Pensieve header: ρ_1 for up to 12 crossings for Radmila.

In[ ]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\People\\Sazdanovic"];
```

In[ ]:=
```
Once[<< KnotTheory`];
```

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
```

In[ ]:=
```
Rot::usage =
  "Rot[K] where K is any n-crossing knot presentation returns {Cs, φ}, where
    Cs is a length n list of crossings as triples {s,i,j}
    and φ is a lengtth 2n list of rotation numbers.";
```

In[ ]:=
```
PD[epd_EPD] := PD @@ epd /. {X_{i_,j_} :> X[j, i + 1, j + 1, i], Overscript[X, -]_{i_,j_} :> X[j, i, j + 1, i + 1]}
```

In[ ]:=
```
Rot[pd_PD] := Module[{n, xs, x, rots, Xp, Xm, front = {1}, k},
   n = Length@pd; rots = Table[0, {2 n}];
   xs = Cases[pd, x_X :> {Xp[x[[4]], x[[1]]] PositiveQ@x
                          Xm[x[[2]], x[[1]]]     True   ]];
   For[k = 1, k ≤ 2 n, ++k,
    If[FreeQ[front, -k],
     front = Flatten@Replace[front, k → (xs /. {
             Xp[k, l_] | Xm[l_, k] :> {l + 1, k + 1, -l},
             Xp[l_, k] | Xm[k, l_] :> (++rots[[l]]; {-l, k + 1, l + 1}),
             _Xp | _Xm :> {}
            }), {1}],
     Cases[front, k | -k] /. {k, -k} :> --rots[[k]];
    ]
   ];
   {xs /. {Xp[i_, j_] :> {+1, i, j}, Xm[i_, j_] :> {-1, i, j}}, rots}];
Rot[K_] := Rot[PD[K]];
```

The Program

```
In[ ]:= R1[s_, i_, j_] := s (g_{ji} (g_{j+,j} + g_{j,j+} - g_{ij}) - g_{ii} (g_{j,j+} - 1) - 1 / 2);
Z[K_] := Module[{Cs, φ, n, A, s, i, j, k, Δ, G, ρ1},
  {Cs, φ} = Rot[K]; n = Length[Cs];
  A = IdentityMatrix[2 n + 1];
  Cases[Cs, {s_, i_, j_} :> (A[[{i, j}, {i + 1, j + 1}]] += ((-T^s  T^s - 1)
                                                             (  0     -1  )))];
  Δ = T^((-Total[φ] - Total[Cs[[All, 1]]]) / 2) Det[A];
  G = Inverse[A];
  ρ1 = Sum_{k=1}^{n} R1 @@ Cs[[k]] - Sum_{k=1}^{2 n} φ[[k]] (g_{kk} - 1 / 2);
  Factor@{Δ, Δ^2 ρ1 /. α_+ :> α + 1 /. g_{α_,β_} :> G[[α, β]]}];
```

The First Few Knots

```
In[ ]:= TableForm[Table[Join[{K[[1]]_{K[[2]]}}, Z[K]], {K, AllKnots[{3, 6}]}], TableAlignments → Center]
```

KnotTheory: Loading precomputed data in PD4Knots`.

Out[ ]//TableForm=

3_1	$\frac{1-T+T^2}{T}$	$\frac{(-1+T)^2\,(1+T^2)}{T^2}$
4_1	$-\frac{1-3\,T+T^2}{T}$	0
5_1	$\frac{1-T+T^2-T^3+T^4}{T^2}$	$\frac{(-1+T)^2\,(1+T^2)\,(2+T^2+2\,T^4)}{T^4}$
5_2	$\frac{2-3\,T+2\,T^2}{T}$	$\frac{(-1+T)^2\,(5-4\,T+5\,T^2)}{T^2}$
6_1	$-\frac{(-2+T)\,(-1+2\,T)}{T}$	$\frac{(-1+T)^2\,(1-4\,T+T^2)}{T^2}$
6_2	$-\frac{1-3\,T+3\,T^2-3\,T^3+T^4}{T^2}$	$\frac{(-1+T)^2\,(1-4\,T+4\,T^2-4\,T^3+4\,T^4-4\,T^5+T^6)}{T^4}$
6_3	$\frac{1-3\,T+5\,T^2-3\,T^3+T^4}{T^2}$	0

In[ ]:=
```
Monitor[
 tab = Table[Join[{K}, Z[K]], {K, AllKnots[{3, 12}]}],
 K]
```

KnotTheory: Loading precomputed data in DTCode4KnotsTo11`.

KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

KnotTheory: Loading precomputed data in KnotTheory/12A.dts.

KnotTheory: Loading precomputed data in KnotTheory/12N.dts.

General: Further output of KnotTheory::loading will be suppressed during this calculation.

Out[ ]=

$$\left\{\left\{\text{Knot}[3,1],\ \frac{1-T+T^2}{T},\ \frac{(-1+T)^2\left(1+T^2\right)}{T^2}\right\},\ \left\{\text{Knot}[4,1],\ -\frac{1-3T+T^2}{T},\ 0\right\},\right.$$
$$\left\langle\cdots 2974 \cdots\right\rangle,\ \left\{\text{Knot}[12,\text{NonAlternating},888],\ \frac{\left(1-T+T^2\right)^2\left(1+T-2T^2+T^3-2T^4+T^5+T^6\right)}{T^5},\right.$$
$$\left.\left.-\frac{(-1+T)^2\left(1+T^2\right)\left(1-T+T^2\right)^2\left(5+10T-9T^2+2T^3+22T^4-30T^5+43T^6-30T^7+22T^8+2T^9-9T^{10}+10T^{11}+5T^{12}\right)}{T^{10}}\right\}\right\}$$

Full expression not available (original memory size: 8.8 MB)

In[ ]:=
```
tab >> "rho1to12xings.m"
```