A question about Interior Multiplication in $\mathbb{O}$

Before executing what follows, one needs to load packages "FreeLie.m'', "AwCalculus.m'', "FAA.m'', "EmergentChordDiagrams.m''

Let us consider the following two elements:

```
In[•]:= T1 = 𝕆AR,{x},{1}[𝒜0[AW1[x] + AW1[x, x]]]
        T2 = 𝕆AR,{x},{1}[𝒜0[AW1[] + AW1[x] + AW1[x, x]]]
```

Out[•]=

$$\mathbb{O}_{AR,\{x\},\{1\}}\left[\mathcal{A}_0\left[AW_1[x] + AW_1[x, x]\right]\right]$$

Out[•]=

$$\mathbb{O}_{AR,\{x\},\{1\}}\left[\mathcal{A}_0\left[AW_1[] + AW_1[x] + AW_1[x, x]\right]\right]$$

```
In[•]:= IM2[T1, T1]
        IM2[T2, T2]
        IM2[T1, T2]
```

Out[•]=

$$\mathbb{O}_{AR,\{x\},\{1\}}\left[\mathcal{A}_0\left[AW_1[x, x] + 2\,AW_1[x, x, x]\right]\right]$$

Out[•]=

$$\mathbb{O}_{AR,\{x\},\{1\}}\left[\mathcal{A}_0\left[AW_1[] + 2\,AW_1[x] + 3\,AW_1[x, x]\right]\right]$$

Out[•]=

$$\mathbb{O}_{AR,\{x\},\{1\}}\left[\mathcal{A}_0\left[AW_1[x] + 2\,AW_1[x, x]\right]\right]$$

The first output, $IM_2[T1, T1]$, should not have the degree 3 part, but it does ... It seems that IM_d does not return the correct answer when both the inputs have the trivial constant term. Why does it happen? Furthermore, if we take powers of such an element, then a bug appears:

```
In[•]:= IM2[T1, T1, T1]
        IM2[T1, T1, T1, T1]
```

Out[•]=

$$\mathbb{O}_{AR,\{x\},\{1\}}\left[\mathcal{A}_0\left[AW_1[x, x, x]\right]\right]$$

Out[•]=

$$sm_{1,\vee\$27214[1]\to 1}[0]$$

It seems that the problem comes from applying the strand multiplication to the zero element in $\mathbb{O}$..

In[•]:=
```
𝕆AR,{x},{1,2}[𝒜0[AW1[] AW2[]]]
𝕆AR,{x},{1,2}[𝒜0[AW1[] AW2[]]] // sm1,2→3
𝕆AR,{x},{1,2}[𝒜0[0 AW1[] AW2[]]]
𝕆AR,{x},{1,2}[𝒜0[0 AW1[] AW2[]]] // sm1,2→3
```

Out[•]=
$$\mathbb{O}_{AR,\{x\},\{1,2\}}[\mathcal{A}_0[AW_1[]\ AW_2[]]]$$

Out[•]=
$$\mathbb{O}_{AR,\{x\},\{3\}}[\mathcal{A}_0[AW_3[]]]$$

Out[•]=
$$0$$

Out[•]=
$$sm_{1,2\to 3}[0]$$