$L: V \to W \qquad L \in V^* \otimes W \qquad V^* = \langle \varphi_i \rangle \quad V = \langle v^i \rangle \quad \varphi_i(v^{i'}) = \delta_i^{i'}$

$W = \langle w^j \rangle$

$L = \sum_{i,j} A^i_j \varphi_i \cdot w^j$

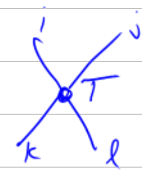
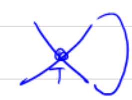
$L(v^{i'}) = \sum_{i,j} A^i_j \varphi_i(v^{i'}) \cdot w^j = \sum_j A^{i'}_j w^j$

Tensor: "A matrix w/ any number of subscripts or superscripts"

$T_{ijkl} \rightsquigarrow \sum_{i,j,k,l} T_{ijkl}\, w^i \otimes w^j \otimes w^k \otimes w^l \in W^{\otimes 4}$

$T^{ij}_{kl} \longrightarrow C^i_k = \sum_j T^{ij}_{kj}$ "Tensor contractions"

Example $\sum_i A^i_i = tr(A)$

T^{ij}_{kl}  $C^i_k = \sum_j T^{ij}_{kj}$  not related 

$T^{ij}_{kl} \cdot A^m_n$ 

$\begin{array}{r} 217 \\ \times\ 67 \\ \hline \end{array}$

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xing#1 ... xing#n

$a_1 a_2 a_3 \ldots a_n = \Delta$

$(a_1 + \epsilon b_1)(a_2 + \epsilon b_2 + \ldots) \ldots\ldots (a_n + \epsilon b_n) = \Delta + \epsilon \Delta_1 + \epsilon^2 (\Delta_2)$

http://drorbn.net/oa22

$\sum_{\text{xings}}$ all is a_i's w/ one b_i

$\sum_{\text{xings} \times \text{xings}}$ all a_i's w/ two b_i's

$n \gg 1 \qquad e^{\sqrt{n}} > n^{20}$ Theory

$e^{\sqrt{n}} < n^{20}$ practice

$\sqrt{n} \overset{?}{=} 20 \log n \qquad \frac{\sqrt{n}}{\log n} = 20$

$n \sim 46{,}000$

3 vectors in $\mathbb{R}^3$ are l.i. generically.

<u>Cayley-Hamilton Thm:</u> $A \in M_{n\times n}(\mathbb{C})$

$\chi(\lambda) = \det(\lambda I - A)$

Then $\chi(A) = 0$.

generically $A = P\begin{pmatrix}\lambda_1 & & \\ & \ddots & \\ & & \lambda_n\end{pmatrix}P^{-1}$