

(Proposed) Agenda. Using Århus-like techniques, construct a map $Z: \mathcal{T}_{vous} \rightarrow \mathcal{A}_{vous}$, where \mathcal{T}_{vous} is the space of VOUS-tangles: Virtual tangles with only Over or Under strands, some labeled as Surgery strands, with a non-singular linking matrix between the surgery strands, modulo acyclic Reidemeister 2 moves and Kirby slide relations, and where \mathcal{A}_{vous} is some space of arrow diagrams modulo appropriate relations. The construction will either fix the definitions of \mathcal{T}_{vous} and \mathcal{A}_{vous} or will allow some flexibility that will be fixed so that the following will hold true:

1. \mathcal{T}_{vous} should have a clearer topological interpretation, perhaps in terms of Heegaard diagrams.
2. \mathcal{A}_{vous} should pair with some kind of Lie bialgebras.
3. \mathcal{A}_{vous} should be the associated graded of \mathcal{T}_{vous} and Z should be an expansion.
4. Ordinary tangles \mathcal{T}_{ord} and ordinary virtual tangles \mathcal{T}_{v-ord} should map into \mathcal{T}_{vous} , and when viewed on $\mathcal{T}_{(v-)ord}$, the invariant Z should explain the Drinfel'd double construction.

It may be better to first construct a Z and only later worry about the numbered properties. Yet property 4 has stand-alone topological content which may be very interesting: \mathcal{T}_{vous} is a space with an $R3$ -free presentation and which contains $\mathcal{T}_{(v-)ord}$, at least nearly faithfully. What does it mean? To what extent does it make $R3$ superfluous in knot theory?

As for constructing Z , the first step should be a $Z: \mathcal{T}_{vou} \rightarrow \mathcal{A}_{vou}$ (no surgery), which would have a prescribed behaviour on strand-doubling.