Date: 2018 · 07 · 28

Setting

Def A marked sutured manifold is a ~~manifold~~ sutured manifold M w/ labelled + arrows on R_+ (red)
- arrows on R_- (blue)

endpoints on sutures

Balanced: $\chi(R_+) = \chi(R_-)$ [Nb Different from bordered sutured]

Combed vector field $\vec{v}$ w/ $\vec{v}$ pointing out on R_+
in on R_-
from R_+ to R_- on sutures.

Framed ~~transverse~~ indep vect field $\vec{v}_x, \vec{v}_y, \vec{v}_z$

$\vec{v}_z$ a combing
+ arrows tangent to $\vec{v}_x$ } at endpoints
- arrows tangent to $\vec{v}_x$ }

Relevant in ~~sutured~~ $S^2 \neq 1$ setting.

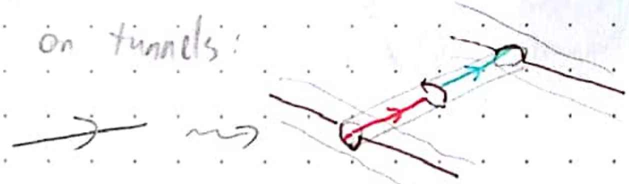
Equivalence • cut tunnels • cut on bigons] a little unclear in framed setting

Embedding virtual tangle:

Given tangle T on Σ

$M = \Sigma \times I \setminus T$

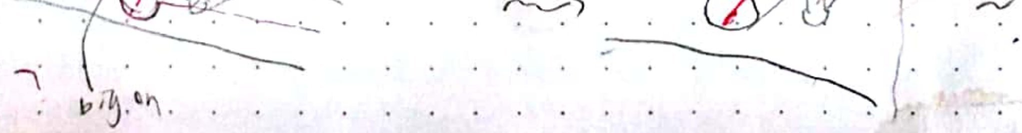
Sutures on $\partial\Sigma \times \{\tfrac{1}{2}\}$ and on tunnels:



~~Framed 3-manifolds are kind of complicated.~~
~~Eg have (at least) 3 different Chern classes, coming from 3 different 2-plane fields: c_{xy}, c_{yz}, c_{zx}~~
~~Maybe a constraint like $c_{xy} + c_{yz} + c_{zx} = 0$.~~
~~Also have more info, like ~~framing~~ combing.~~

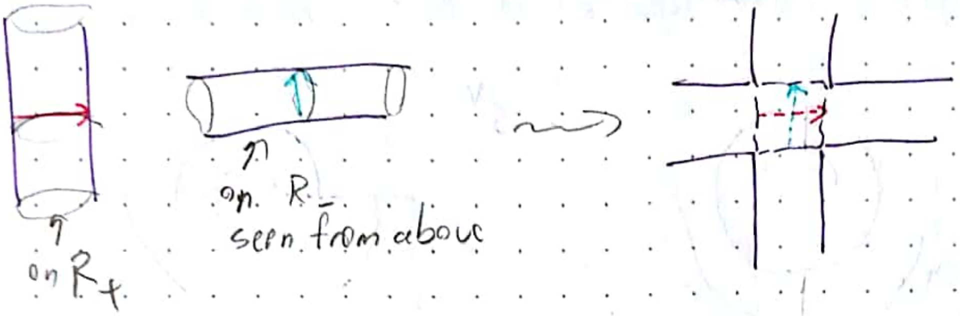
Nope: for combing v_x, $c_1(v_x^\perp) = 0$, since it has a non-zero section.

If tangle is an $\cap$ then $\cup$ tangle, can simplify:



This is hard to draw...
will be harder to get the framings right..

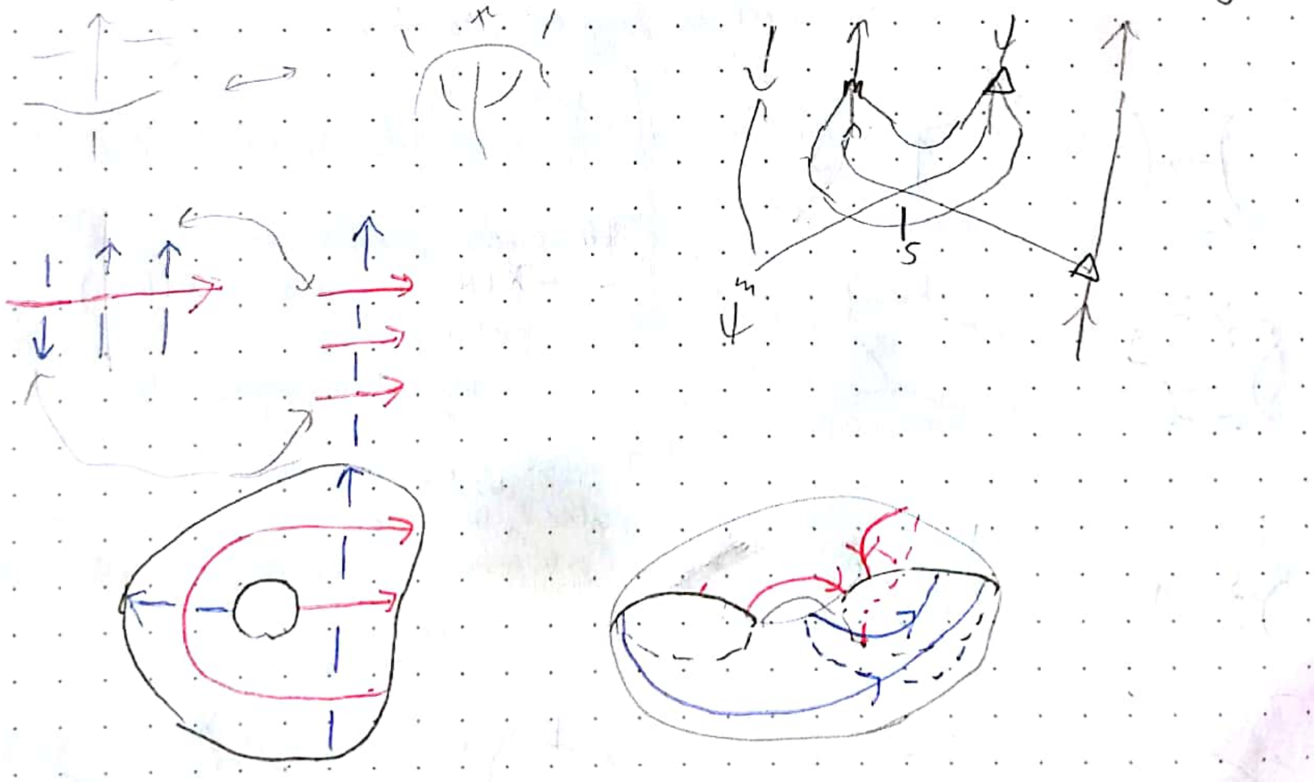
Gluings: Attach like so:



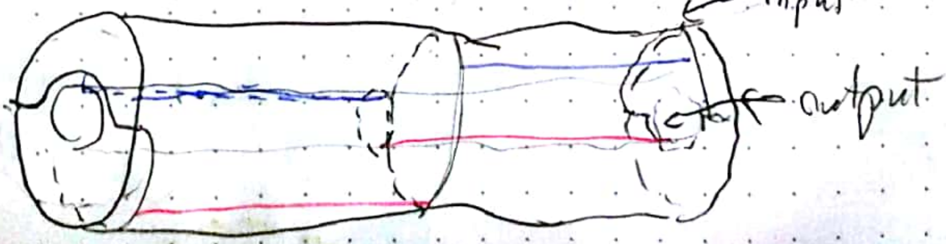
Basic objects: Drawn earlier

Drinfeld Double

Basically want to reverse a tube - just like doing S



What do I want? Some gluing that switches "red"/"s"

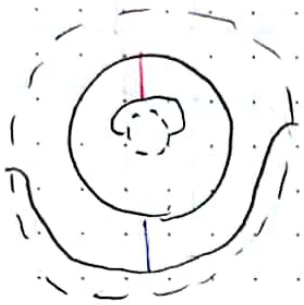


Need a "duality" operation

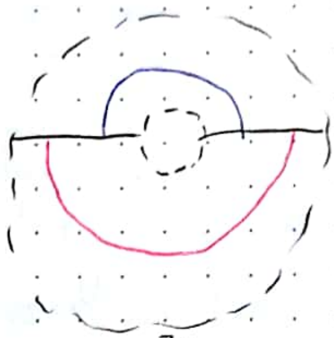
Given a marked subsurface S, find another surface $S^\vee$ that gives the effect of gluing with S. $S^{\vee\vee} = -S \rightsquigarrow$ reverse all arrows

In this example, need both + & − markings.

subsurface S:



$S^\vee$

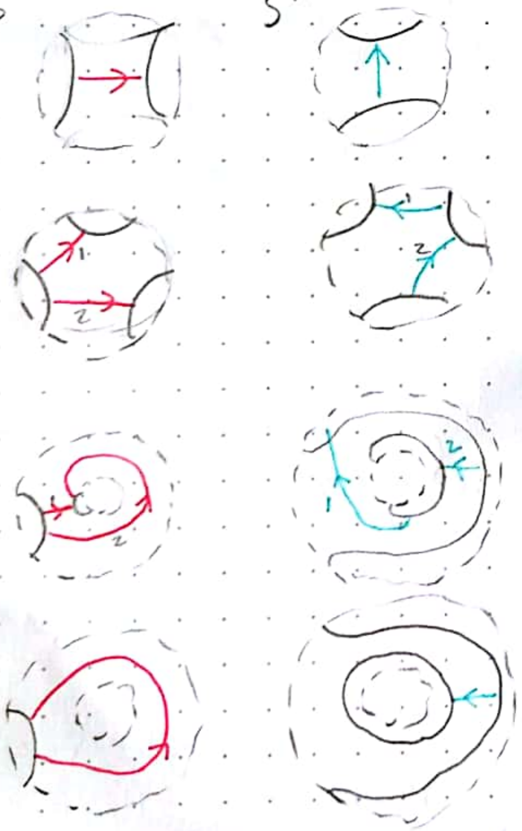


(maybe?)

This would give the result...

What's the rule? Which surfaces are fully parametrized?
- probably: spine of the surface & sutures touch each boundary?

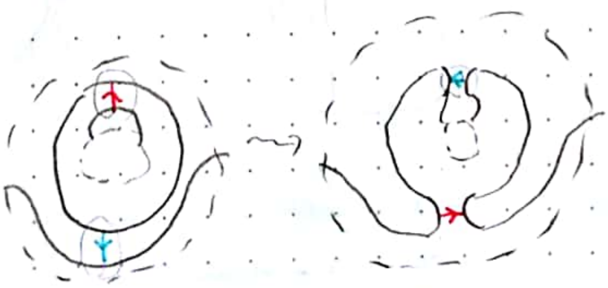
S $S^\vee$



What happens to χ?
- should switch $\chi(R_+)$, $\chi(R_-)$

How many arcs do you need?
- $-\chi(R_+)$ and $-\chi(R_-)$ probably (counting ∂ appropriately).

I guess answer is clear: replace nbhd of each arc by a dual arc (presumably getting another spine)



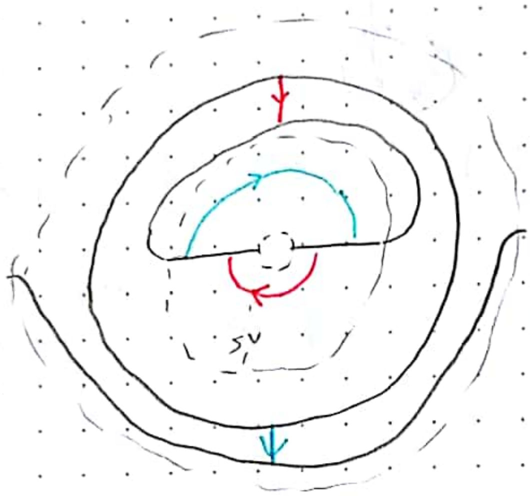
orientations seem to be coming out wrong..

Overall: For representing tangles, on tunnels, put following sutures:

$S_1 =$ 

"Dualizing" operation (for Drinfeld double)

is $S_1^\vee \cup S_2$ filled by solid torus



Dotted circles bounding circles (as $S_2 \times I$).

↳ this matches! yay.

Q. Do you get every sutured, marked 3-manifold?

- What's a more standard representation? Heegaard ~~surface~~ diagram?

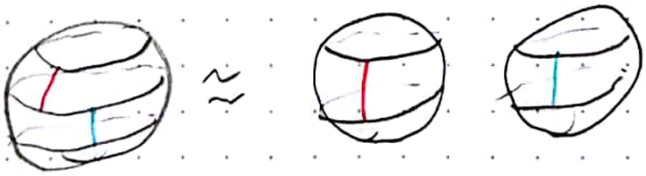
Take $\Sigma \times I$, attach α and β handles (equal #s to be balanced)

can you arrange for marking to be disjoint from attaching circles? Probably.

So then Represent (markings) $\cup$ (α circles) $\cup$ (β circles)

as a virtual^{ou} tangle

Attach a gadget to α-β pairs (picked arbitrarily)

Greg's gadget: 

so, yes.