

$T_{\text{Obj}} = \mathbb{N}$   
 $\text{Hom}(h, s) = T_{h, s}$

Composition  $A \circ B$   
 take the bundles of B and put them into strands of A.

Habiro: Quantum  $\pi_1$  (of  $\mathbb{S}^1$ )  $\mathbb{Z}\langle \tau \rangle$   
 Idea: replace homology classes by isotopy classes (in a thickening)

$\mathbb{Z}$  - Representation variety  $G$   
 $\{ \tau_i(k) \rightarrow \mathbb{S}(e_i) \}$

Braided Hopf algebra  
 $R_{i,j} = m_{i,j}^1 \dots$   
 $\delta_{i,j,k} = m_{i,j}^1 \circ \psi_{k,1,3} \circ \phi_3 \circ \Delta_{1,2,3}$

$\psi_{i,j} = \psi_{i,j}^{\text{Hopf}}$   
 $\psi_{i,j}^{\text{Hopf}} = \psi_{i,j}^{\text{Hopf}} \circ \psi_{i,j}^{\text{Hopf}}$   
 $\psi_{i,j}^{\text{Hopf}} = \psi_{i,j}^{\text{Hopf}} \circ \psi_{i,j}^{\text{Hopf}}$

$B = \langle \text{LINE} \rangle$   $\psi(x \circ x) = -q x \circ x$   
 $\Delta(x) = x \circ 1 + 1 \circ x$   
 $S(x) = -q x$   $m: B \otimes B \rightarrow B$   
 $\Delta: B \rightarrow B \otimes B$   $B^* = \langle \text{LINE} \rangle$   
 $R: B \otimes B \rightarrow B \otimes B$   $B^* = \langle \text{LINE} \rangle$   
 $\text{Exp}(\dots) = \dots$

In AP/People/VanDerVeen/2024-02\_Visit\_to\_Toronto/GFB6@.nb:  
 The new objects:

$\$n = 1$ ; Define [

- $n\bar{1}_i = \mathbb{E}_{\{1\} \rightarrow \{1\}} [\theta, \xi_1 x_1, 1]$ ,
- $nm_{i,j \rightarrow k} = \mathbb{E}_{\{1, j\} \rightarrow \{k\}} [\theta, (\xi_i + \xi_j) x_k, 1]$ ,
- $n\tau_{i,j} = \mathbb{E}_{\{1, j\} \rightarrow \{1, j\}} [\theta, \xi_i x_1 + \xi_j x_j, 1 + \hbar \xi_i x_1 \xi_j x_j + \mathcal{O}[\hbar]^2]$ ,
- $\bar{n}\tau_{i,j} = \mathbb{E}_{\{1, j\} \rightarrow \{1, j\}} [\theta, \xi_i x_1 + \xi_j x_j, 1 - \hbar \xi_i x_1 \xi_j x_j + \mathcal{O}[\hbar]^2] (**)$ ,
- $ns_i = \mathbb{E}_{\{1\} \rightarrow \{1\}} [\theta, -\xi_i x_1, 1 + \hbar \frac{\xi_i^2 x_1^2}{2} + \mathcal{O}[\hbar]^2]$ ,
- $\bar{ns}_i = \mathbb{E}_{\{1\} \rightarrow \{1\}} [\theta, -\xi_i x_1, 1 - \hbar \frac{\xi_i^2 x_1^2}{2} + \mathcal{O}[\hbar]^2]$ ,
- $n\Delta_{i \rightarrow j, k} = \mathbb{E}_{\{1\} \rightarrow \{j, k\}} [\theta, (x_j + x_k) \xi_i, 1 + \hbar \frac{\xi_i^2 x_j x_k}{2} + \mathcal{O}[\hbar]^2]$ ,
- $n\phi_i = \mathbb{E}_{\{1\} \rightarrow \{1\}} [\theta, T \xi_i x_1, 1]$ ,
- $\bar{n}\phi_i = \mathbb{E}_{\{1\} \rightarrow \{1\}} [\theta, T^{-1} \xi_i x_1, 1]$ ,
- $n\delta_{i \rightarrow j, k} = n\Delta_{i \rightarrow 1, 2} // n\Delta_{2 \rightarrow k, 3} // ns_3 // n\phi_3 // n\tau_{k, 3} // nm_{1, 3 \rightarrow j}$ ,
- $nR_{i,j} = n\phi_j // n\delta_{i \rightarrow 1, i} // n\tau_{i, j} // nm_{1, j \rightarrow j}$ ,
- $\bar{n}R_{i,j} = n\delta_{i \rightarrow 2, 1} // \bar{n}\tau_{2, 1, j} // \bar{ns}_{2, 1} // \bar{n}\tau_{i, 2, 1} // \bar{n}\tau_{i, 2, 1} // nm_{2, 1, j} // \bar{n}\phi_j$ ,

$R(\omega) = (1-t)\omega + t_2$   
 $H_1(\tilde{D}_c) \rightarrow H_1(\tilde{D}_c)$   $\begin{pmatrix} 1+t & \\ & 1 \end{pmatrix}$

$\ell \in H_1(\tilde{D}_c)$   $\ell \otimes \omega$   
 $q: B \rightarrow B$   $x \mapsto tx$   
 $\mathbb{Z}(\dots) = \text{lid}$   $R: \dots$   
 $\text{End}(B) = H$

- $nc_i = \mathbb{E}_{\{1\} \rightarrow \{1\}} [\theta, \xi_i x_1, 1 + \xi_i x_1 \hbar + \mathcal{O}[\hbar]^2]$ ,
- $\bar{nc}_i = \mathbb{E}_{\{1\} \rightarrow \{1\}} [\theta, \xi_i x_1, 1 - \xi_i x_1 \hbar + \mathcal{O}[\hbar]^2]$ ,
- $n\text{Kink}r_i = (nR_{1,3} \bar{nc}_2 // n\mu_{1,2 \rightarrow 1} // n\mu_{1,3 \rightarrow 1})$ ,
- $n\text{Kink}l_i = (nR_{3,1} nc_2 // n\mu_{1,2 \rightarrow 1} // n\mu_{1,3 \rightarrow 1})$ ,
- $\bar{n}\text{Kink}r_i = (\bar{n}R_{3,1} \bar{nc}_2 // n\mu_{1,2 \rightarrow 1} // n\mu_{1,3 \rightarrow 1})$ ,
- $\bar{n}\text{Kink}l_i = (\bar{n}R_{1,3} nc_2 // n\mu_{1,2 \rightarrow 1} // n\mu_{1,3 \rightarrow 1})$

In Monoblog:

170317 Wikipedia:  $q$ -derivative:  $D_{q,x} f(x) = \frac{f(qx) - f(x)}{qx - x}$ ; has  $D_{q,x} e_q^x = e_q^x$  (and  $e_q^0 = 1$ ); seek it and  $e_q^x$  and  $xy = qyx$  in nature. Finds:  $[a, x] = x \Rightarrow e^{ta} x = e^t x e^{ta}$ . Also, in tensor powers with  $X_k := e^{t(a_1 + \dots + a_{k-1})} x_k$ , have  $X_k X_l = e^t X_l X_k$  for  $k < l$ .

In Notebook-2024:

$(GX \mathbb{Z} \text{ at } \mathbb{Z}\text{-level } -1) \Leftrightarrow$

Commutative by  $\phi^v$

$\langle ybx \rangle = \langle a \rangle \otimes \langle ybx \rangle$

(\*For composing tensors, leg by leg\*)  
 $n\mu_{i,j \rightarrow k} [\mathbb{E}_{d \rightarrow r} [\theta, Q, P]] := \text{Module}[\{v\},$   
 $\text{QZip}_{\{\xi_v\}} [\mathbb{E}[\theta, Q / \cdot \{ \xi_j \rightarrow \xi_v, x_i \rightarrow x_v, \xi_i \rightarrow \xi_k, x_j \rightarrow x_k \},$   
 $P / \cdot \{ \xi_j \rightarrow \xi_v, x_i \rightarrow x_v, \xi_i \rightarrow \xi_k, x_j \rightarrow x_k \}]]$   
 $] / \cdot \mathbb{E} \rightarrow \mathbb{E} (d / \cdot \{i \rightarrow \text{Nothing}, j \rightarrow k\}) \rightarrow (r / \cdot \{i \rightarrow \text{Nothing}, j \rightarrow k\})$

$nR_{i,j}$   
 $\mathbb{E}_{\{1, j\} \rightarrow \{1, j\}} [\theta, x_i \xi_i + (1-T) x_j \xi_i + T x_j \xi_j,$   
 $1 + \left( \frac{1}{2} (1-3T) x_i x_j \xi_i^2 + \frac{1}{2} (-T+T^2) x_j^2 \xi_i^2 + T x_i x_j \xi_i \xi_j \right) \hbar + \mathcal{O}[\hbar]^2]$

$n\delta_{i \rightarrow j, k}$   
 $\mathbb{E}_{\{1\} \rightarrow \{j, k\}} [\theta, (1-T) x_j \xi_i + x_k \xi_i,$   
 $1 + \left( \frac{1}{2} (-T+T^2) x_j^2 \xi_i^2 + \frac{1}{2} (1-3T) x_j x_k \xi_i^2 \right) \hbar + \mathcal{O}[\hbar]^2]$

$\tau(x_1, x_2) = q x_2 x_1$   $S(x) = q \binom{2}{1} x$   
 $x_1 A_1 x_2 = q A_1 x_2 x_1$   $S(A - Aq) = \text{same}$   
 $R_D^{-1} X_1' X_2' R_D = R_B (X_1 \otimes X_2')$