A question about Interior Multiplication in $\mathbb{O}$

Before executing what follows, one needs to load packages “FreeLie.m”, “AwCalculus.m”, “FAA.m”, “EmergentChordDiagrams.m”

In[•]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\People\\Kuno"];
<< FreeLie.m
<< AwCalculus.m
<< FAA.m
<< EmergentChordDiagrams.m
```

```
FreeLie` implements / extends
 {*, +, **, $SeriesShowDegree, 〈〉, ∫, ≡, ad, Ad, adSeries, AllCyclicWords, AllLyndonWords,
  AllWords, Arbitrator, AS, ASeries, AW, b, BCH, BooleanSequence, BracketForm, BS, CC, Crop,
  cw, CW, CWS, CWSeries, D, Deg, DegreeScale, DerivationSeries, div, DK, DKS, DKSeries, EulerE,
  Exp, Inverse, j, J, JA, LieDerivation, LieMorphism, LieSeries, LS, LW, LyndonFactorization,
  Morphism, New, RandomCWSeries, Randomizer, RandomLieSeries, RC, SeriesSolve, Support,
  t, tb, TopBracketForm, tr, UndeterminedCoefficients, αMap, Γ, ι, Λ, σ, ℏ, ⌐, ⌢}.
```

```
FreeLie` is in the public domain. Dror Bar-Natan is committed
  to support it within reason until July 15, 2022. This is version 150814.
```

```
AwCalculus` implements / extends {*, **, ≡, dA, dc, deg, dm, dS, d△, dη, dσ, El, Es, hA,
  hm, hS, h△, hη, hσ, RandomElSeries, RandomEsSeries, tA, tha, tm, tS, t△, tη, tσ, Γ, Λ}.
```

```
AwCalculus` is in the public domain. Dror Bar-Natan is committed
  to support it within reason until July 15, 2022. This is version 150909.
```


```
FreeLie` implements / extends
 {*, +, **, $SeriesShowDegree, 〈〉, ∫, ≡, ad, Ad, adSeries, AllCyclicWords, AllLyndonWords,
  AllWords, Arbitrator, AS, ASeries, AW, b, BCH, BooleanSequence, BracketForm, BS, CC, Crop,
  cw, CW, CWS, CWSeries, D, Deg, DegreeScale, DerivationSeries, div, DK, DKS, DKSeries, EulerE,
  Exp, Inverse, j, J, JA, LieDerivation, LieMorphism, LieSeries, LS, LW, LyndonFactorization,
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```
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```


Let us consider the following two elements:

In[•]:=

$T1 = \mathbb{O}_{AR,\{x\},\{1\}}[\mathcal{A}_0[AW_1[x] + AW_1[x, x]]]$

$T2 = \mathbb{O}_{AR,\{x\},\{1\}}[\mathcal{A}_0[AW_1[] + AW_1[x] + AW_1[x, x]]]$

Out[•]=

$\mathbb{O}_{AR,\{x\},\{1\}}[\mathcal{A}_0[AW_1[x] + AW_1[x, x]]]$

Out[•]=

$\mathbb{O}_{AR,\{x\},\{1\}}[\mathcal{A}_0[AW_1[] + AW_1[x] + AW_1[x, x]]]$

In[∘]:=
```
IM_2[T1, T1]
IM_2[T2, T2]
IM_2[T1, T2]
```

Out[∘]=

$\mathbb{O}_{AR,\{x\},\{1\}}[\mathcal{A}_0[AW_1[x, x]]]$

Out[∘]=

$\mathbb{O}_{AR,\{x\},\{1\}}[\mathcal{A}_0[AW_1[] + 2\, AW_1[x] + 3\, AW_1[x, x]]]$

Out[∘]=

$\mathbb{O}_{AR,\{x\},\{1\}}[\mathcal{A}_0[AW_1[x] + 2\, AW_1[x, x]]]$

The first output, $IM_2[T1, T1]$, should not have the degree 3 part,
but it does ... It seems that IM_d does not return the correct answer when both the inputs have the trivial constant term. Why does it happen? Furthermore,
if we take powers of such an element, then a bug appears :

In[∘]:=
```
IM_2[T1, T1, T1]
IM_2[T1, T1, T1, T1]
```

Out[∘]=

0

Out[∘]=

$IM_2[0, \mathbb{O}_{AR,\{x\},\{1\}}[\mathcal{A}_0[AW_1[x] + AW_1[x, x]]]]$

It seems that the problem comes from applying the strand multiplication to the zero element in $\mathbb{O}$..

In[∘]:=
```
O_{AR,{x},{1,2}}[A_0[AW_1[] AW_2[]]]
O_{AR,{x},{1,2}}[A_0[AW_1[] AW_2[]]] // sm_{1,2→3}
O_{AR,{x},{1,2}}[A_0[0 AW_1[] AW_2[]]]
O_{AR,{x},{1,2}}[A_0[0 AW_1[] AW_2[]]] // sm_{1,2→3}
```

Out[∘]=

$\mathbb{O}_{AR,\{x\},\{1,2\}}[\mathcal{A}_0[AW_1[]\, AW_2[]]]$

Out[∘]=

$\mathbb{O}_{AR,\{x\},\{3\}}[\mathcal{A}_0[AW_3[]]]$

Out[∘]=

0

Out[∘]=

0