$uK,\ vK,\ u\mathcal{D},\ v\mathcal{D}\ \ldots$

$$
\begin{tikzcd}
v\mathcal{D} \arrow[drr, "s"] \arrow[d, shift right, "\rho"'] & & \\
u\mathcal{D} \arrow[u, shift right, "\iota"'] \arrow[rr, "s"', "\text{subsets}"] \arrow[d] & & v\mathcal{D} \arrow[ull, bend right, "s^{-1}"'] \arrow[d, "\exists W"] \\
uK \arrow[rr, "V", "\text{type } n"'] & & A
\end{tikzcd}
$$

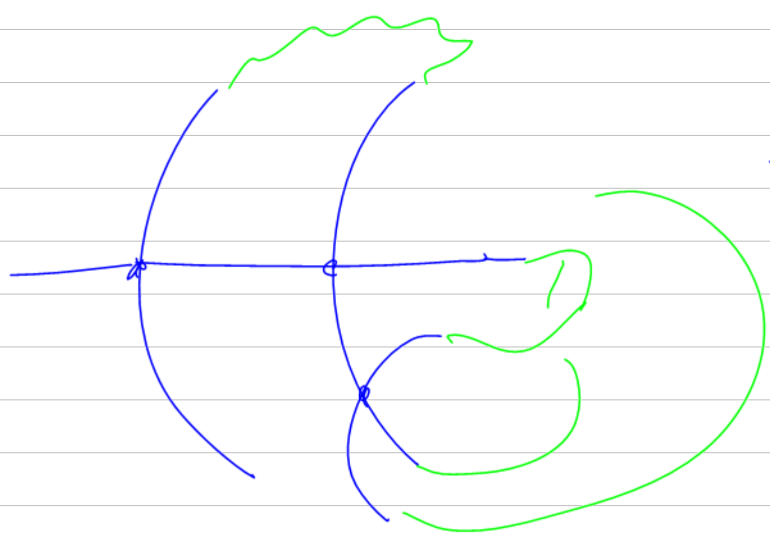
Vanishes $>n$ xings

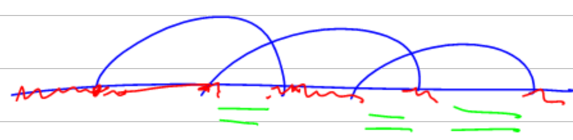
$$
\begin{tikzcd}
v\mathcal{D}/\_ \arrow[drr, "s"] \arrow[d, shift right, "\rho"'] & & \\
u\mathcal{D}/u\mathcal{D}_{n+1} \arrow[u, shift right, "\iota"'] \arrow[rr, "s"', "\text{subsets}"] \arrow[d] & & v\mathcal{D}/\_ \arrow[ull, bend right, "s^{-1}"'] \arrow[d, "\exists\sigma"] \\
uK/\_ \arrow[rr, "\Pi", "\text{type } n"'] & & uK/uK_{n+1}
\end{tikzcd}
$$

Vanishes $>n$ xings

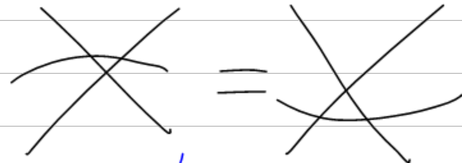
Does σ determine ρ

which σ's come from a ρ?





$D \longmapsto K_D$



$$\begin{tikzcd}
u\mathcal{D} \arrow[r, hook] \arrow[d, "p_u"] & v\mathcal{D} \arrow[r, shift left, "s"] \arrow[dl, "p_v"] & v\mathcal{D} \arrow[l, shift left, "s^{-1}"] \arrow[dl, "w"] \\
\langle K_D \rangle \arrow[r, "V"] & A &
\end{tikzcd}$$

vanishes on $>n$ xings.

$$xyz$$
$$\begin{tikzcd}
x(y-1)(z-1) \arrow[d, "p"] & (x+1)yz - yz \arrow[l, mapsto, "s^{-1}"] \\
(\bar{x} \pm \dot{x})(y-1)(z-1) \arrow[r, mapsto, "s"] & (\bar{x} \pm \dot{x}+1)yz - yz
\end{tikzcd}$$
$$\bar{x}yz \pm \dot{x}yz$$

x

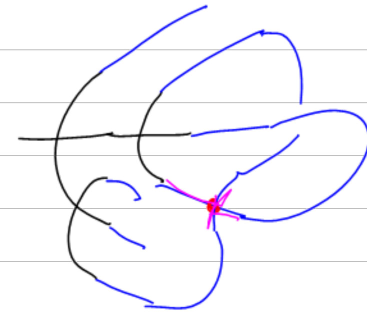
$$xyz$$
$$\begin{tikzcd}
x(y-1)(z-1) \arrow[d] & (x+1)yz - yz \arrow[l, mapsto, "s^{-1}"] \\
x_1x_2x_3(y-1)(z-1) \arrow[r, mapsto, "s"] &
\end{tikzcd}$$

Proof of

$$\begin{tikzcd} u\mathcal{D} \arrow[r, hook] \arrow[d, "P_u"] & v\mathcal{D} \arrow[r, shift left, "S"'] \arrow[dl, "P_v"] & v\mathcal{D} \arrow[l, shift left, "S^{-1}"'] \arrow[d, "w"] & \supset v\mathcal{D}_{>n} \\ \langle K_D \rangle \arrow[rr, "\pi"] & & uK/uK_{n+1} & \end{tikzcd}$$

vanishes if $\#\text{X} + \#\text{X}' > n$

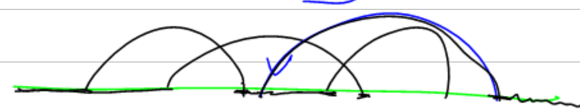
$\langle K_D \rangle$ ← "Canonical form diagrams, CF"



P_v: 1. Brush xings off the maximal tree for the X

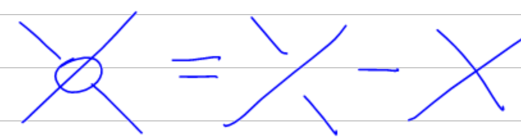
2. Flip the 1st non-desc xing, paying w/ X

3. Realize ("v-xings") as descending



NTS: $W = S^{-1} // P_v // \pi$ vanishes on $v\mathcal{D}_{>n}$

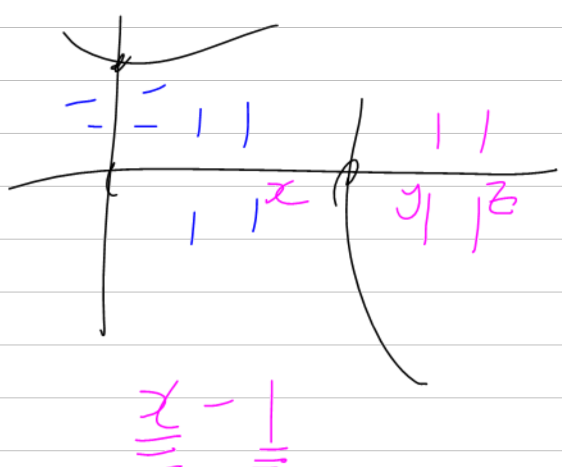
Use downward induction on #X.



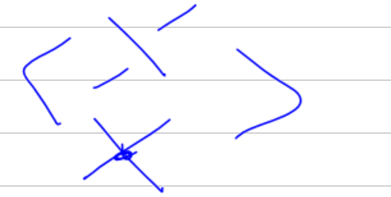
Step 1 W vanishes on CF/VCF diags w/ #X' > 0.

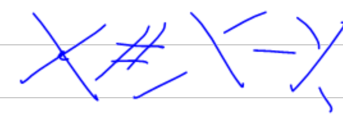
Step 2 W vanishes on clear-tree diagrams with a non-descending xing.

Step 3 The rest.



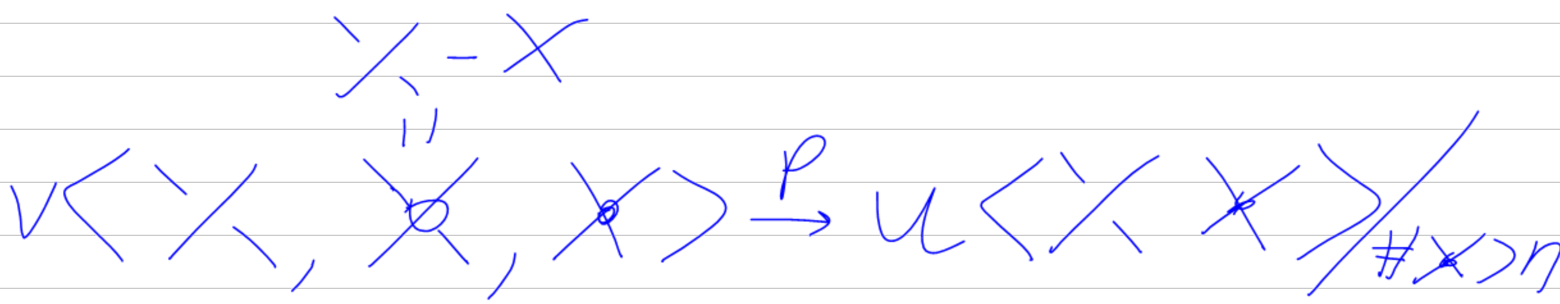
$$\begin{tikzcd}
 & vD \arrow[r, bend right, "s"'] \arrow[dl, "P_K"] & vD \arrow[l, bend right, "s^{-1}"'] \arrow[d, "\sim"] \\
uD \arrow[ur] \arrow[d, "P_u"] & & \\
\text{canonical knots} \arrow[rr, "\pi"] & & K/K_{n+1}
\end{tikzcd}$$

$vD =$ 

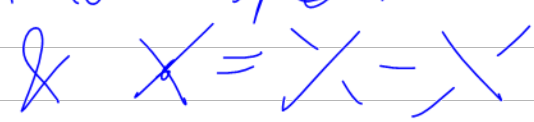


$P_K = $ "P"$\big|_{\#\times \geq K}$

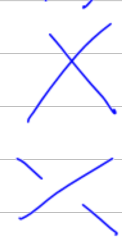
Assume P_{K+1} is def & has right props, def P_K



s.t. 1. $P|_{u<\ \ >}$ should respect R-moves



2. $P : \{\#\times + \#\times > n\} \longrightarrow \{\#\times > n\}$



Goal: Construct $\qquad \dot{\times} := \times - X$

$$P : vD\langle \dot{\times}, \times' \rangle \longrightarrow uD\langle \dot{\times}, \times \rangle / \#\dot{\times} > n$$

s.t. 1. $P|_{uD} = Id$ mod R-moves & $\dot{\times} = \times - \times'$

2. $P|_{\#\dot{\times} + \#\dot{\times} > n} = 0$

if $\#\dot{\times} > n$ in D $\quad P(D) = 0$; suppose P is defined & satisfies 1&2 if $\#\dot{\times} > K$, and D has $\#\dot{\times} = K$.

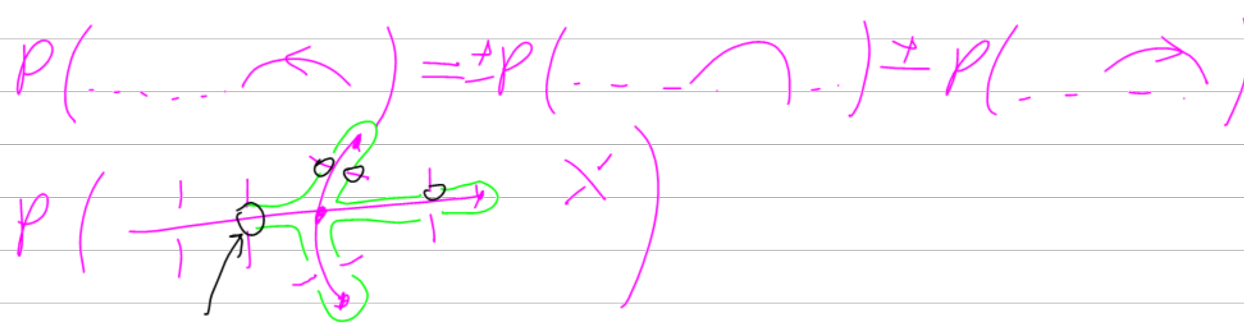
* For every D choose a tree $T(D)$ (for $\times$), indep of $\times'$.

* Aside A "good choice function" on $S(X)$, the set of non-empty subsets of a finite set X, is $c : S(X) \to X$ s.t. 1. $c(A) \in A$

2. If $b \in A \setminus \{c(A)\}$ then $\underline{c(A \setminus \{b\}) = c(A)}$.

Thm $\{\text{good choice Fnctns}\} \longleftrightarrow \{\text{total orderings of } X\}$

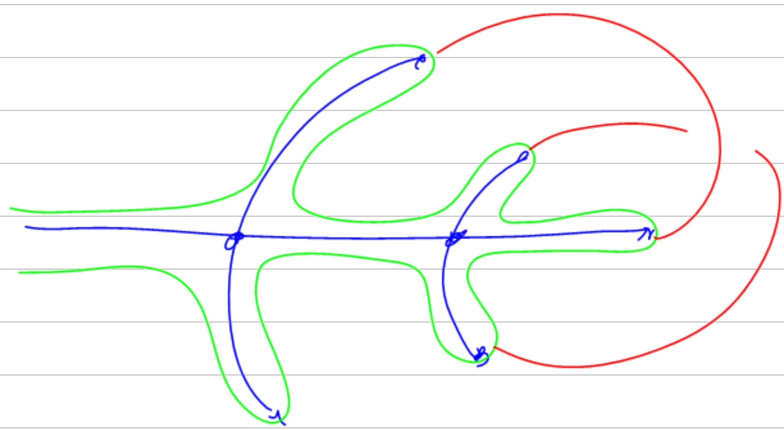
* Define P by fixing the first issue.

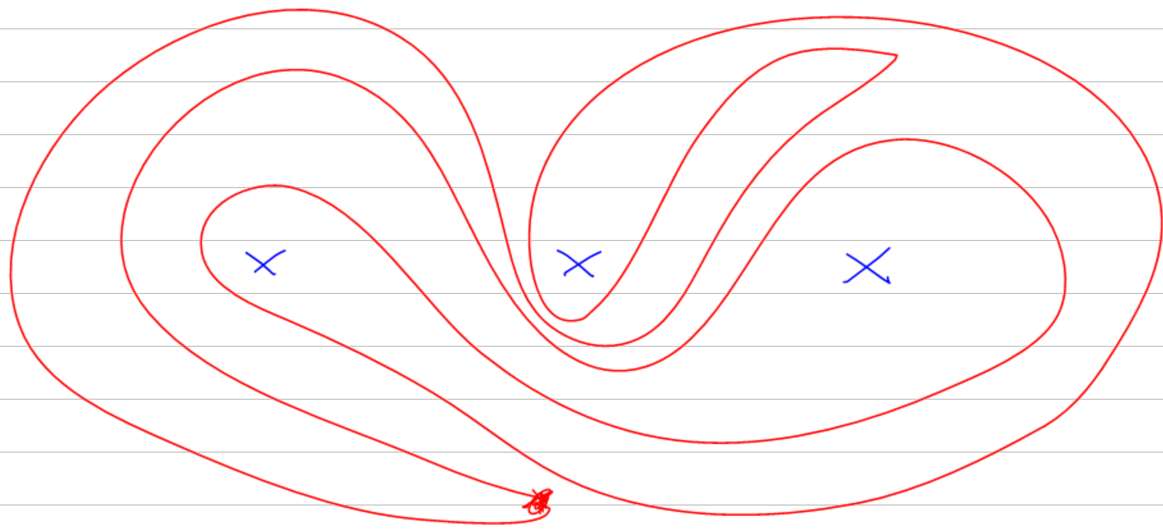


$$V(D) = V(PD) = V\left(P \sum_{D' \subset D} D'|_{\times \to \dot{\times}}\right) =$$

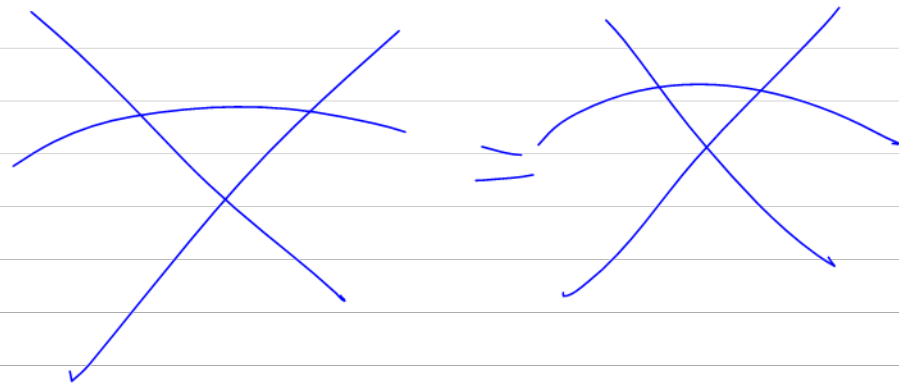
$$= V\Big(P \sum_{\substack{D' \subset D \\ |D'| \le n}} D'\big|_{x \to x'}\Big)$$

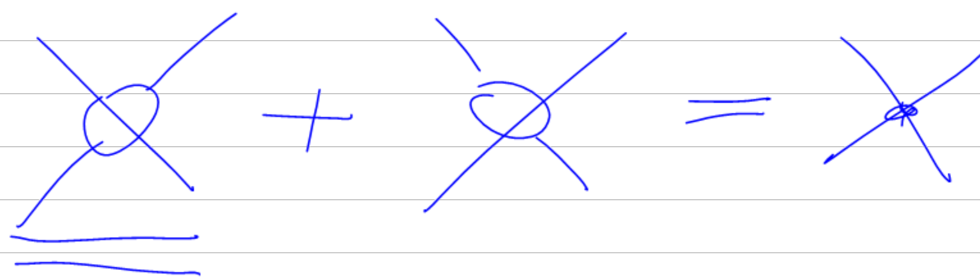
$$= \sum_{\substack{D' \subset D \\ |D'| \le n}} V\big(P(D'|_{x \to x'})\big) = \sum W(D')$$

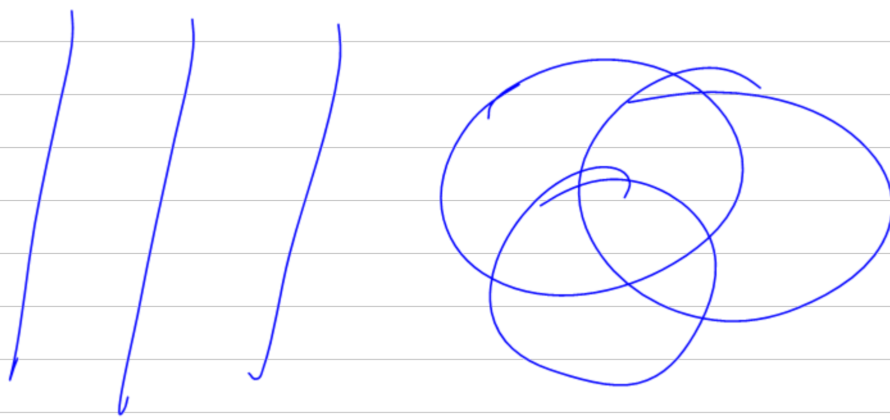




$a b a^{-1} c a b b a^{-1} c$



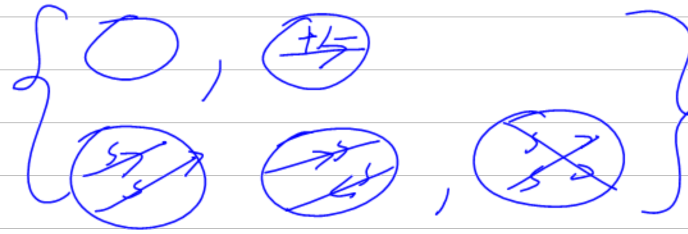




A poly of deg 3 on $\mathbb{R}^{26}$.

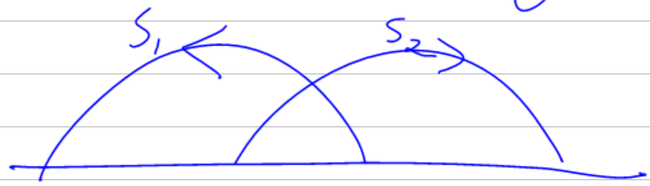
1. $\underline{a}\,\underline{b}\,\underline{c} \quad \underline{q}\,\underline{r}\,\underline{s}$.

2. $F : \mathbb{R}^{26} \to \mathbb{R}$ s.t. $\partial_{aqzv} F = 0$

$\exists\ W :$  $\longrightarrow \mathbb{Q}$

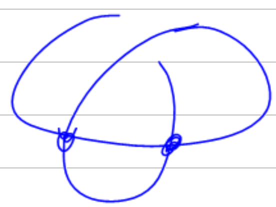
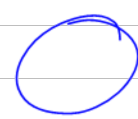
s.t. $V_2(D) = W \circ S(D)$

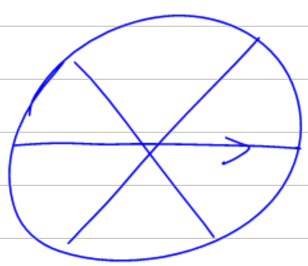
knot diag

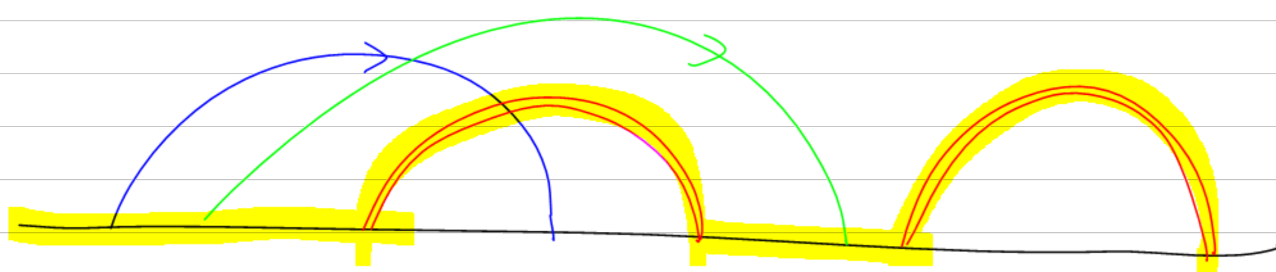


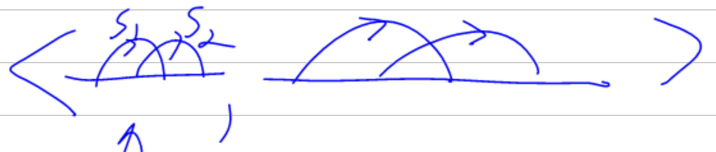
$\longrightarrow S_1, S_2$

$V_2(\otimes) = 1$

 $\xrightarrow[?]{S/\!/\Pi_2}$ 





 $= s_1 s_2$

SK

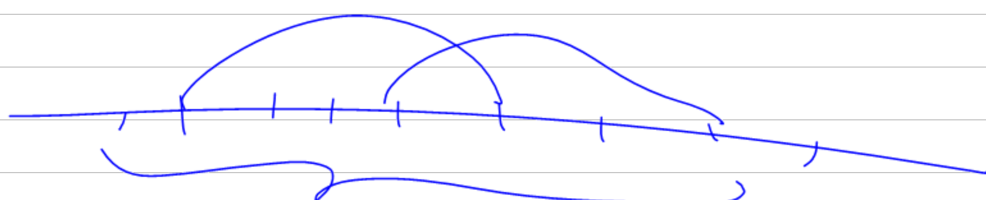
$\bar{v}\mathcal{D} = v\mathcal{D} \big/$  $= s$ 

$$\begin{tikzcd}
v\mathcal{D} \arrow[r, "S"] & v\mathcal{D} \arrow[r, "\pi"] \arrow[d, "w"] & \bar{v}\mathcal{D} \arrow[d, "\bar{w}"] \\
u\mathcal{D} \arrow[u] \arrow[ur, "uS"'] & Q \arrow[r, equal] & Q
\end{tikzcd}$$

$$\operatorname{im}(uS)^* \overset{?}{=} \operatorname{im}(uS/\!/\pi)^*$$

$$\ker(uS)^\perp \overset{?}{\subseteq} \ker(uS/\!/\pi)^\perp$$

$$\ker uS \overset{?}{\supseteq} \ker(uS/\!/\pi)$$



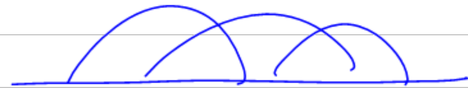
$2n$

$\binom{2n}{n} n! 2^n \qquad \frac{2n!}{n!} 2^n$

$6\cdot 7\cdot 8\cdot 9\cdot 10\cdot 32$

$Z : K \longrightarrow \mathcal{A}$ "Expansion"

$\cong$ UFTI

 $/ 4T$

<u>Exercise</u> Prove GPV for w-knots.

$PwB_n = \Big\langle \sigma_{ij} : 1 \leq i \neq j \leq n \Big) : \begin{matrix} R\text{-moves} \\ OC \end{matrix} \Big\rangle$

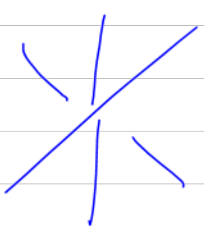
"overcrossings commute"

$=$ motion group of flying rings in $\mathbb{R}^3$
horizontal

$\sigma_{ij}^{\pm} =$ "ring j flies through ring i in the $\pm$ direction.

<u>Relations</u> 1. $\sigma_{ij}\sigma_{kl} = \sigma_{kl}\sigma_{ij}$ $\quad |\{i,j,k,l\}| = 4$

2. $\sigma_{ij}^{+}\sigma_{ij}^{-} = 1$



3. $\sigma_{jk}\sigma_{ik}\sigma_{ij} = \sigma_{ij}\sigma_{ik}\sigma_{jk}$ $\quad$ i = big, j = med, k = small

4 OC: $\sigma_{ij}\sigma_{jk} = \sigma_{jk}\sigma_{ij}$

$$G \rightsquigarrow \mathbb{Q}G = \left\{\sum a_i g_i\right\} \supset I = \left\{\sum a_i g_i : \sum a_i = 0\right\}$$

$$I = \langle g-1 \rangle$$

$$\mathbb{Q}G = I^0 \supset I^1 \supset I^2 \supset I^3 \supset \dots$$

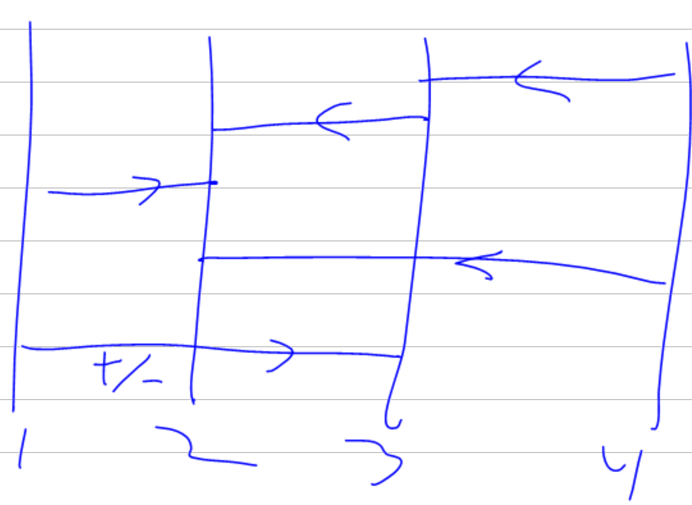
Def $\varphi : G \longrightarrow \mathbb{Q}$ is of type m if

$$\varphi\big|_{I^{m+1}} \equiv 0$$

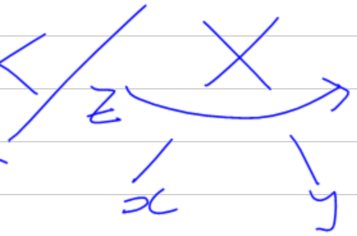
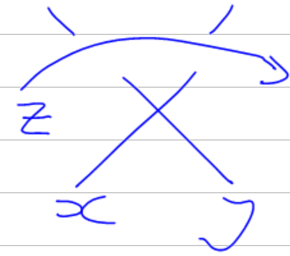
$$\varphi\big(\dots(g_1-1)\dots(g_2-1)\dots(g_{m+1}-1)\dots\big)=0$$

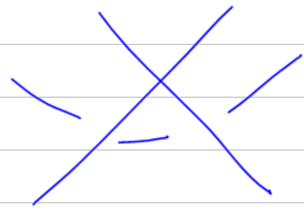
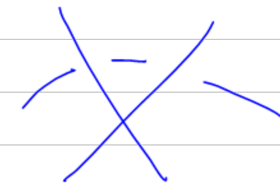
Exercise GPV holds over $\mathbb{Q}$

over $\mathbb{Z}$?

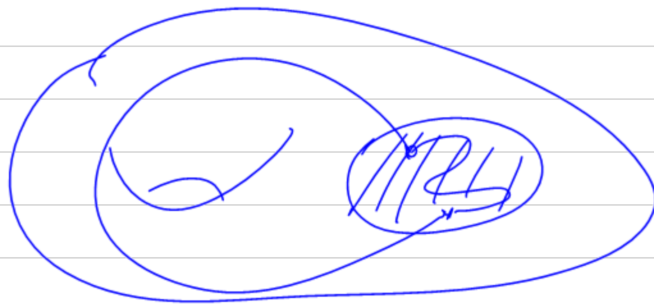


[WKO I]

$wK = vK /$  $=$  "OC"

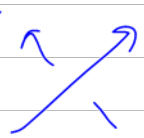
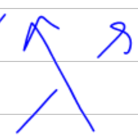
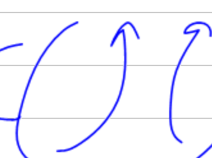
"Forbidden moves"  $\neq$  "VC"

$$PwB_n = \pi_1\left(\text{conf space of hor rings in } \mathbb{R}^3\right)$$



$$\pi_1\left((\text{rings})^c\right)$$
$$\|$$
$$PwB_n \longrightarrow \mathrm{Aut}(F_n)$$

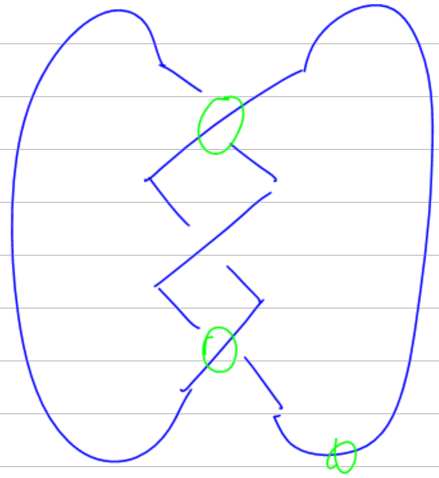
$$C(O^k) = \delta_{k1}$$

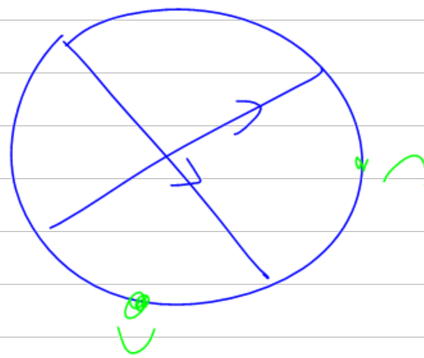
$C($  $) - C($  $) = z \cdot C($  $)$

$$C(K) = \sum_{m=0}^{\infty} V_m(K) z^k$$

V_2

GPV

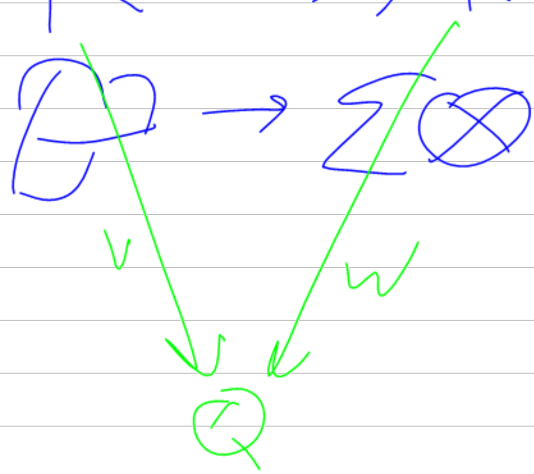


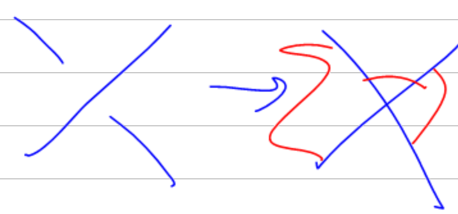


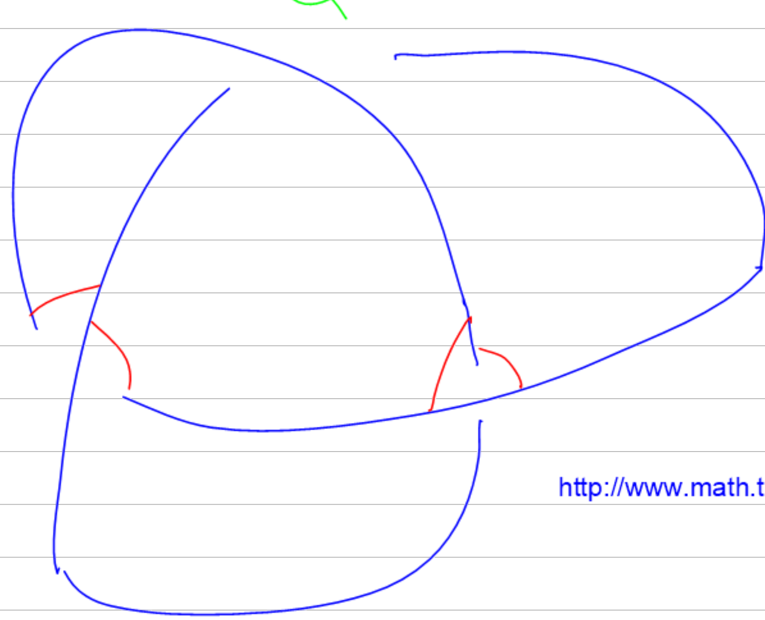
$V = W \circ S'$

Expansion $\Leftrightarrow$ UFTI

$Z: \mathcal{K} \longrightarrow \mathcal{A}$ $\qquad Z(K_D) = D + \dots$







http://www.math.toronto.edu/~drorbn/papers/ktgs/

$$\mathrm{ad}_a x := [a,x] = ax - xa$$
$$= L_a x - R_a x$$

$$e^{\mathrm{ad}_a} x = x + [a,x] + \frac{1}{2}[a,[a,x]] + \cdots$$
$$\| $$
$$e^{L_a - R_a} x = e^{L_a} e^{R_{-a}} x$$
$$= e^a x e^{-a}$$

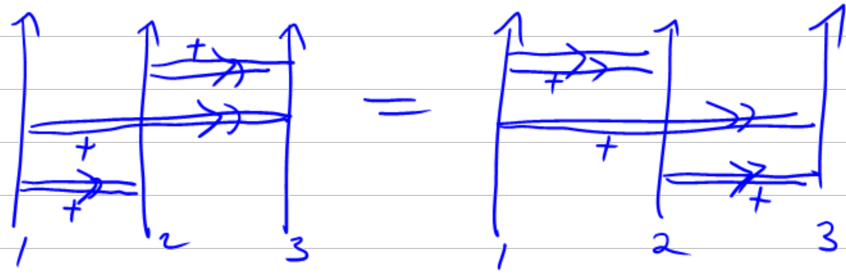
$$\dot{g}_s(x_i) = \frac{d}{ds} g_s(x_i)$$

$$L_s \in M_{n\times n}(\ )$$

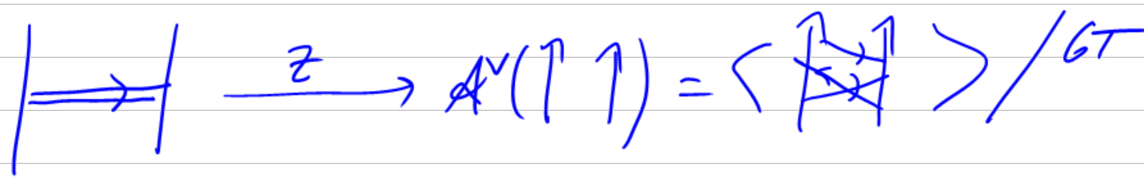
$$l\ (\dot{L}_s)(V) = \frac{d}{ds}(L_s V)$$
$$\|$$
$$E = \sum x_i \frac{\partial}{\partial x_i}$$

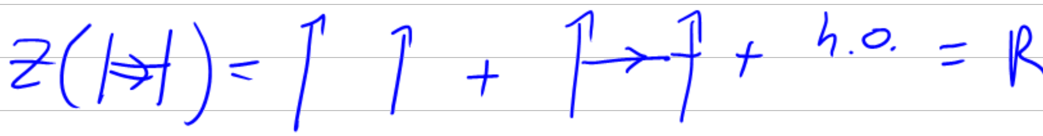
$$A_s(x^d) = s^d x^d \qquad A_s^{-1}(x^d) = s^{-d} x^d$$
$$\dot{A}_s(x^d) = d s^{d-1} x^d$$
$$\dot{A}_s A_s^{-1}(x^d) = \dot{A}_s(s^{-d} x^d) = d s^{-1} x^d$$

http://drorbn.net/AcademicPensieve/2012-12/nb/BCH-Lyndon_Question.pdf
http://drorbn.net/dbnvp/wClips-120418.php



$R_{12} R_{13} R_{23} = R_{23} R_{13} R_{12}$





$\square(V) = V \otimes V$ $\qquad$ $V = e^{P}$ up to incl. deg 7.

$V e^{-P} = 1 + \underset{\deg 8}{\rho} + \text{h.o.}$

$\square(V e^{-P}) = V e^{-P} \otimes V e^{-P}$

$\square(1+\rho) = (1+\rho) \otimes (1+\rho)$ $\quad$ up to deg 8

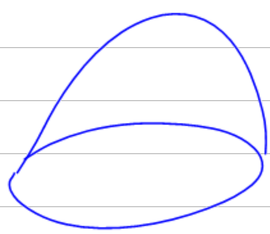
$\square \rho = 1 \otimes \rho + \rho \otimes 1$ $\quad$ so ρ is primitive.

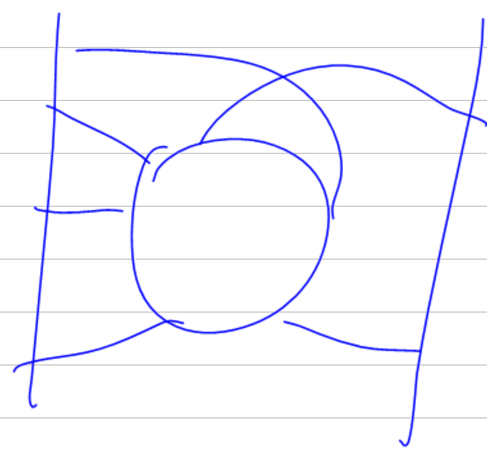
$P' = P + \rho$ $\qquad$ $V e^{-P'} = V e^{-P} e^{-\rho} = (1+\rho)(1-\rho) = 1$ $\quad$ up to deg 8

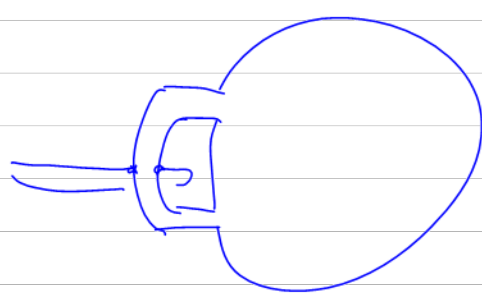
$$\mathfrak{g}^* \otimes \mathfrak{g}^* \otimes \mathfrak{g}^*$$

$$\langle A, B \rangle = tr(A \cdot B)$$

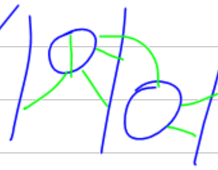
$$\langle X_{ij}, X_{kl} \rangle = \delta_{jk} \delta_{li}$$







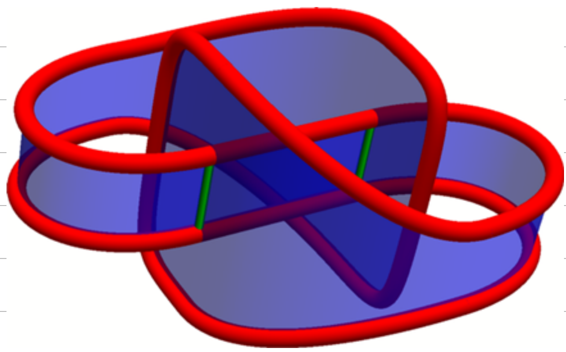
$[\,,\,]$ K_n : knots in comp of n poles.

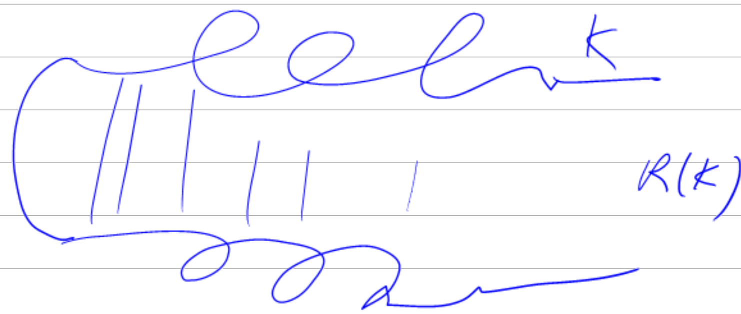
$[\,,\,]$ A_n : $gr\, K_n = \{$  $\}$ / rels

$[\,,\,]$ M_n : A_n in $gl(N)$: surface diagrams / rels

$$\begin{array}{c} M_n/hC \\ \uparrow \varphi \\ C_n \end{array} \qquad [\varphi(w_1), \varphi(w_2)] = \varphi([w_1, w_2])$$

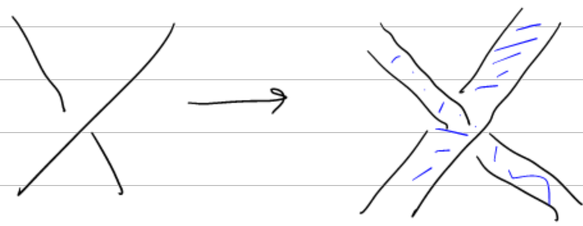
$[\,,\,]_{Gold}$ C_n : cyclic words

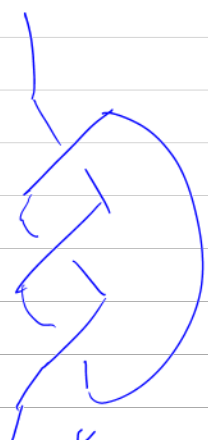




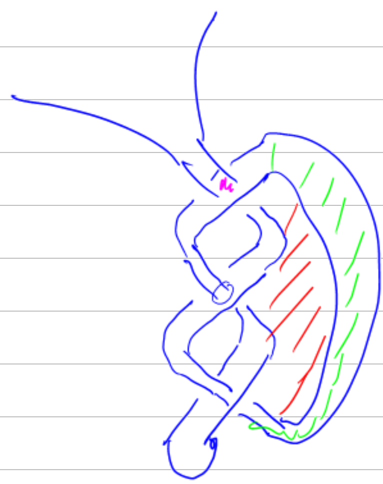
$$\Delta(R(K)) = \Delta(K)\Delta(\bar{K}) = \Delta^2$$

W/ ⟶ U



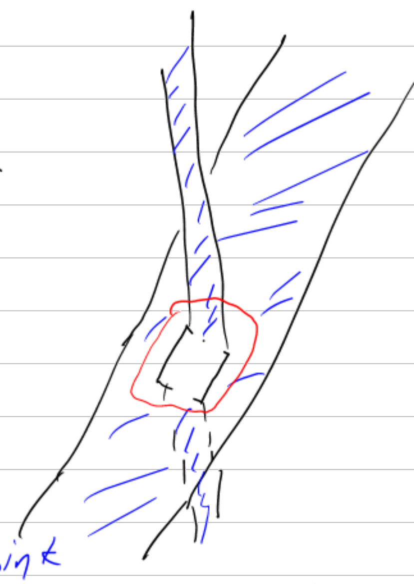


K



R(K)

	small loops	faces
small loops	0	A
faces	A^T	C



I think

$$\Delta(R(K)) = |\Delta(K)|^2$$

$$\begin{tikzcd} \gamma^*B \arrow[r, "\alpha"] \arrow[d, "\gamma"] & B \arrow[d, "\beta"] \\ C \arrow[r, "\gamma"'] & A \end{tikzcd}$$

"Equalizer"

$$\gamma^*B = \{(c,b) \in C \times B : \gamma c = \beta b\} \overset{\sim}{\cong} \beta^*C = B \oplus_A C$$

$$B \oplus_{\beta, A, \gamma} C$$

Claim? pullbacks of pushforward scenes are pushforward scenes

"push & pull commute"

$$\begin{tikzcd} \alpha^*S \ni B \oplus_A C \arrow[r, "\alpha"] \arrow[d] & B \ni S \arrow[d, "\beta"] \\ \gamma^*\beta_*S \quad C \arrow[r, "\gamma"] & A \ni \beta_*S \end{tikzcd}$$

if

$$\begin{tikzcd} B \oplus_A C \arrow[r] \arrow[d] & B \arrow[d] \\ C \arrow[r] & A \end{tikzcd}$$

I think

$$\begin{tikzcd} D \arrow[r, "\alpha"] \arrow[d, "\delta"] \arrow[dr, "\pi"] & B \arrow[d, "\beta"] \\ C \arrow[r, "\gamma"'] & A \end{tikzcd}$$

is a pullback scene, ie, $D \cong C \oplus_A B$

iff

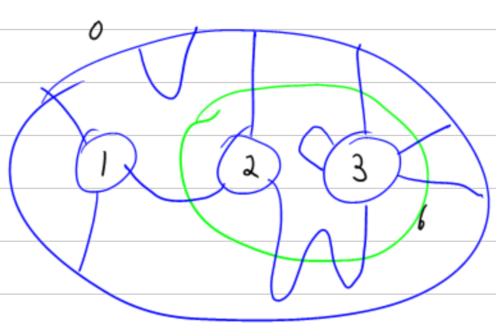
1. $\ker \alpha \cap \ker \delta = 0$
2. $\mathrm{im}\, \alpha \supset \ker \beta$ and $\mathrm{im}\, \delta \supset \ker \gamma$
3. $\ker \pi = \ker \alpha + \ker \delta$

Proof?

$$\begin{tikzcd} & A \oplus E \oplus F \arrow[dl, "\alpha"'] \arrow[dr, "\delta"] \arrow[dd, "\pi"] & \\ A \oplus B \oplus E \arrow[dr, "\beta"'] & & A \oplus C \oplus F \arrow[dl, "\gamma"] \\ & A \oplus B \oplus C \oplus D & \end{tikzcd}$$

This is the most general pullback diagram!

$$\begin{tikzcd} V_1 \arrow[dr, "\beta"'] & & V_2 \arrow[dl, "\gamma"] \\ & V_0 & \end{tikzcd}$$



$$\begin{tikzcd}[column sep=small] & & \langle \text{inner faces} \rangle \oplus_{\langle 6\text{-gaps} \rangle} \langle \text{outer faces} \rangle \arrow[d, equal] & & \\ & & \langle \text{big faces} \rangle \arrow[dll, bend right, "\psi"'] \arrow[dl, "\gamma"'] \arrow[dr, "\alpha"] \arrow[drr, bend left, "\phi"] & & \\ \langle 1\&2\&3 \text{ gaps} \rangle & \langle 1 \text{ gaps} \rangle \oplus \langle \text{inner faces} \rangle \arrow[l, "\mu"'] \arrow[dr, "\delta"'] & \text{good?} & \langle \text{outer faces} \rangle \arrow[r, "\nu"] \arrow[dl, "\beta"] & \langle 0 \text{ gaps} \rangle \\ & & \langle 1\&6 \text{ gaps} \rangle & & \end{tikzcd}$$