The testing of the 4 methods' effectiveness, on the 200th knot in Dunfield' s list of knots .

In[ ]:= **<< KnotTheory`**

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.

In[ ]:= **Ks = ReadList["http://drorbn.net/AcademicPensieve/People/Dunfield/nmd_random_knots"]**

Out[ ]=

```
{PD[X[3, 1, 4, 0], X[5, 3, 0, 2], X[1, 5, 2, 4]], ⟨ … 996 … ⟩,
 PD[X[555, 537, 556, 536], X[1662, 1693, 1663, 1694], X[422, 426, 423, 425],
   X[1962, 1888, 1963, 1887], X[760, 338, 761, 337], X[331, 790, 332, 791],
   X[783, 775, 784, 774], X[723, 709, 724, 708], X[1728, 1570, 1729, 1569],
   X[1047, 1031, 1048, 1030], X[175, 199, 176, 198], X[62, 66, 63, 65],
   X[962, 926, 963, 925], X[1150, 1155, 1151, 1156], X[611, 628, 612, 629], ⟨ … 970 … ⟩,
   X[1849, 1795, 1850, 1794], X[820, 1346, 821, 1345], X[596, 592, 597, 591],
   X[1174, 1129, 1175, 1130], X[1154, 1181, 1155, 1182], X[1489, 1505, 1490, 1504],
   X[1478, 1452, 1479, 1451], X[1453, 1702, 1454, 1703], X[956, 959, 957, 960],
   X[1429, 478, 1430, 479], X[1257, 1255, 1258, 1254], X[814, 1294, 815, 1293],
   X[1699, 1484, 1700, 1485], X[1156, 1133, 1157, 1134], X[1444, 486, 1445, 485]]}
```

large output   show less   show more   show all   set size limit...

In[ ]:= **testingKnotBig = Ks〚198〛;**

In[ ]:= **Length @ testingKnotBig**

Out[ ]= 200

In[ ]:= **Timing[bestGreedy[testingKnotBig, {factor → JonesResult, tries → 1000}]]〚1〛**

Out[ ]= 60.8594

■ This is the unparallelized version, comparing the run time with the 100 crossing knot, the estimate is that it will take about 20 seconds.

In[ ]:= **unselTableBig = Flatten[Table[bestGreedySamples[PD @@ testingKnotBig,**
      {factor → JonesResult, neededSamples → 1, tries → 1000}], {i, 50}], 1];

In[ ]:= **unselTableJonesBig = JonesResult[#] & /@ unselTableBig**

Out[ ]= {20 643 678, 17 737 092, 14 487 716, 10 274 132, 11 666 702, 18 323 248, 11 277 284,
   8 937 148, 20 876 540, 15 112 114, 18 408 240, 16 735 460, 18 132 226, 17 624 460,
   9 555 582, 13 782 588, 15 512 144, 10 826 198, 13 780 290, 6 309 212, 15 597 528, 9 168 216,
   19 701 188, 13 002 066, 21 570 700, 20 116 094, 18 696 238, 19 602 358, 17 641 584,
   11 638 034, 16 755 692, 16 621 406, 6 746 568, 16 725 952, 14 165 042, 13 257 714,
   15 122 284, 13 996 470, 16 239 004, 15 694 454, 10 891 276, 17 666 734, 14 459 002,
   14 619 618, 8 964 022, 13 338 504, 14 900 492, 14 193 440, 22 208 018, 11 914 044}

In[ ]:= **Mean[unselTableJonesBig] * RunTimeConstant**

Out[ ]= 193.756

- Estimation of Run Time, similar to ModelRunTime
- LM

In[ ]:= `Timing[localOptimization[PD @@ #, {startPt → convertToList,`
`        invariantCriteria → JonesOptimize, optSize → 6}] & /@ unselTableBig][[1]]`

Out[ ]= 366.594

In[ ]:= `LOunselTableBig = localOptimization[PD @@ #, {startPt → convertToList,`
`        invariantCriteria → JonesOptimize, optSize → 6}] & /@ unselTableBig;`

In[ ]:= `LOJonesBig = JonesResult[#] & /@ LOunselTableBig;`

In[ ]:= `Mean[LOJonesBig] * RunTimeConstant`

Out[ ]= 105.708

In[ ]:= **105.71 + 366.59 / 50**

Out[ ]= 113.042

In[ ]:= `Timing[localOptimization[PD @@ #, {startPt → convertToList,`
`        invariantCriteria → JonesOptimize, optSize → 5}] & /@ unselTableBig][[1]]`

Out[ ]= 49.8906

In[ ]:= `LOunselTableSmallBig = localOptimization[PD @@ #, {startPt → convertToList,`
`        invariantCriteria → JonesOptimize, optSize → 5}] & /@ unselTableBig;`

In[ ]:= `LOJonesSmallBig = JonesResult[#] & /@ LOunselTableSmallBig;`

In[ ]:= `Mean[LOJonesSmallBig] * RunTimeConstant`

Out[ ]= 118.146

In[ ]:= **118.1457394 + 49.890625` / 50**

Out[ ]= 119.144

- PLM

In[ ]:= `Timing[partialLO[PD @@ #,`
`        {startPt → convertToList, invariantCriteria → PartialJonesOptimize,`
`         optSize → 6, partialPortion → 0.1, rounds → 10}] & /@ unselTableBig][[1]]`

Out[ ]= 398.109

In[ ]:= `PLOunselTableBig = partialLO[PD @@ #,`
`        {startPt → convertToList, invariantCriteria → PartialJonesOptimize,`
`         optSize → 6, partialPortion → 0.1, rounds → 10}] & /@ unselTableBig;`

In[ ]:= `PLOJonesBig = JonesResult[#] & /@ PLOunselTableBig;`

In[ ]:= `Mean[PLOJonesBig] * RunTimeConstant`

Out[ ]= 86.472

In[●]:= **86.472 + 398.109375 / 50**

Out[●]= 94.4342

In[●]:= **Timing[partialLO[PD @@ #,**
**   {startPt → convertToList, invariantCriteria → PartialJonesOptimize,**
**   optSize → 5, partialPortion → 0.1, rounds → 10}] & /@ unselTableBig][[1]]**

Out[●]= 72.1875

In[●]:= **PLOunselTableSmallBig = partialLO[PD @@ #,**
**   {startPt → convertToList, invariantCriteria → PartialJonesOptimize,**
**   optSize → 5, partialPortion → 0.1, rounds → 10}] & /@ unselTableBig;**

In[●]:= **PLOJonesSmallBig = JonesResult[#] & /@ PLOunselTableSmallBig;**

In[●]:= **Mean[PLOJonesSmallBig] * RunTimeConstant**

Out[●]= 103.304

In[●]:= **103.30386716` + 72.1875` / 50**

Out[●]= 104.748

In[●]:= **Timing[partialLO[PD @@ #,**
**   {startPt → convertToList, invariantCriteria → PartialJonesOptimize,**
**   optSize → 5, partialPortion → 0.2, rounds → 5}] & /@ unselTableBig][[1]]**

Out[●]= 62.7344

In[●]:= **PLOunselTableSmallModBig = partialLO[PD @@ #,**
**   {startPt → convertToList, invariantCriteria → PartialJonesOptimize,**
**   optSize → 5, partialPortion → 0.2, rounds → 5}] & /@ unselTableBig;**

In[●]:= **PLOJonesSmallModBig = JonesResult[#] & /@ PLOunselTableSmallModBig;**

In[●]:= **Mean[PLOJonesSmallModBig] * RunTimeConstant**

Out[●]= 97.4728

In[●]:= **97.47 + 62.73 / 50**

Out[●]= 98.7246

■ RLM

In[●]:= **Timing[reverseLocalOptimization[PD @@ #, {startPt → convertToList,**
**   invariantCriteria → JonesOptimize, optSize → 6}] & /@ unselTableBig][[1]]**

Out[●]= 430.141

In[●]:= **RLOunselTableBig = reverseLocalOptimization[PD @@ #, {startPt → convertToList,**
**   invariantCriteria → JonesOptimize, optSize → 6}] & /@ unselTableBig;**

In[●]:= **RLOJonesBig = JonesResult[#] & /@ RLOunselTableBig;**

In[ ]:= **Mean[RLOJonesBig] * RunTimeConstant**

Out[ ]= 120.666

In[ ]:= **120.66 + 430.141 / 50**

Out[ ]= 129.263

In[ ]:= **Timing[reverseLocalOptimization[PD @@ #, {startPt → convertToList,**
**   invariantCriteria → JonesOptimize, optSize → 5}] & /@ unselTableBig][[1]]**

Out[ ]= 125.688

In[ ]:= **RLOunselTableSmallBig = reverseLocalOptimization[PD @@ #, {startPt → convertToList,**
**   invariantCriteria → JonesOptimize, optSize → 5}] & /@ unselTableBig;**

In[ ]:= **RLOJonesSmallBig = JonesResult[#] & /@ RLOunselTableSmallBig;**

In[ ]:= **Mean[RLOJonesSmallBig] * RunTimeConstant**

Out[ ]= 133.617

In[ ]:= **133.61722867999998` + 125.6875` / 50**

Out[ ]= 136.131

▪ RPLM

In[ ]:= **Protect[startPt, invariantCriteria, optSize, partialPortion, rounds];**

In[ ]:= **Options[backwardsPLO] :=**
**   {startPt → convertToList, invariantCriteria → PartialJonesOptimize,**
**   optSize → 3, partialPortion → 0.1, rounds → 3};**

```
In[ ]:= backwardsPLO[pd_PD, opts : OptionsPattern[backwardsPLO]] :=
         Module[{s, k, sBest, optimizeList, optimizeResult, widths, width, round},
           s = OptionValue[startPt][pd];
           k = Length[s] - OptionValue[optSize] + 1;
           round = 1;
           sBest = s;
           While[round ≤ OptionValue[rounds],
            widths = FoldList[Complement[#1 ⋃ #2, #1 ⋂ #2] &, {}, List @@@ sBest];
            While[k ≥ 1,
             optimizeList = sBest[[k ;; k + OptionValue[optSize] - 1]];
             width = widths[[k]];
             optimizeResult = OptionValue[invariantCriteria][
               width, optimizeList, k, OptionValue[partialPortion]];
             sBest = Join[If[k == 1, {}, sBest[[1 ;; k - 1]]], optimizeResult,
               If[k == Length[s] - OptionValue[optSize] + 1,
                {}, sBest[[k + OptionValue[optSize] ;; -1]]]];
             k = k - 1;
            ];
            round = round + 1;
            k = Length[s] - OptionValue[optSize] + 1;
           ];
           sBest]

In[ ]:= Timing[backwardsPLO[PD @@ #,
          {startPt → convertToList, invariantCriteria → PartialJonesOptimize,
           optSize → 6, partialPortion → 0.1, rounds → 10}] & /@ unselTableBig][[1]]

Out[ ]= 379.

In[ ]:= BPLOunselTableBig = backwardsPLO[PD @@ #,
          {startPt → convertToList, invariantCriteria → PartialJonesOptimize,
           optSize → 6, partialPortion → 0.1, rounds → 10}] & /@ unselTableBig;

In[ ]:= BPLOJonesBig = JonesResult[#] & /@ BPLOunselTableBig;

In[ ]:= Mean[BPLOJonesBig] * RunTimeConstant

Out[ ]= 80.7753

In[ ]:= 80.77528264` + 379 / 50

Out[ ]= 88.3553

In[ ]:= Timing[backwardsPLO[PD @@ #,
          {startPt → convertToList, invariantCriteria → PartialJonesOptimize,
           optSize → 5, partialPortion → 0.1, rounds → 10}] & /@ unselTableBig][[1]]

Out[ ]= 65.375

In[ ]:= BPLOunselTableSmallBig = backwardsPLO[PD @@ #,
          {startPt → convertToList, invariantCriteria → PartialJonesOptimize,
           optSize → 5, partialPortion → 0.1, rounds → 10}] & /@ unselTableBig;
```

In[ ]:= **BPLOJonesSmallBig = JonesResult[#] & /@ BPLOunselTableSmallBig;**

In[ ]:= **Mean[BPLOJonesSmallBig] * RunTimeConstant**

Out[ ]= 92.3882

In[ ]:= **92.38820032` + 65.375` / 50**

Out[ ]= 93.6957

In[ ]:= **Timing[backwardsPLO[PD @@ #,**
    {startPt → convertToList, invariantCriteria → PartialJonesOptimize,
     optSize → 5, partialPortion → 0.2, rounds → 5}] & /@ unselTableBig][[1]]

Out[ ]= 59.5

In[ ]:= **BPLOunselTableSmallModBig = backwardsPLO[PD @@ #,**
    {startPt → convertToList, invariantCriteria → PartialJonesOptimize,
     optSize → 5, partialPortion → 0.2, rounds → 5}] & /@ unselTableBig;

In[ ]:= **BPLOJonesSmallModBig = JonesResult[#] & /@ BPLOunselTableSmallModBig;**

In[ ]:= **Mean[BPLOJonesSmallModBig] * RunTimeConstant**

Out[ ]= 95.9417

In[ ]:= **95.94167544` + 59.5` / 50**

Out[ ]= 97.1317

- And similarly, what is the actual run time estimates looking like?

In[ ]:= **unselTableBigFKB = (# /. Global`X → X) & /@ unselTableBig;**

In[ ]:= **Timing[FKB[#]][[1]] & /@ unselTableBigFKB**

Out[ ]= {191.422, 155.734, 195.844, 124.156, 160.5, 258.703, 158.797, 123.406, 183.078, 222.672,
246.172, 210.188, 181.141, 157.938, 124.531, 162.813, 199., 139.531, 170.734, 86.9375,
250.578, 136.578, 231.172, 178.531, 224.797, 253.422, 243.344, 200.516, 220.188, 131.188,
124.578, 149.094, 64.1875, 241.906, 183.016, 164.75, 200.484, 190.547, 155.422, 155.453,
151.734, 224.5, 176.594, 135.5, 123.594, 162.516, 172.984, 149.016, 295.359, 153.656}

In[ ]:= **Mean[{191.421875`, 155.734375`, 195.84375`, 124.15625`, 160.5`, 258.703125`, 158.796875`,**
123.40625`, 183.078125`, 222.671875`, 246.171875`, 210.1875`, 181.140625`, 157.9375`,
124.53125`, 162.8125`, 199.`, 139.53125`, 170.734375`, 86.9375`, 250.578125`, 136.578125`,
231.171875`, 178.53125`, 224.796875`, 253.421875`, 243.34375`, 200.515625`, 220.1875`,
131.1875`, 124.578125`, 149.09375`, 64.1875`, 241.90625`, 183.015625`, 164.75`,
200.484375`, 190.546875`, 155.421875`, 155.453125`, 151.734375`, 224.5`, 176.59375`,
135.5`, 123.59375`, 162.515625`, 172.984375`, 149.015625`, 295.359375`, 153.65625`}]

Out[ ]= 177.97

In[ ]:= **FKBTimeLOBig = Timing[FKB[#]][[1]] & /@ ((# /. Global`X → X) & /@ LOunselTableBig);**

In[ ]:= **Mean[FKBTimeLOBig]**

Out[ ]= 108.122

In[ ]:= **366.59375` / 50 + 108.121875`**

Out[ ]= 115.454

In[ ]:= **FKBTimeLOSmallBig =**
      Timing[FKB[#]]〚1〛 & /@ ((# /. Global`X → X) & /@ LOunselTableSmallBig);

In[ ]:= **Mean[FKBTimeLOSmallBig]**

Out[ ]= 117.238

In[ ]:= **117.2378125` + 49.890625` / 50**

Out[ ]= 118.236

In[ ]:= **FKBTimePLOBig = Timing[FKB[#]]〚1〛 & /@ ((# /. Global`X → X) & /@ PLOunselTableBig);**

In[ ]:= **Mean[FKBTimePLOBig]**

Out[ ]= 86.6306

In[ ]:= **86.630625` + 398.109375` / 50**

Out[ ]= 94.5928

In[ ]:= **FKBTimePLOSmallBig =**
      Timing[FKB[#]]〚1〛 & /@ ((# /. Global`X → X) & /@ PLOunselTableSmallBig);

In[ ]:= **Mean[FKBTimePLOSmallBig]**

Out[ ]= 103.982

In[ ]:= **103.9821875` + 72.1875` / 50**

Out[ ]= 105.426

In[ ]:= **FKBTimePLOSmallModBig =**
      Timing[FKB[#]]〚1〛 & /@ ((# /. Global`X → X) & /@ PLOunselTableSmallModBig);

In[ ]:= **Mean[FKBTimePLOSmallModBig]**

Out[ ]= 100.735

In[ ]:= **100.735` + 62.734375` / 50**

Out[ ]= 101.99

In[ ]:= **FKBTimeRLOBig = Timing[FKB[#]]〚1〛 & /@ ((# /. Global`X → X) & /@ RLOunselTableBig);**

In[ ]:= **Mean[FKBTimeRLOBig]**

Out[ ]= 123.032

In[ ]:= **123.03 + 430.140625` / 50**

Out[ ]= 131.633

In[ ]:= **FKBTimeRLOSmallBig =**
      Timing[FKB[#]]〚1〛 & /@ ((# /. Global`X → X) & /@ RLOunselTableSmallBig);

In[ ]:= **Mean[FKBTimeRLOSmallBig]**

Out[ ]= 132.262

In[ ]:= **132.26 + 125.6875 / 50**

Out[ ]= 134.774

In[ ]:= **FKBTimeBPLOBig = Timing[FKB[#]]〚1〛 & /@ ((# /. Global`X → X) & /@ BPLOunselTableBig);**

In[ ]:= **Mean[FKBTimeBPLOBig]**

Out[ ]= **82.74625`**

In[ ]:= **82.74625` + 379.` / 50**

Out[ ]= 90.3263

In[ ]:= **FKBTimeBPLOSmallBig =**
Timing[FKB[#]]〚1〛 & /@ ((# /. Global`X → X) & /@ BPLOunselTableSmallBig);

In[ ]:= **Mean[FKBTimeBPLOSmallBig]**

Out[ ]= 94.2606

In[ ]:= **94.260625` + 65.375` / 50**

Out[ ]= 95.5681

In[ ]:= **FKBTimeBPLOSmallModBig =**
Timing[FKB[#]]〚1〛 & /@ ((# /. Global`X → X) & /@ BPLOunselTableSmallModBig);

In[ ]:= **Mean[FKBTimeBPLOSmallModBig]**

Out[ ]= 97.5688

In[ ]:= **97.56875` + 59.5` / 50**

Out[ ]= 98.7587