```
In[•]:= δ /: δ_i_,j_ := If[i == j, 1, 0];
        LKB_i_[ξ_] := ξ /. v_j_,k_ :>
            (1 - δ_i,j-1) (1 - δ_i,k-1) (1 - δ_i,j) (1 - δ_i,k) v_j,k + δ_i,j-1 (q v_i,k + (q^2 - q) v_i,j + (1 - q) v_j,k) +
             δ_i,j (1 - δ_i,k-1) (v_i+1,k) + δ_i,k-1 (1 - δ_i,j) (q v_j,i + (1 - q) v_j,i+1 - (q^2 - q) t v_i,k) +
             δ_i,k (v_j,i+1) + δ_i,j δ_i,k-1 (-t q^2 v_j,k) // Expand

In[•]:= bas[n_] := Flatten@Table[v_i,j, {i, 1, n - 1}, {j, i + 1, n}]

In[•]:= bas[5]

Out[•]= {v_1,2, v_1,3, v_1,4, v_1,5, v_2,3, v_2,4, v_2,5, v_3,4, v_3,5, v_4,5}

In[•]:= M_i_[n_] := Module[{inp, out, z},
          Table[
           z = LKB_i[inp];
           Table[Coefficient[z, out], {out, bas[n]}],
           {inp, bas[n]}
          ]
         ]

In[•]:= M_2[3]

Out[•]= {{0, 1, 0}, {q, 1 - q, q t - q^2 t}, {0, 0, -q^2 t}}

In[•]:= iM_i_[n_] := Simplify@Inverse[M_i[n]]

In[•]:= iM_2[3] // MatrixForm

Out[•]//MatrixForm=
        ⎛ (-1+q)/q   1/q   (1-q)/q^2 ⎞
        ⎜     1      0        0     ⎟
        ⎝     0      0   -1/(q^2 t) ⎠

In[•]:= Thread[bas[3] → iM_2[3].bas[3]]

Out[•]= {v_1,2 → (-1 + q) v_1,2/q + v_1,3/q + (1 - q) v_2,3/q^2, v_1,3 → v_1,2, v_2,3 → -v_2,3/(q^2 t)}

In[•]:= iLKB_i_[n_, ξ_] := Expand[ξ /. Thread[bas[n] → iM_i[n].bas[n]]]

In[•]:= iLKB_2[3, bas[3]]

Out[•]= {v_1,2 - v_1,2/q + v_1,3/q + v_2,3/q^2 - v_2,3/q, v_1,2, -v_2,3/(q^2 t)}

In[•]:= iLKB_3[5, bas[5]] // LKB_3

Out[•]= {v_1,2, v_1,3, v_1,4, v_1,5, v_2,3, v_2,4, v_2,5, v_3,4, v_3,5, v_4,5}
```

In[•]:=
```
Thread[bas[7] → iM3[7].bas[7]]
```

Out[•]=
$$\left\{v_{1,2}\to v_{1,2},\ v_{1,3}\to \frac{(-1+q)\ v_{1,3}}{q}+\frac{v_{1,4}}{q}+\frac{(1-q)\ v_{3,4}}{q^2},\ v_{1,4}\to v_{1,3},\ v_{1,5}\to v_{1,5},\right.$$
$$v_{1,6}\to v_{1,6},\ v_{1,7}\to v_{1,7},\ v_{2,3}\to \frac{(-1+q)\ v_{2,3}}{q}+\frac{v_{2,4}}{q}+\frac{(1-q)\ v_{3,4}}{q^2},\ v_{2,4}\to v_{2,3},\ v_{2,5}\to v_{2,5},$$
$$v_{2,6}\to v_{2,6},\ v_{2,7}\to v_{2,7},\ v_{3,4}\to -\frac{v_{3,4}}{q^2\,t},\ v_{3,5}\to \frac{(-1+q)\ v_{3,4}}{q^2\,t}+\frac{(-1+q)\ v_{3,5}}{q}+\frac{v_{4,5}}{q},$$
$$v_{3,6}\to \frac{(-1+q)\ v_{3,4}}{q^2\,t}+\frac{(-1+q)\ v_{3,6}}{q}+\frac{v_{4,6}}{q},\ v_{3,7}\to \frac{(-1+q)\ v_{3,4}}{q^2\,t}+\frac{(-1+q)\ v_{3,7}}{q}+\frac{v_{4,7}}{q},$$
$$\left.v_{4,5}\to v_{3,5},\ v_{4,6}\to v_{3,6},\ v_{4,7}\to v_{3,7},\ v_{5,6}\to v_{5,6},\ v_{5,7}\to v_{5,7},\ v_{6,7}\to v_{6,7}\right\}$$

In[•]:=
```
Table[bas[7][[i]] → (iM3[7].bas[7])[[i]],
 {i, Length[bas[7]]}]
```

Out[•]=
$$\left\{v_{1,2}\to v_{1,2},\ v_{1,3}\to \frac{(-1+q)\ v_{1,3}}{q}+\frac{v_{1,4}}{q}+\frac{(1-q)\ v_{3,4}}{q^2},\ v_{1,4}\to v_{1,3},\ v_{1,5}\to v_{1,5},\right.$$
$$v_{1,6}\to v_{1,6},\ v_{1,7}\to v_{1,7},\ v_{2,3}\to \frac{(-1+q)\ v_{2,3}}{q}+\frac{v_{2,4}}{q}+\frac{(1-q)\ v_{3,4}}{q^2},\ v_{2,4}\to v_{2,3},\ v_{2,5}\to v_{2,5},$$
$$v_{2,6}\to v_{2,6},\ v_{2,7}\to v_{2,7},\ v_{3,4}\to -\frac{v_{3,4}}{q^2\,t},\ v_{3,5}\to \frac{(-1+q)\ v_{3,4}}{q^2\,t}+\frac{(-1+q)\ v_{3,5}}{q}+\frac{v_{4,5}}{q},$$
$$v_{3,6}\to \frac{(-1+q)\ v_{3,4}}{q^2\,t}+\frac{(-1+q)\ v_{3,6}}{q}+\frac{v_{4,6}}{q},\ v_{3,7}\to \frac{(-1+q)\ v_{3,4}}{q^2\,t}+\frac{(-1+q)\ v_{3,7}}{q}+\frac{v_{4,7}}{q},$$
$$\left.v_{4,5}\to v_{3,5},\ v_{4,6}\to v_{3,6},\ v_{4,7}\to v_{3,7},\ v_{5,6}\to v_{5,6},\ v_{5,7}\to v_{5,7},\ v_{6,7}\to v_{6,7}\right\}$$