

Pensieve header: A program to compute the Alexander polynomial using matrices coming from the Dehn presentation. Written by Daniel Martchenkov in June 2023.

```
In[ ]:= << KnotTheory`
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.  
Read more at <http://katlas.org/wiki/KnotTheory>.

```
In[ ]:= AlexanderUsingDehn[K_] := t ↦ Module[{XingsByArmpits, bends, faces, p, A, is, poly},
  XingsByArmpits =
    List@@PD[K] /. x : X[i_, j_, k_, L_] =>
      If[PositiveQ[x], X+[-i, j, k, -L], X_-[-j, k, L, -i]];
  bends = Times@@XingsByArmpits /.
    _[X][a_, b_, c_, d_] => pa,-d pb,-a pc,-b pd,-c;
  faces = bends // . px_,y_ py_,z_ => px,y,z;
  A = Table[0, Length@XingsByArmpits, Length@faces];
  Do[is = Position[faces, #][[1, 1]] & /@ List@@XingsByArmpits[[j]];
  A[[j], is] += If[Head[XingsByArmpits[[j]]] === X+,
    (1 - 1 t - t), (-t 1 - 1 t)],
    {j, Length@XingsByArmpits}];
  poly = A[[All,
    Delete[Range[Length@faces], {Position[faces, #][[1, 1]] & /@ {1, -1}}]] // Det;
  Mean[{Exponent[poly, t, Max], Exponent[poly, t, Min]}] // Apart];
```

```
In[ ]:= AlexanderUsingDehn[Knot[3, 1]][t]
```

 KnotTheory: Loading precomputed data in PD4Knots`.

```
Out[ ]:=
```

$$-1 + \frac{1}{t} + t$$

```
In[ ]:= Sum[Alexander[K][t] == AlexanderUsingDehn[K][t], {K, AllKnots[{3, 10}}]]
```

```
Out[ ]:=
```

249 True