Pensieve header: Comparing exponentials of q-commuting variables.

In[∘]:= **Series[$e^{\beta e^{n\epsilon} X_1}$, {ε, 0, 4}]**

Out[∘]=

$$e^{\beta X_1} + e^{\beta X_1} n \beta X_1 \epsilon + \frac{1}{2} e^{\beta X_1} n^2 \beta X_1 (1 + \beta X_1) \epsilon^2 + \frac{1}{6} e^{\beta X_1} n^3 \beta X_1 \left(1 + 3 \beta X_1 + \beta^2 X_1^2\right) \epsilon^3 + \frac{1}{24} e^{\beta X_1} n^4 \beta X_1 \left(1 + 7 \beta X_1 + 6 \beta^2 X_1^2 + \beta^3 X_1^3\right) \epsilon^4 + O[\epsilon]^5$$

In[∘]:= **Series[$\frac{\alpha^n}{n!} e^{\beta e^{n\epsilon} X_1} X_2^n$, {ε, 0, 4}]**

Out[∘]=

$$\frac{e^{\beta X_1} \alpha^n X_2^n}{n!} + \frac{e^{\beta X_1} n \alpha^n \beta X_1 X_2^n \epsilon}{n!} + \frac{e^{\beta X_1} n^2 \alpha^n \beta X_1 (1 + \beta X_1) X_2^n \epsilon^2}{2\, n!} + \frac{e^{\beta X_1} n^3 \alpha^n \beta X_1 \left(1 + 3 \beta X_1 + \beta^2 X_1^2\right) X_2^n \epsilon^3}{6\, n!} + \frac{e^{\beta X_1} n^4 \alpha^n \beta X_1 \left(1 + 7 \beta X_1 + 6 \beta^2 X_1^2 + \beta^3 X_1^3\right) X_2^n \epsilon^4}{24\, n!} + O[\epsilon]^5$$

In[∘]:= **Collect[Sum[Normal@Series[$\frac{\alpha^n}{n!} e^{\beta e^{n\epsilon} X_1} X_2^n$, {ε, 0, 8}], {n, 0, ∞}], ε, Simplify]**

Out[∘]=

$$e^{\beta X_1 + \alpha X_2} + e^{\beta X_1 + \alpha X_2} \alpha \beta \epsilon X_1 X_2 + \frac{1}{2} e^{\beta X_1 + \alpha X_2} \alpha \beta \epsilon^2 X_1 (1 + \beta X_1) X_2 (1 + \alpha X_2) +$$
$$\frac{1}{6} e^{\beta X_1 + \alpha X_2} \alpha \beta \epsilon^3 X_1 \left(1 + 3 \beta X_1 + \beta^2 X_1^2\right) X_2 \left(1 + 3 \alpha X_2 + \alpha^2 X_2^2\right) +$$
$$\frac{1}{24} e^{\beta X_1 + \alpha X_2} \alpha \beta \epsilon^4 X_1 \left(1 + 7 \beta X_1 + 6 \beta^2 X_1^2 + \beta^3 X_1^3\right) X_2 \left(1 + 7 \alpha X_2 + 6 \alpha^2 X_2^2 + \alpha^3 X_2^3\right) + \frac{1}{120} e^{\beta X_1 + \alpha X_2} \alpha \beta \epsilon^5 X_1 \left(1 + 15 \beta X_1 + 25 \beta^2 X_1^2 + 10 \beta^3 X_1^3 + \beta^4 X_1^4\right) X_2 \left(1 + 15 \alpha X_2 + 25 \alpha^2 X_2^2 + 10 \alpha^3 X_2^3 + \alpha^4 X_2^4\right) +$$
$$\frac{1}{720} e^{\beta X_1 + \alpha X_2} \alpha \beta \epsilon^6 X_1 \left(1 + 31 \beta X_1 + 90 \beta^2 X_1^2 + 65 \beta^3 X_1^3 + 15 \beta^4 X_1^4 + \beta^5 X_1^5\right) X_2 \left(1 + 31 \alpha X_2 + 90 \alpha^2 X_2^2 + 65 \alpha^3 X_2^3 + 15 \alpha^4 X_2^4 + \alpha^5 X_2^5\right) +$$
$$\frac{1}{5040} e^{\beta X_1 + \alpha X_2} \alpha \beta \epsilon^7 X_1 \left(1 + 63 \beta X_1 + 301 \beta^2 X_1^2 + 350 \beta^3 X_1^3 + 140 \beta^4 X_1^4 + 21 \beta^5 X_1^5 + \beta^6 X_1^6\right) X_2 \left(1 + 63 \alpha X_2 + 301 \alpha^2 X_2^2 + 350 \alpha^3 X_2^3 + 140 \alpha^4 X_2^4 + 21 \alpha^5 X_2^5 + \alpha^6 X_2^6\right) +$$
$$\frac{1}{40320} e^{\beta X_1 + \alpha X_2} \alpha \beta \epsilon^8 X_1 \left(1 + 127 \beta X_1 + 966 \beta^2 X_1^2 + 1701 \beta^3 X_1^3 + 1050 \beta^4 X_1^4 + 266 \beta^5 X_1^5 + 28 \beta^6 X_1^6 + \beta^7 X_1^7\right) X_2 \left(1 + 127 \alpha X_2 + 966 \alpha^2 X_2^2 + 1701 \alpha^3 X_2^3 + 1050 \alpha^4 X_2^4 + 266 \alpha^5 X_2^5 + 28 \alpha^6 X_2^6 + \alpha^7 X_2^7\right)$$