,r_{29}-4212\,T^4\,r_{29}+30\,888\,F\,T^4\,r_{29}-98\,748\,F^2\,T^4\,r_{29}+185\,328\,F^3\,T^4\,r_{29}-226\,980\,F^4\,T^4\,r_{29}+185\,328\,F^5\,T^4\,r_{29}-98\,748\,F^6\,T^4\,r_{29}+30\,888\,F^7\,T^4\,r_{29}-4212\,F^8\,T^4\,r_{29}+7308\,T^5\,r_{29}-53\,592\,F\,T^5\,r_{29}+171\,332\,F^2\,T^5\,r_{29}-321\,552\,F^3\,T^5\,r_{29}+393\,820\,F^4\,T^5\,r_{29}-321\,552\,F^5\,T^5\,r_{29}+171\,332\,F^6\,T^5\,r_{29}-53\,592\,F^7\,T^5\,r_{29}+7308\,F^8\,T^5\,r_{29}-6192\,T^6\,r_{29}+45\,408\,F\,T^6\,r_{29}-145\,168\,F^2\,T^6\,r_{29}+272\,448\,F^3\,T^6\,r_{29}-333\,680\,F^4\,T^6\,r_{29}+272\,448\,F^5\,T^6\,r_{29}-145\,168\,F^6\,T^6\,r_{29}+45\,408\,F^7\,T^6\,r_{29}-6192\,F^8\,T^6\,r_{29}+2682\,T^7\,r_{29}-19\,668\,F\,T^7\,r_{29}+62\,878\,F^2\,T^7\,r_{29}-118\,008\,F^3\,T^7\,r_{29}+144\,530\,F^4\,T^7\,r_{29}-118\,008\,F^5\,T^7\,r_{29}+62\,878\,F^6\,T^7\,r_{29}-19\,668\,F^7\,T^7\,r_{29}+2682\,F^8\,T^7\,r_{29}-468\,T^8\,r_{29}+3432\,F\,T^8\,r_{29}-10\,972\,F^2\,T^8\,r_{29}+20\,592\,F^3\,T^8\,r_{29}-25\,220\,F^4\,T^8\,r_{29}+20\,592\,F^5\,T^8\,r_{29}-10\,972\,F^6\,T^8\,r_{29}+3432\,F^7\,T^8\,r_{29}-468\,F^8\,T^8\,r_{29}+180\,r_{35}-1320\,F\,r_{35}+4220\,F^2\,r_{35}-7920\,F^3\,r_{35}+9700\,F^4\,r_{35}-7920\,F^5\,r_{35}+4220\,F^6\,r_{35}-1320\,F^7\,r_{35}+180\,F^8\,r_{35}-882\,T\,r_{35}+6468\,F\,T\,r_{35}-20\,678\,F^2\,T\,r_{35}+38\,808\,F^3\,T\,r_{35}-47\,530\,F^4\,T\,r_{35}+38\,808\,F^5\,T\,r_{35}-20\,678\,F^6\,T\,r_{35}+6468\,F^7\,T\,r_{35}-882\,F^8\,T\,r_{35}+1404\,T^2\,r_{35}-10\,296\,F\,T^2\,r_{35}+32\,916\,F^2\,T^2\,r_{35}-61\,776\,F^3\,T^2\,r_{35}+75\,660\,F^4\,T^2\,r_{35}-61\,776\,F^5\,T^2\,r_{35}+32\,916\,F^6\,T^2\,r_{35}-10\,296\,F^7\,T^2\,r_{35}+1404\,F^8\,T^2\,r_{35}+180\,T^3\,r_{35}-1320\,F\,T^3\,r_{35}+4220\,F^2\,T^3\,r_{35}-7920\,F^3\,T^3\,r_{35}+9700\,F^4\,T^3\,r_{35}-7920\,F^5\,T^3\,r_{35}+4220\,F^6\,T^3\,r_{35}-1320\,F^7\,T^3\,r_{35}+180\,F^8\,T^3\,r_{35}-4212\,T^4\,r_{35}+30\,888\,F\,T^4\,r_{35}-98\,748\,F^2\,T^4\,r_{35}+185\,328\,F^3\,T^4\,r_{35}-226\,980\,F^4\,T^4\,r_{35}+185\,328\,F^5\,T^4\,r_{35}-98\,748\,F^6\,T^4\,r_{35}+30\,888\,F^7\,T^4\,r_{35}-4212\,F^8\,T^4\,r_{35}+7308\,T^5\,r_{35}-53\,592\,F\,T^5\,r_{35}+171\,332\,F^2\,T^5\,r_{35}-321\,552\,F^3\,T^5\,r_{35}+393\,820\,F^4\,T^5\,r_{35}-321\,552\,F^5\,T^5\,r_{35}+171\,332\,F^6\,T^5\,r_{35}-53\,592\,F^7\,T^5\,r_{35}+7308\,F^8\,T^5\,r_{35}-6192\,T^6\,r_{35}+45\,408\,F\,T^6\,r_{35}-145\,168\,F^2\,T^6\,r_{35}+272\,448\,F^3\,T^6\,r_{35}-333\,680\,F^4\,T^6\,r_{35}+272\,448\,F^5\,T^6\,r_{35}-145\,168\,F^6\,T^6\,r_{35}+45\,408\,F^7\,T^6\,r_{35}-6192\,F^8\,T^6\,r_{35}+2682\,T^7\,r_{35}-19\,668\,F\,T^7\,r_{35}+62\,878\,F^2\,T^7\,r_{35}-118\,008\,F^3\,T^7\,r_{35}+144\,530\,F^4\,T^7\,r_{35}-118\,008\,F^5\,T^7\,r_{35}+62\,878\,F^6\,T^7\,r_{35}-19\,668\,F^7\,T^7\,r_{35}+2682\,F^8\,T^7\,r_{35}-468\,T^8\,r_{35}+3432\,F\,T^8\,r_{35}-10\,972\,F^2\,T^8\,r_{35}+20\,592\,F^3\,T^8\,r_{35}-25\,220\,F^4\,T^8\,r_{35}+20\,592\,F^5\,T^8\,r_{35}-10\,972\,F^6\,T^8\,r_{35}+3432\,F^7\,T^8\,r_{35}-468\,F^8\,T^8\,r_{35}-594\,F\,r_{37}+3798\,F^2\,r_{37}-10\,692\,F^3\,r_{37}+17\,460\,F^4\,r_{37}-17\,820\,F^5\,r_{37}+11\,394\,F^6\,r_{37}-4158\,F^7\,r_{37}+648\,F^8\,r_{37}+594\,T\,r_{37}-594\,F\,T\,r_{37}-10\,128\,F^2\,T\,r_{37}+41\,580\,F^3\,T\,r_{37}-78\,570\,F^4\,T\,r_{37}+86\,724\,F^5\,T\,r_{37}-58\,236\,F^6\,T\,r_{37}+21\,978\,F^7\,T\,r_{37}-3510\,F^8\,T\,r_{37}-3204\,T^2\,r_{37}+13\,926\,F\,T^2\,r_{37}-13\,926\,F^2\,T^2\,r_{37}-31\,284\,F^3\,T^2\,r_{37}+108\,640\,F^4\,T^2\,r_{37}-146\,124\,F^5\,T^2\,r_{37}+108\,454\,F^6\,T^2\,r_{37}-43\,494\,F^7\,T^2\,r_{37}+7236\,F^8\,T^2\,r_{37}+6894\,T^3\,r_{37}-38\,346\,F\,T^3\,r_{37}+83\,556\,F^2\,T^3\,r_{37}-83\,556\,F^3\,T^3\,r_{37}+12\,610\,F^4\,T^3\,r_{37}+62\,964\,F^5\,T^3\,r_{37}-72\,584\,F^6\,T^3\,r_{37}+34\,914\,F^7\,T^3\,r_{37}-6426\,F^8\,T^3\,r_{37}-6768\,T^4\,r_{37}+43\,758\,F\,T^4\,r_{37}-121\,114\,F^2\,T^4\,r_{37}+192\,060\,F^3\,T^4\,r_{37}-192\,060\,F^4\,T^4\,r_{37}+121\,572\,F^5\,T^4\,r_{37}-45\,998\,F^6\,T^4\,r_{37}+8514\,F^7\,T^4\,r_{37}-360\,F^8\,T^4\,r_{37}+360\,T^5\,r_{37}-8514\,F\,T^5\,r_{37}+45\,998\,F^2\,T^5\,r_{37}-121\,572\,F^3\,T^5\,r_{37}+192\,060\,F^4\,T^5\,r_{37}-192\,060\,F^5\,T^5\,r_{37}+121\,114\,F^6\,T^5\,r_{37}-43\,758\,F^7\,T^5\,r_{37}+6768\,F^8\,T^5\,r_{37}+6426\,T^6\,r_{37}-34\,914\,F\,T^6\,r_{37}+72\,584\,F^2\,T^6\,r_{37}-62\,964\,F^3\,T^6\,r_{37}-12\,610\,F^4\,T^6\,r_{37}+83\,556\,F^5\,T^6\,r_{37}-83\,556\,F^6\,T^6\,r_{37}+38\,346\,F^7\,T^6\,r_{37}-6894\,F^8\,T^6\,r_{37}-7236\,T^7\,r_{37}+43\,494\,F\,T^7\,r_{37}-108\,454\,F^2\,T^7\,r_{37}+146\,124\,F^3\,T^7\,r_{37}-108\,640\,F^4\,T^7\,r_{37}+31\,284\,F^5\,T^7\,r_{37}+13\,926\,F^6\,T^7\,r_{37}-13\,926\,F^7\,T^7\,r_{37}+3204\,F^8\,T^7\,r_{37}+3510\,T^8\,r_{37}-21\,978\,F\,T^8\,r_{37}+58\,236\,F^2\,T^8\,r_{37}-86\,724\,F^3\,T^8\,r_{37}+78\,570\,F^4\,T^8\,r_{37}-41\,580\,F^5\,T^8\,r_{37}+10\,128\,F^6\,T^8\,r_{37}+594\,F^7\,T^8\,r_{37}-594\,F^8\,T^8\,r_{37}-648\,T^9\,r_{37}+4158\,F\,T^9\,r_{37}-11\,394\,F^2\,T^9\,r_{37}+17\,820\,F^3\,T^9\,r_{37}-17\,460\,F^4\,T^9\,r_{37}+10\,692\,F^5\,T^9\,r_{37}-3798\,F^6\,T^9\,r_{37}+594\,F^7\,T^9\,r_{37}\Big)\Big\}$$

In[ ]:=
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Select[Tally[ρ11na], #[[2]] > 1 &]
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Out[•]=

$$\left\{\left\{\left\{1, -\frac{2\,(-1+T)^2\,\left(1+T^4\right)}{T^3}\right\}, 2\right\}, \left\{\left\{-\frac{2-10\,T+20\,T^2-25\,T^3+20\,T^4-10\,T^5+2\,T^6}{T^3},\right.\right.\right.$$
$$\left.\left.\left.-\frac{(-1+T)^2\,\left(9-74\,T+248\,T^2-514\,T^3+768\,T^4-868\,T^5+768\,T^6-514\,T^7+248\,T^8-74\,T^9+9\,T^{10}\right)}{T^6}\right\}, 2\right\},$$
$$\left\{\left\{-\frac{1-4\,T+8\,T^2-13\,T^3+15\,T^4-13\,T^5+8\,T^6-4\,T^7+T^8}{T^4}, -\frac{1}{T^8}(-1+T)^2\,\left(1-6\,T+19\,T^2-42\,T^3+\right.\right.\right.$$
$$\left.\left.\left.64\,T^4-76\,T^5+85\,T^6-84\,T^7+85\,T^8-76\,T^9+64\,T^{10}-42\,T^{11}+19\,T^{12}-6\,T^{13}+T^{14}\right)\right\}, 2\right\},$$
$$\left\{\left\{\frac{\left(2-2\,T+T^2\right)\,\left(1-2\,T+2\,T^2\right)}{T^2}, -\frac{(-1+T)^2\,\left(1-3\,T+T^2\right)\,\left(1-T+T^2\right)\,\left(3-4\,T+3\,T^2\right)}{T^4}\right\}, 2\right\},$$
$$\left\{\left\{\frac{2-8\,T+18\,T^2-23\,T^3+18\,T^4-8\,T^5+2\,T^6}{T^3},\right.\right.$$
$$\left.\left.-\frac{(-1+T)^2\,\left(5-30\,T+113\,T^2-264\,T^3+438\,T^4-512\,T^5+438\,T^6-264\,T^7+113\,T^8-30\,T^9+5\,T^{10}\right)}{T^6}\right\}, 2\right\},$$
$$\left\{\left\{\frac{1-4\,T+8\,T^2-9\,T^3+9\,T^4-9\,T^5+8\,T^6-4\,T^7+T^8}{T^4}, -\frac{1}{T^8}(-1+T)^2\,\left(2-12\,T+36\,T^2-64\,T^3+\right.\right.\right.$$
$$\left.\left.\left.83\,T^4-88\,T^5+93\,T^6-92\,T^7+93\,T^8-88\,T^9+83\,T^{10}-64\,T^{11}+36\,T^{12}-12\,T^{13}+2\,T^{14}\right)\right\}, 2\right\},$$
$$\left\{\left\{\frac{\left(1-T+T^2\right)^2}{T^2}, \frac{4\,(-1+T)^2\,\left(1-T+T^2\right)^2}{T^3}\right\}, 2\right\}, \left\{\left\{-\frac{2-6\,T+7\,T^2-6\,T^3+2\,T^4}{T^2},\right.\right.$$
$$\left.\left.\left.-\frac{(-1+T)^2\,\left(7-28\,T+50\,T^2-60\,T^3+50\,T^4-28\,T^5+7\,T^6\right)}{T^4}\right\}, 2\right\}\right\}$$

In[•]:=
```
AllKnots[{3, 12}] // Length
```

Out[•]=

2977

In[•]:=
```
ρ12 = ρ /@ AllKnots[{3, 12}];
```

In[•]:=
```
ρρ12 = Factor[ρρ /@ AllKnots[{3, 12}]];
```

KnotTheory: Loading precomputed data in KnotTheory/12A.dts.

KnotTheory: Loading precomputed data in KnotTheory/12N.dts.

In[•]:=
```
Tally[ρ12] // Length
Tally[ρρ12] // Length
```

Out[•]=

2882

Out[•]=

2882