$$I^2 = \langle \underbrace{(g_i - 1)(g_j - 1)}^{g_{ij}} \rangle$$

$$F - 1 = \prod_{\alpha=1}^{K} g_{i_\alpha} - 1 = \sum_{\beta=1}^{K} \prod_{\alpha=1}^{\beta-1} g_{i_\alpha} (g_{i_\beta} - 1)$$

$$(F-1)(F'-1) = \sum c_\alpha (g_{i_\alpha} - 1) d_\alpha (g_{j_\alpha} - 1)$$

$$(g-1)d = \sum e_\alpha (g_{i_\alpha} - 1)$$

$$\shortparallel$$

$$(gd - 1) - (d - 1)$$

$$\sigma((F-1)(F'-1)) = ((\sigma F - 1)(\sigma F' - 1))$$

$$\sigma g_{ij} = (\sigma g_i - 1)(\sigma g_j - 1)$$

Aug 10, 2020.

$$g_{Kj} g_i = (g_K - 1)(g_j - 1) g_i = (g_K - 1)(g_j g_i - g_i)$$

$$= (g_K - 1)(g_j g_i - 1 - (g_i - 1))$$

$$= (g_K - 1)(g_j (g_i - 1) + g_j - 1) - (g_i - 1))$$

$$= (g_K - 1) g_j (g_i - 1) + g_{Kj} - g_{Ki}$$

$$= (g_K g_j - 1 - (g_j - 1))(g_i - 1) + g_{Kj} - g_{Ki}$$

$$= g_K g_{ji} + g_{Ki} - g_{ji} + g_{Kj} - g_{Ki}$$

Q. How does σ_{ij} act on $(\mathrm{Aug}\,F_n)\otimes_{F_n} V$

Ans. $\sigma_{ij}(\gamma\otimes v) = \sigma_{ij}(\gamma)\otimes\sigma_{ij}(v)$

Wada   B/G

$t\ (t_i)$

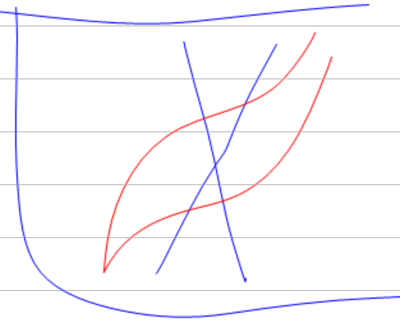
August 13, 2020:

Bellingeri-Soulie, contains stuff about Wada: https://arxiv.org/pdf/2001.04272.pdf

$$G = N \rtimes H$$

http://www.math.toronto.edu/~drorbn/Talks/Georgetown-1503/

Aug 17, 2020



$$\mathbb{Q}G/I^{n+1} \qquad I^n/I^{n+1}$$

$$I^n ?$$

$$I = \langle g-1\rangle \qquad \bar{g} = g-1 \qquad g = \bar{g}+1$$

$$\mathbb{Q}G = \langle 1, \bar{g} : g\neq 1\rangle = \{1, \bar{g}_1\bar{g}_2, \bar{g}_1\bar{g}_2\bar{g}_3\}$$

$I = \langle \bar{g}\rangle$ = words of length at least 1 in $\{\bar{g}\}$

I^n = words of length at least n in $\{\bar{g}\}$

n rest

$$I^n = \langle \text{words of length } n\rangle\cdot\mathbb{Q}F_n$$

$$= \mathbb{Q}F_n\,\langle \text{words of length } n\rangle$$

$n=5$

$a \cdot n n n c y = a n a n c \cdot y$

Aug 31, 2020

$$FA\langle a_1 \dots a_n \rangle \xrightarrow[\epsilon]{a_i \mapsto 1} \mathbb{Q}$$

A $\quad I = \ker(\epsilon)$

$\bar{a}_i = a_i - 1 \qquad A = FA\langle \bar{a}_i \rangle$

$I = \langle \text{Words of length} > 0 \rangle$

$$\mathbb{Q}FG\langle g_1 \dots g_n \rangle = FA\langle g_i^{\pm 1} \rangle \big/ g_i g_i^{-1} = 1 = g_i^{-1} g_i$$

$$= FA\langle \overline{g_i^{\pm 1}} \rangle \big/ (\bar{g}_i + 1)(\overline{g_i^{-1}} + 1) = 1 = \dots$$

$$\bar{g}_i \overline{g_i^{-1}} + \bar{g}_i + \overline{g_i^{-1}} + 1 = 1$$

$$\bar{g}_i + \overline{g_i^{-1}} = -\bar{g}_i \overline{g_i^{-1}} \in I^2$$

Magnus Expansion: $Z: \mathbb{Q}FG_n \longrightarrow \prod I^n / I^{n+1}$

$\cong ?$

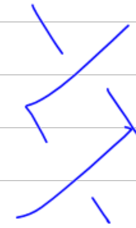
$\widehat{FA}\langle \bar{g}_i \rangle$

$\bar{g}_i = t_i \quad$ " $FA\langle t_i \rangle$

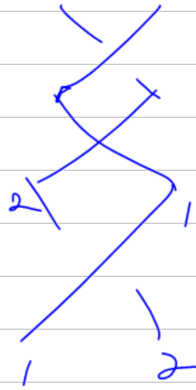
$$\underbrace{\overline{g_i^{-1}}}_{I} = -\underbrace{\bar{g}_i}_{I} - \underbrace{\bar{g}_i \overline{g_i^{-1}}}_{I^2}$$

σ_{12}

σ_{21}



$\sigma_{12}\sigma_{21} \neq \sigma_{12}\sigma_{12}$

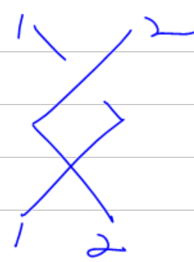


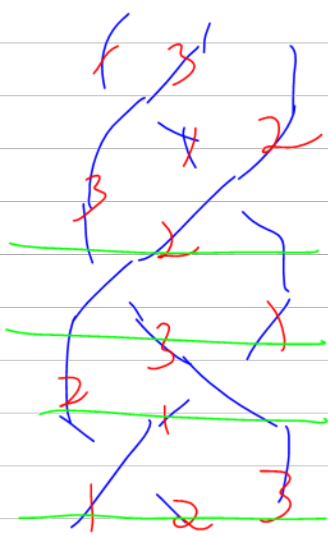
$= (\sigma_{12})^2$

$\sigma_{12} = \sigma_1 \tau_1$

$\| \; \sigma_1 \tau_1 \sigma_1 \tau_1$

$\sigma_{21} = \tau_1 \sigma_1$





σ_{31}

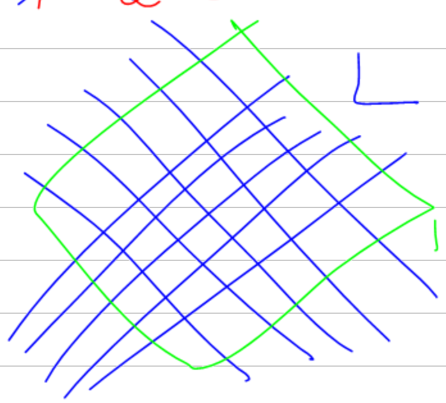
σ_{21}

σ_{23}^{-1}

σ_{31}

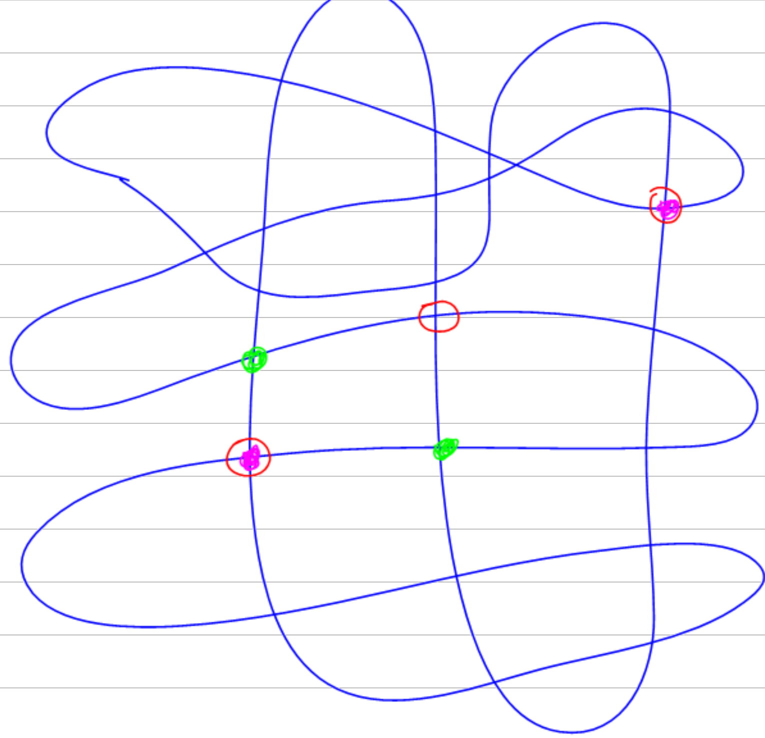
σ_{12}

$\rho_1 = A^{-1} \rho_2 A$



L

$d=2$



$$w(\beta^k) = k\,w(\beta)$$

$$\beta^k = \gamma^{-1}(\beta')^k\gamma$$

$$k\,w(\beta) = w(\beta^k) = w(\gamma^{-1}(\beta')^k\gamma)$$

$$= \pm 2 + w(\beta'^k) = \pm 2 + k\,w(\beta')$$

$\beta \in B_n$

$$w(\beta) = \sum_{\emptyset \neq A \subset \underline{n}} (-1)^{|A|+n}\, w_r(\beta_A)/|A|$$

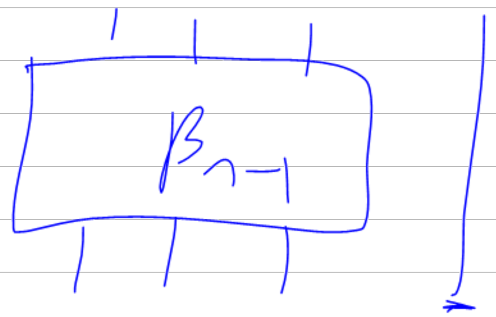
$$w(\times) = \frac{1}{2} - 0 - 0 = \frac{1}{2} \qquad w_r(\Delta_n) = \binom{n}{2}$$

$$W(\Delta_n) = \sum_{k=1}^{n} (-1)^{k+n} \binom{n}{k}\binom{k}{2}\frac{1}{k}$$

$$= \frac{(-1)^n}{2}\sum_{k=1}^{n} (-1)^k (k-1)\binom{n}{k} =$$

$$= \frac{(-1)^n}{2}\left[\frac{d}{dx}(1-x)^n - (1-x)^n \Big|_{x=1} - (-1)\right]$$

$$\sum_{k=0}^{n} (-1)^k \binom{n}{k} x^k$$

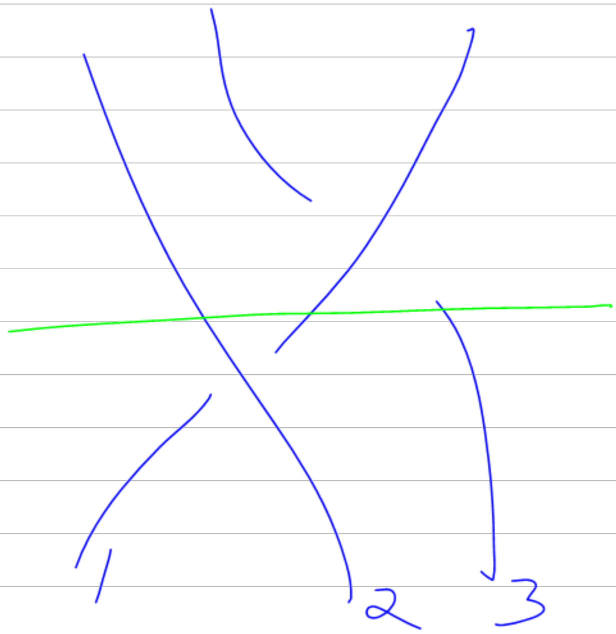


$$W(\beta) = \frac{(-1)^n}{n}\sum_{\emptyset \neq A \subset \underline{n}} (-1)^{|A|}\, Wr(\beta_A)$$

$$W(\Delta) = \frac{(-1)^n}{n}\sum_{k\geq 1}\binom{n}{k}(-1)^k\binom{k}{2}$$

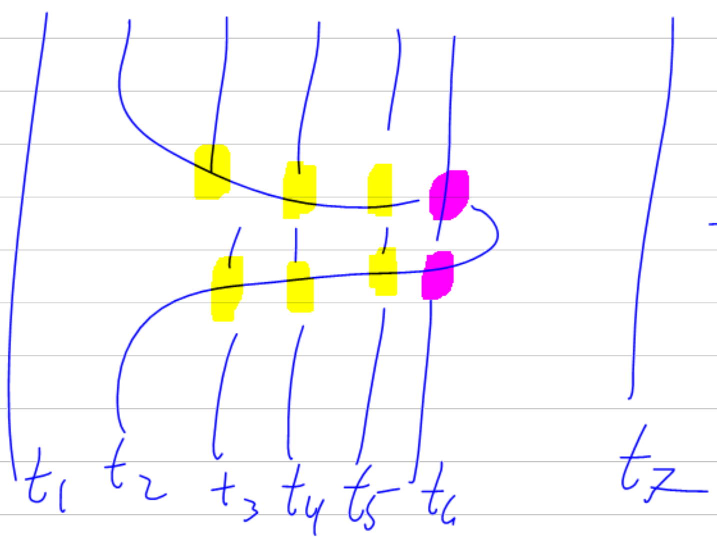
$$= \frac{(-1)^n}{2n}\frac{d^2}{dx^2}(1-x)^n\Big|_{x=1}$$

Expand $(t-1)t - (t^{-1}-1)t^{2}(t-1)$



$\sigma_{21}^{-1}\sigma_{13}$

bns/ $t_i \cancel{\to} \mathbb{Z}$



$\longrightarrow t_2 t_1$

$$Z: \quad g_i \longmapsto 1+\bar{g}_i = 1+t_i$$
$$g_i^{-1} \longmapsto \frac{1}{1+t_i} = 1-t_i+t_i^2-t_i^3+\cdots$$

$$Z: F G_n \xrightarrow[1-1?]{} \hat{FA}_n$$

$x^\infty = xxx\cdots$ ✗

$e^x = \sum \frac{x^n}{n!}$ ✓

$$\hat{FG}_n = \varprojlim_{p\to\infty} \mathbb{Q}FG_n / I^p$$

$$Z: \hat{FG}_n \xrightarrow{\sim} \hat{FA}_n$$

$\langle$ words in t_i of length $\geq p \rangle$

$$\varprojlim_{q\geq p} I^p/I^q = \hat{I}^p_G \xrightarrow{\sim} \hat{I}^p_A = \prod_{q\geq p} \underbrace{I^q/I^{q+1}}_{\text{words of length } q}$$

$$= \{(w_p, w_{p+1}, w_{p+2}, \ldots\ldots)\}$$

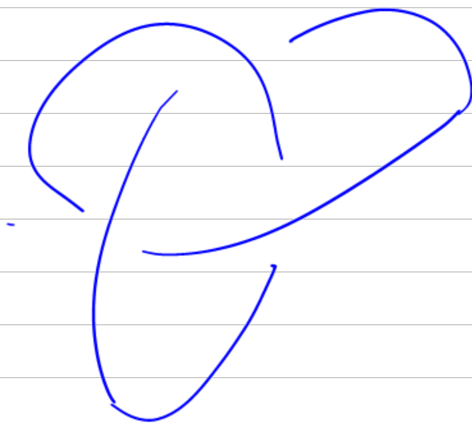
magnus karrass solitar

Injectivity of Z: If $w = g_{i_1}^{k_1}\cdots g_{i_n}^{k_n}$, $n\geq 1$, $\forall \alpha\ i_\alpha \neq i_{\alpha+1}$, $k_\alpha \neq 0$ NTS, $Z(w)\neq 1$.

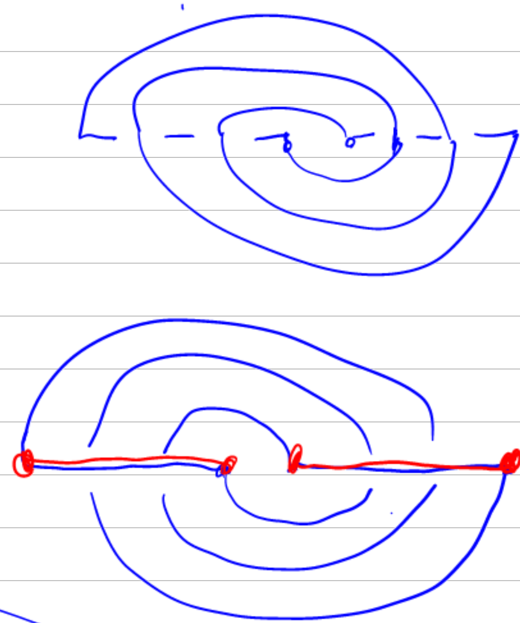
Pf Consider the coeff of $t_{i_1}\cdots t_{i_n}$ in $Z(w)$. It is $k_1 k_2 \cdots k_n \neq 0$ $\square$

$$\begin{pmatrix} t_i & t_i(1-t_j) \\ 0 & t_i^2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & (1-t_j) \\ 0 & t_i \end{pmatrix} \qquad \begin{pmatrix} 1-t_i & \\ & 1-t_j \end{pmatrix}$$



bridge # = 3



$\rho : PwB_{0n} \longrightarrow End(V)$; $K \quad F_n \triangleleft PwB_{0n}$

$g_j = \sigma_{j0}$

$\rho^{+K} : PwB_n \longrightarrow End(I^K \otimes_{F_n} V)$

$\rho^{+K}(\beta)(x \otimes v) = x^\beta \otimes \rho(\beta) v$