

# A Poly-Time Program for some HOMFLYPT Coefficients

(Ledvinka following Przytycki following Vertigan)

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Here's is some real mathematics written in TeX:  $e^x$ .

```
<< KnotTheory`
NormalizePD[K_PD] := K /. i_Integer => Mod[i, 2 Crossings[K]];
FlipCrossing[X[i_, j_, k_, L_]] :=
  If[PositiveQ[X[i, j, k, L]], X[L, i, j, k], X[j, k, L, i]];
FirstEdge[i_, j_] := If[Abs[j - i] == 1, Min[i, j], Max[i, j]];
OrderKnot[K_PD] :=
  Association @@
  Flatten @
  (List @@ K /. X[i_, j_, k_, L_] =>
    {i -> X[i, j, k, L], FirstEdge[j, L] -> X[i, j, k, L]});
```

Now comes the main program:

```
HomflyptCoefficient[K_PD, n_] :=
Module[{m, L, OK, UnderCrossingQ, P, Linking},
  m = 2 Crossings[K];
  L = NormalizePD[K];
  OK = OrderKnot[L];
  UnderCrossingQ[i_] := (First @ OK[i] == i);
  P[k_, i_, j_] := P[k, i, j] = Expand @ Module[{t},
    Piecewise[{
      {If[k == 0, 1, 0], j - i <= 4},
      {P[k, i + 1, j], t = FirstEdge @@ OK[i][[2, 4]]};
      Not @ UnderCrossingQ[i] || Not[i < t <= j]}],
    If[PositiveQ[OK[i]],
      -a-2 P[k, i + 1, j] + a-1 P[k, i, j, t],
      -a2 P[k, i + 1, j] + a P[k, i, j, t]
    ]];
  Linking[i_, j_, t_] :=
    Sum[If[UnderCrossingQ[l] &&
      (i + 1 <= FirstEdge @@ OK[l][[2, 4]] <= t - 1),
      If[PositiveQ[OK[l]], 1, -1], 0], {l, t + 1, j - 1}];
  P[0, i_, j_, t_] := (-a-2)Linking[i, j, t] (a + a-1)
    P[0, i + 1, t] P[0, t + 1, j];
  P[k_, i_, j_, t_] :=
    Module[{FlipCheck, e, Smooth, D, T, D0},
      FlipCheck[X[a_, b_, c_, d_]] := (e = FirstEdge[b, d];
        (a < i && Not[a < e < i]) || (a >= j && a > e > i));
      Smooth[D_, s_] := Module[{r, d1, d2, d3, A},
        r = FirstEdge @@ OK[s][[2, 4]];
        d1 = i + s - (t + 1); d2 = d1 + t - (r + 1);
        d3 = d2 + r - (i + 1);
        A = AssociationThread[
          Range[0, i] ~Join~ Range[t + 1, s] ~Join~
            Range[r + 1, t] ~Join~ Range[i + 1, r] ~Join~
              Range[s + 1, m - 1] ->
            Range[0, i] ~Join~ Range[i, d1] ~Join~
              Range[d1, d2] ~Join~ Range[d2, d3] ~Join~
                Range[d3, m - 5]];
        D /. X[a_, b_, c_, d_] =>
          If[MatchQ[X[a, b, c, d], OK[s]], ## &[],
            X[A[a], A[b], A[c], A[d]]];
        T[j] := (a + a-1) Sum[P[2 r, i + 1, t] P[k - 2 r, t + 1, j],
          {r, 0,  $\frac{k}{2}$ }]];
      T[s_] :=
        If[UnderCrossingQ[s] &&
          i + 1 <= FirstEdge @@ OK[s][[2, 4]] <= t - 1,
          D0 = Smooth[D, s];
          D = D /. OK[s] => FlipCrossing[OK[s]];
          If[PositiveQ[OK[s]],
            a-1 HomflyptCoefficient[D0, k - 2] - a-2 T[s + 1],
            a HomflyptCoefficient[D0, k - 2] - a2 T[s + 1]],
          T[s + 1]];
      D =
        L /. x_X => If[MatchQ[x, OK[i]], ## &[],
          If[FlipCheck[x], FlipCrossing[x], x]];
      T[t + 1]
    ];
  P[n, 0, m]
];
```

```
HomflyptCoefficient[K_, n_] :=  
  HomflyptCoefficient[PD@K, n];
```