

$a_n + 2u a_{n+1} + a_{n+2} = 0$

$$\frac{\left(\frac{u + \sqrt{u^2 - 1}}{2\sqrt{u^2 - 1}}\right)^n \left(-u + \sqrt{u^2 - 1}\right) + \left(\frac{-u + \sqrt{u^2 - 1}}{2\sqrt{u^2 - 1}}\right)^n \left(-u - \sqrt{u^2 - 1}\right)}{1}$$

$$a_n = \sum_{k=0}^{\lfloor n/2 \rfloor} (-1)^{n-k} \binom{n-k}{k} (2u)^{n-2k}$$

$B(v, e_i) = B \left(\begin{matrix} a_{n-2} (\dots \text{level } n-2) \\ + \dots \\ + a_{n+2} (\text{level } n+2, e_i) \end{matrix} \right)$

$$= A_R \left(\dots \right) + A_L \left(\dots \right) + B \left(\dots \right)$$

3	2u	2u	2u	1	0	0	0	1	0	1	0
2u	4u ² -6	1	1	0	-2u	-2u	-2u	0	0	0	-4

$\frac{u + \sqrt{u^2 - 1}}{2\sqrt{u^2 - 1}}$	$\frac{2u}{-1}$
$\frac{-u + \sqrt{u^2 - 1}}{2\sqrt{u^2 - 1}}$	$\frac{2u}{-1}$

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In[*]:= a[0] = 1; a[1] = 0; a[n_] := -2 u a[n - 1] - a[n - 2]; Factor@Table[a[n], {n, 0, 20}]
Out[*]=
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{1, 0, -1, 2 u, -((-1 + 2 u) (1 + 2 u)), 4 u (-1 + 2 u2), -((-1 - 2 u + 4 u2) (-1 + 2 u + 4 u2)),
2 u (-1 + 2 u) (1 + 2 u) (-3 + 4 u2), -((1 - 4 u - 4 u2 + 8 u3) (-1 - 4 u + 4 u2 + 8 u3)),
8 u (-1 + 2 u2) (1 - 8 u2 + 8 u4), -((-1 + 2 u) (1 + 2 u) (-1 - 6 u + 8 u3) (1 - 6 u + 8 u3)),
2 u (-1 - 2 u + 4 u2) (-1 + 2 u + 4 u2) (5 - 20 u2 + 16 u4),
-((-1 + 6 u + 12 u2 - 32 u3 - 16 u4 + 32 u5) (1 + 6 u - 12 u2 - 32 u3 + 16 u4 + 32 u5)),
4 u (-1 + 2 u) (1 + 2 u) (-1 + 2 u2) (-3 + 4 u2) (1 - 16 u2 + 16 u4),
-((-1 - 6 u + 24 u2 + 32 u3 - 80 u4 - 32 u5 + 64 u6) (-1 + 6 u + 24 u2 - 32 u3 - 80 u4 + 32 u5 + 64 u6)),
2 u (1 - 4 u - 4 u2 + 8 u3) (-1 - 4 u + 4 u2 + 8 u3) (-7 + 56 u2 - 112 u4 + 64 u6), -((-1 + 2 u) (1 + 2 u)
(-1 - 2 u + 4 u2) (-1 + 2 u + 4 u2) (1 + 8 u - 16 u2 - 8 u3 + 16 u4) (1 - 8 u - 16 u2 + 8 u3 + 16 u4)),
16 u (-1 + 2 u2) (1 - 8 u2 + 8 u4) (1 - 32 u2 + 160 u4 - 256 u6 + 128 u8),
-((1 + 8 u - 40 u2 - 80 u3 + 240 u4 + 192 u5 - 448 u6 - 128 u7 + 256 u8)
(1 - 8 u - 40 u2 + 80 u3 + 240 u4 - 192 u5 - 448 u6 + 128 u7 + 256 u8)),
2 u (-1 + 2 u) (1 + 2 u) (-3 + 4 u2) (-1 - 6 u + 8 u3) (1 - 6 u + 8 u3) (-3 + 36 u2 - 96 u4 + 64 u6),
-((-1 + 10 u + 40 u2 - 160 u3 - 240 u4 + 672 u5 + 448 u6 - 1024 u7 - 256 u8 + 512 u9)
(1 + 10 u - 40 u2 - 160 u3 + 240 u4 + 672 u5 - 448 u6 - 1024 u7 + 256 u8 + 512 u9)) }
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In[*]:= **a[0] = 0; a[1] = 1; a[n_] := -2 u a[n - 1] - a[n - 2]; Factor@Table[a[n], {n, 0, 20}]**

Out[*]=

$$\{0, 1, -2 u, (-1 + 2 u) (1 + 2 u), -4 u (-1 + 2 u^2), (-1 - 2 u + 4 u^2) (-1 + 2 u + 4 u^2),$$

$$-2 u (-1 + 2 u) (1 + 2 u) (-3 + 4 u^2), (1 - 4 u - 4 u^2 + 8 u^3) (-1 - 4 u + 4 u^2 + 8 u^3),$$

$$-8 u (-1 + 2 u^2) (1 - 8 u^2 + 8 u^4), (-1 + 2 u) (1 + 2 u) (-1 - 6 u + 8 u^3) (1 - 6 u + 8 u^3),$$

$$-2 u (-1 - 2 u + 4 u^2) (-1 + 2 u + 4 u^2) (5 - 20 u^2 + 16 u^4),$$

$$(-1 + 6 u + 12 u^2 - 32 u^3 - 16 u^4 + 32 u^5) (1 + 6 u - 12 u^2 - 32 u^3 + 16 u^4 + 32 u^5),$$

$$-4 u (-1 + 2 u) (1 + 2 u) (-1 + 2 u^2) (-3 + 4 u^2) (1 - 16 u^2 + 16 u^4),$$

$$(-1 - 6 u + 24 u^2 + 32 u^3 - 80 u^4 - 32 u^5 + 64 u^6) (-1 + 6 u + 24 u^2 - 32 u^3 - 80 u^4 + 32 u^5 + 64 u^6),$$

$$-2 u (1 - 4 u - 4 u^2 + 8 u^3) (-1 - 4 u + 4 u^2 + 8 u^3) (-7 + 56 u^2 - 112 u^4 + 64 u^6),$$

$$(-1 + 2 u) (1 + 2 u) (-1 - 2 u + 4 u^2) (-1 + 2 u + 4 u^2) (1 + 8 u - 16 u^2 - 8 u^3 + 16 u^4)$$

$$(1 - 8 u - 16 u^2 + 8 u^3 + 16 u^4), -16 u (-1 + 2 u^2) (1 - 8 u^2 + 8 u^4) (1 - 32 u^2 + 160 u^4 - 256 u^6 + 128 u^8),$$

$$(1 + 8 u - 40 u^2 - 80 u^3 + 240 u^4 + 192 u^5 - 448 u^6 - 128 u^7 + 256 u^8)$$

$$(1 - 8 u - 40 u^2 + 80 u^3 + 240 u^4 - 192 u^5 - 448 u^6 + 128 u^7 + 256 u^8),$$

$$-2 u (-1 + 2 u) (1 + 2 u) (-3 + 4 u^2) (-1 - 6 u + 8 u^3) (1 - 6 u + 8 u^3) (-3 + 36 u^2 - 96 u^4 + 64 u^6),$$

$$(-1 + 10 u + 40 u^2 - 160 u^3 - 240 u^4 + 672 u^5 + 448 u^6 - 1024 u^7 - 256 u^8 + 512 u^9)$$

$$(1 + 10 u - 40 u^2 - 160 u^3 + 240 u^4 + 672 u^5 - 448 u^6 - 1024 u^7 + 256 u^8 + 512 u^9), -4 u (-1 + 2 u^2)$$

$$(-1 - 2 u + 4 u^2) (-1 + 2 u + 4 u^2) (5 - 20 u^2 + 16 u^4) (1 - 48 u^2 + 304 u^4 - 512 u^6 + 256 u^8) \}$$

In[*]:= **a[0] = u; a[1] = -1; a[n_] := -2 u a[n - 1] - a[n - 2]; Factor@Table[a[n], {n, 0, 10}]**

Out[*]=

$$\{u, -1, u, 1 - 2 u^2, u (-3 + 4 u^2), -1 + 8 u^2 - 8 u^4, u (5 - 20 u^2 + 16 u^4),$$

$$-((-1 + 2 u^2) (1 - 16 u^2 + 16 u^4)), u (-7 + 56 u^2 - 112 u^4 + 64 u^6),$$

$$-1 + 32 u^2 - 160 u^4 + 256 u^6 - 128 u^8, u (-3 + 4 u^2) (-3 + 36 u^2 - 96 u^4 + 64 u^6) \}$$

In[*]:= **a[0] = u; a[1] = -1; a[n_] := -2 u a[n - 1] - a[n - 2];**
Factor@PowerExpand@

$$\text{FullSimplify@Table}\left[\frac{2 a[n]}{\sqrt{-\frac{(-1+\omega)^2}{\omega}}}\right] /. u \rightarrow \sqrt{(v+1) / 2} /. v \rightarrow -(\omega + \omega^{-1}) / 2, \{n, 0, 10\}$$

Out[*]=

$$\left\{1, \frac{2 i \sqrt{\omega}}{(-1 + \sqrt{\omega}) (1 + \sqrt{\omega})}, 1, -\frac{i (1 + \omega^2)}{(-1 + \sqrt{\omega}) (1 + \sqrt{\omega}) \sqrt{\omega}}, -\frac{1 + \omega + \omega^2}{\omega},\right.$$

$$\frac{i (1 + \omega^4)}{(-1 + \sqrt{\omega}) (1 + \sqrt{\omega}) \omega^{3/2}}, \frac{1 + \omega + \omega^2 + \omega^3 + \omega^4}{\omega^2}, -\frac{i (1 + \omega^2) (1 - \omega^2 + \omega^4)}{(-1 + \sqrt{\omega}) (1 + \sqrt{\omega}) \omega^{5/2}},$$

$$\left. -\frac{1 + \omega + \omega^2 + \omega^3 + \omega^4 + \omega^5 + \omega^6}{\omega^3}, \frac{i (1 + \omega^8)}{(-1 + \sqrt{\omega}) (1 + \sqrt{\omega}) \omega^{7/2}}, \frac{(1 + \omega + \omega^2) (1 + \omega^3 + \omega^6)}{\omega^4}\right\}$$