

Pensieve Header: Converting the PPS associator to EmergentChordDiagrams language.

```
In[*]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\People\\Kuno"];
<< FreeLie.m
<< AwCalculus.m
<< FAA.m
<< EmergentChordDiagrams.m
```

FreeLie` implements / extends
 {*, +, **, \$SeriesShowDegree, <>, ∫, ≡, ad, Ad, adSeries, AllCyclicWords, AllLyndonWords, AllWords, Arbitrator, AS, ASeries, AW, b, BCH, BooleanSequence, BracketForm, BS, CC, Crop, cw, CW, CWS, CWSeries, D, Deg, DegreeScale, DerivationSeries, div, DK, DKS, DKSeries, EulerE, Exp, Inverse, j, J, JA, LieDerivation, LieMorphism, LieSeries, LS, LW, LyndonFactorization, Morphism, New, RandomCWSeries, Randomizer, RandomLieSeries, RC, SeriesSolve, Support, t, tb, TopBracketForm, tr, UndeterminedCoefficients, αMap, Γ, ℓ, Δ, σ, ħ, ↦, ↪}.

FreeLie` is in the public domain. Dror Bar-Natan is committed to support it within reason until July 15, 2022. This is version 150814.

AwCalculus` implements / extends {*, **, ≡, dA, dc, deg, dm, dS, dΔ, dη, dσ, El, Es, hA, hm, hS, hΔ, hη, hσ, RandomElSeries, RandomEsSeries, tA, tha, tm, tS, tΔ, tη, tσ, Γ, Δ}.

AwCalculus` is in the public domain. Dror Bar-Natan is committed to support it within reason until July 15, 2022. This is version 150909.

FreeLie` implements / extends
 {*, +, **, \$SeriesShowDegree, <>, ∫, ≡, ad, Ad, adSeries, AllCyclicWords, AllLyndonWords, AllWords, Arbitrator, AS, ASeries, AW, b, BCH, BooleanSequence, BracketForm, BS, CC, Crop, cw, CW, CWS, CWSeries, D, Deg, DegreeScale, DerivationSeries, div, DK, DKS, DKSeries, EulerE, Exp, Inverse, j, J, JA, LieDerivation, LieMorphism, LieSeries, LS, LW, LyndonFactorization, Morphism, New, RandomCWSeries, Randomizer, RandomLieSeries, RC, SeriesSolve, Support, t, tb, TopBracketForm, tr, UndeterminedCoefficients, αMap, Γ, ℓ, Δ, σ, ħ, ↦, ↪}.

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AwCalculus` implements / extends {*, **, ≡, dA, dc, deg, dm, dS, dΔ, dη, dσ, El, Es, hA, hm, hS, hΔ, hη, hσ, RandomElSeries, RandomEsSeries, tA, tha, tm, tS, tΔ, tη, tσ, Γ, Δ}.

AwCalculus` is in the public domain. Dror Bar-Natan is committed to support it within reason until July 15, 2022. This is version 150909.

```
In[*]:=  $\mathfrak{e}_0[2, 1] = \mathfrak{e}_0[3, 1] = \mathfrak{e}_0[3, 2] = 0;$ 
 $\mathfrak{e}_0[3, 1, 2] = 1 / 24; \mathfrak{e}_0 = \text{DKS}[3, \mathfrak{e}_0];$ 
SeriesSolve[ $\mathfrak{e}_0, \mathfrak{e}_0^{\sigma[3,2,1]} \equiv -\mathfrak{e}_0 \&\& \mathfrak{e}_0 ** \mathfrak{e}_0^{\sigma[1,23,4]} ** \mathfrak{e}_0^{\sigma[2,3,4]} \equiv \mathfrak{e}_0^{\sigma[12,3,4]} ** \mathfrak{e}_0^{\sigma[1,2,34]}$ ];
 $\varphi[n_] := \text{Sum}[\mathfrak{e}_0[k], \{k, n\}][[2]]$ 
```

In[*]:= $\varphi[6]$

SeriesSolve: In degree 3 arbitrarily setting $\{\Phi_0s[3, 1, 1, 2] \rightarrow 0\}$.

SeriesSolve: In degree 5 arbitrarily setting $\{\Phi_0s[3, 1, 1, 1, 1, 2] \rightarrow 0\}$.

Out[*]=

$$\frac{12}{24} - \frac{1 \ 1 \ 1 \ 2}{1440} + \frac{7 \ 1 \ 1 \ 2 \ 2}{5760} - \frac{7 \ 1 \ 2 \ 2 \ 2}{5760} + \frac{1 \ 1 \ 1 \ 1 \ 1 \ 2}{60480} - \frac{13 \ 1 \ 1 \ 1 \ 1 \ 2 \ 2}{241920} + \frac{11 \ 1 \ 1 \ 1 \ 2 \ 1 \ 2}{290304} +$$

$$\frac{83 \ 1 \ 1 \ 1 \ 2 \ 2 \ 2}{967680} + \frac{31 \ 1 \ 1 \ 2 \ 1 \ 2 \ 2}{725760} - \frac{157 \ 1 \ 1 \ 2 \ 2 \ 1 \ 2}{1935360} - \frac{31 \ 1 \ 1 \ 2 \ 2 \ 2 \ 2}{483840} - \frac{31 \ 1 \ 2 \ 1 \ 2 \ 2 \ 2}{387072} + \frac{31 \ 1 \ 2 \ 2 \ 2 \ 2 \ 2}{967680}$$

In[*]:= $\mathcal{L}[\varphi[4]]$

Out[*]=

$$\frac{1}{24} \text{AW}[1, 2] - \frac{1}{24} \text{AW}[2, 1] - \frac{\text{AW}[1, 1, 1, 2]}{1440} + \frac{1}{480} \text{AW}[1, 1, 2, 1] + \frac{7 \text{AW}[1, 1, 2, 2]}{5760} -$$

$$\frac{1}{480} \text{AW}[1, 2, 1, 1] - \frac{7 \text{AW}[1, 2, 1, 2]}{2880} - \frac{7 \text{AW}[1, 2, 2, 2]}{5760} + \frac{\text{AW}[2, 1, 1, 1]}{1440} +$$

$$\frac{7 \text{AW}[2, 1, 2, 1]}{2880} + \frac{7 \text{AW}[2, 1, 2, 2]}{1920} - \frac{7 \text{AW}[2, 2, 1, 1]}{5760} - \frac{7 \text{AW}[2, 2, 1, 2]}{1920} + \frac{7 \text{AW}[2, 2, 2, 1]}{5760}$$

```
In[*]:= AWExp_d[_L_] := Module[{t = AW[]},
  AW[] + Sum[t = Expand[t ** L / k] /. a_AW /; Length[a] > d => 0, {k, d}]
]
```

In[*]:= $\text{AWExp}_4[\mathcal{L}[\varphi[4]]]$

Out[*]=

$$\text{AW}[] + \frac{1}{24} \text{AW}[1, 2] - \frac{1}{24} \text{AW}[2, 1] - \frac{\text{AW}[1, 1, 1, 2]}{1440} + \frac{1}{480} \text{AW}[1, 1, 2, 1] +$$

$$\frac{7 \text{AW}[1, 1, 2, 2]}{5760} - \frac{1}{480} \text{AW}[1, 2, 1, 1] - \frac{1}{640} \text{AW}[1, 2, 1, 2] - \frac{\text{AW}[1, 2, 2, 1]}{1152} -$$

$$\frac{7 \text{AW}[1, 2, 2, 2]}{5760} + \frac{\text{AW}[2, 1, 1, 1]}{1440} - \frac{\text{AW}[2, 1, 1, 2]}{1152} + \frac{19 \text{AW}[2, 1, 2, 1]}{5760} +$$

$$\frac{7 \text{AW}[2, 1, 2, 2]}{1920} - \frac{7 \text{AW}[2, 2, 1, 1]}{5760} - \frac{7 \text{AW}[2, 2, 1, 2]}{1920} + \frac{7 \text{AW}[2, 2, 2, 1]}{5760}$$

$$In[*]:= AWExp_4[L[\varphi[4]]] /. aw_AW \to AW_1 @@ (aw /. {1 \to x, 2 \to y})$$

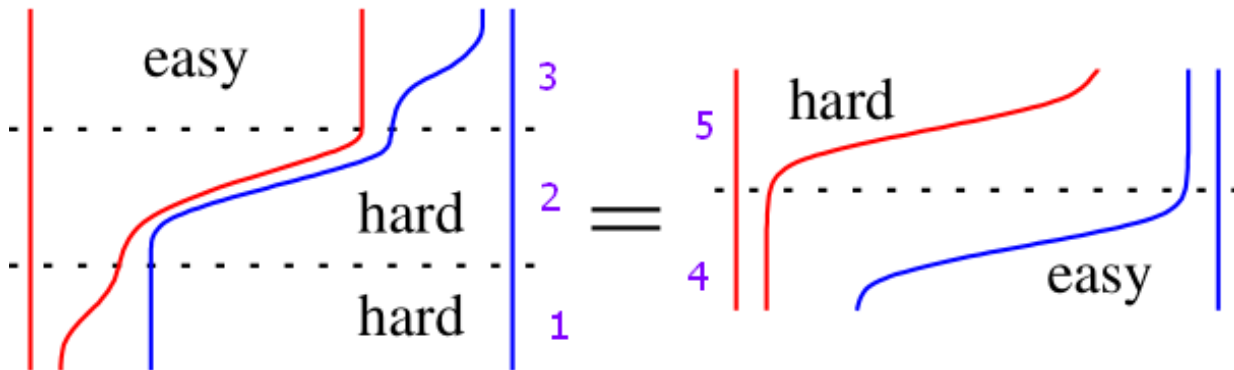
Out[*]=

$$\begin{aligned} & AW_1[] + \frac{1}{24} AW_1[x, y] - \frac{1}{24} AW_1[y, x] - \frac{AW_1[x, x, x, y]}{1440} + \frac{1}{480} AW_1[x, x, y, x] + \\ & \frac{7 AW_1[x, x, y, y]}{5760} - \frac{1}{480} AW_1[x, y, x, x] - \frac{1}{640} AW_1[x, y, x, y] - \frac{AW_1[x, y, y, x]}{1152} - \\ & \frac{7 AW_1[x, y, y, y]}{5760} + \frac{AW_1[y, x, x, x]}{1440} - \frac{AW_1[y, x, x, y]}{1152} + \frac{19 AW_1[y, x, y, x]}{5760} + \\ & \frac{7 AW_1[y, x, y, y]}{1920} - \frac{7 AW_1[y, y, x, x]}{5760} - \frac{7 AW_1[y, y, x, y]}{1920} + \frac{7 AW_1[y, y, y, x]}{5760} \end{aligned}$$

$$In[*]:= \Phi_{em} = O_{AR, \{x, y\}, \{1\}} [\mathcal{A}_0 [AWExp_4[L[\varphi[4]]] /. aw_AW \to AW_1 @@ (aw /. {1 \to x, 2 \to y})]]$$

Out[*]=

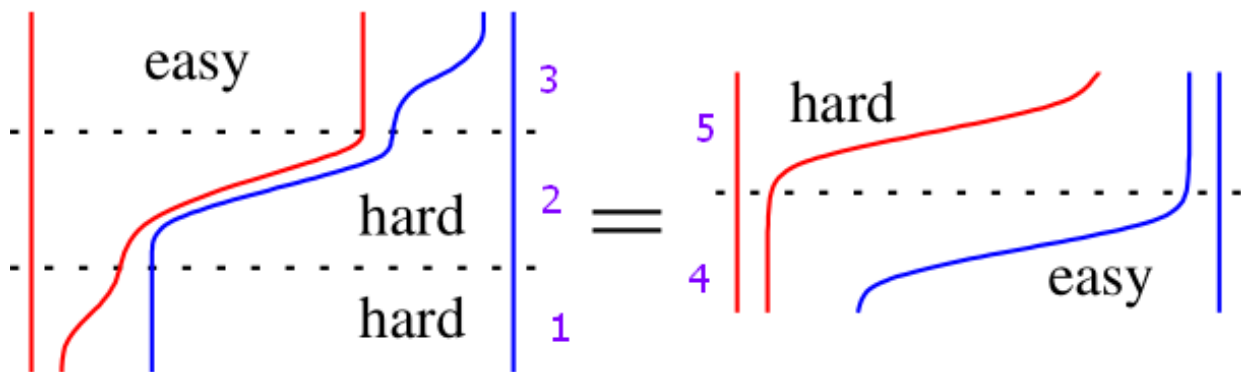
$$\begin{aligned} & O_{AR, \{x, y\}, \{1\}} \left[\mathcal{A}_0 \left[AW_1[] + \frac{1}{24} AW_1[x, y] - \frac{1}{24} AW_1[y, x] - \frac{AW_1[x, x, x, y]}{1440} + \frac{1}{480} AW_1[x, x, y, x] + \right. \right. \\ & \left. \frac{7 AW_1[x, x, y, y]}{5760} - \frac{1}{480} AW_1[x, y, x, x] - \frac{1}{640} AW_1[x, y, x, y] - \frac{AW_1[x, y, y, x]}{1152} - \right. \\ & \left. \frac{7 AW_1[x, y, y, y]}{5760} + \frac{AW_1[y, x, x, x]}{1440} - \frac{AW_1[y, x, x, y]}{1152} + \frac{19 AW_1[y, x, y, x]}{5760} + \right. \\ & \left. \frac{7 AW_1[y, x, y, y]}{1920} - \frac{7 AW_1[y, y, x, x]}{5760} - \frac{7 AW_1[y, y, x, y]}{1920} + \frac{7 AW_1[y, y, y, x]}{5760} \right] \end{aligned}$$



In[*]:= Pent1 = $\Phi_{em} // s\eta_2$

Out[*]=

$$\begin{aligned} & \mathcal{O}_{AR, \{x,y\}, \{1,2\}} \left[\mathcal{A}_0 \left[AW_1[] AW_2[] + \frac{1}{24} AW_1[x, y] AW_2[] - \right. \right. \\ & \frac{1}{24} AW_1[y, x] AW_2[] - \frac{AW_1[x, x, x, y] AW_2[]}{1440} + \frac{1}{480} AW_1[x, x, y, x] AW_2[] + \\ & \frac{7 AW_1[x, x, y, y] AW_2[]}{5760} - \frac{1}{480} AW_1[x, y, x, x] AW_2[] - \frac{1}{640} AW_1[x, y, x, y] AW_2[] - \\ & \frac{AW_1[x, y, y, x] AW_2[]}{1152} - \frac{7 AW_1[x, y, y, y] AW_2[]}{5760} + \frac{AW_1[y, x, x, x] AW_2[]}{1440} - \\ & \frac{AW_1[y, x, x, y] AW_2[]}{1152} + \frac{19 AW_1[y, x, y, x] AW_2[]}{5760} + \frac{7 AW_1[y, x, y, y] AW_2[]}{1920} - \\ & \left. \left. \frac{7 AW_1[y, y, x, x] AW_2[]}{5760} - \frac{7 AW_1[y, y, x, y] AW_2[]}{1920} + \frac{7 AW_1[y, y, y, x] AW_2[]}{5760} \right] \right] \end{aligned}$$

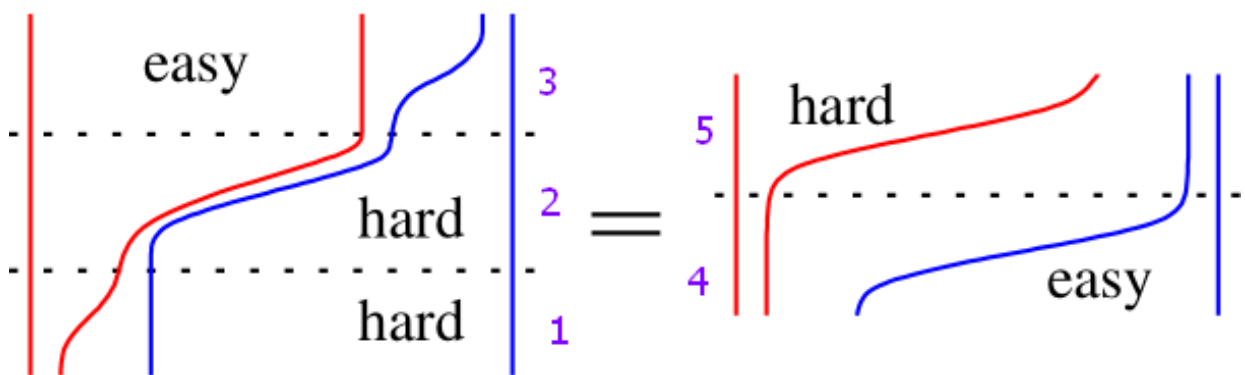


In[*]:= Pent2 = $\Phi_{em} // s\sigma_{1 \rightarrow 2} // p\Delta_{y \rightarrow y, z} // p2s_{z \rightarrow 1}$

Out[*]=

$$\begin{aligned} & \mathcal{O}_{AR, \{x,y\}, \{1,2\}} \left[\mathcal{A}_0 \left[AW_1[] AW_2[] + \frac{1}{24} AW_1[] AW_2[x, y] - \right. \right. \\ & \frac{1}{24} AW_1[] AW_2[y, x] - \frac{AW_1[] AW_2[x, x, x, y]}{1440} + \frac{1}{480} AW_1[] AW_2[x, x, y, x] + \\ & \frac{7 AW_1[] AW_2[x, x, y, y]}{5760} - \frac{1}{480} AW_1[] AW_2[x, y, x, x] - \frac{1}{640} AW_1[] AW_2[x, y, x, y] - \\ & \frac{AW_1[] AW_2[x, y, y, x]}{1152} - \frac{7 AW_1[] AW_2[x, y, y, y]}{5760} + \frac{AW_1[] AW_2[y, x, x, x]}{1440} - \\ & \frac{AW_1[] AW_2[y, x, x, y]}{1152} + \frac{19 AW_1[] AW_2[y, x, y, x]}{5760} + \frac{7 AW_1[] AW_2[y, x, y, y]}{1920} - \\ & \left. \left. \frac{7 AW_1[] AW_2[y, y, x, x]}{5760} - \frac{7 AW_1[] AW_2[y, y, x, y]}{1920} + \frac{7 AW_1[] AW_2[y, y, y, x]}{5760} \right] \right] + \\ & \mathcal{A}_C[1,2] \left[-\frac{1}{24} AW_1[x] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[] + \frac{AW_1[x, x, x] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{1440} - \right. \\ & \frac{AW_1[x, x, y] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{1152} + \frac{19 AW_1[x, y, x] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{5760} + \\ & \left. \frac{7 AW_1[x, y, y] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{1920} - \frac{7 AW_1[y, x, x] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{5760} \right] \end{aligned}$$

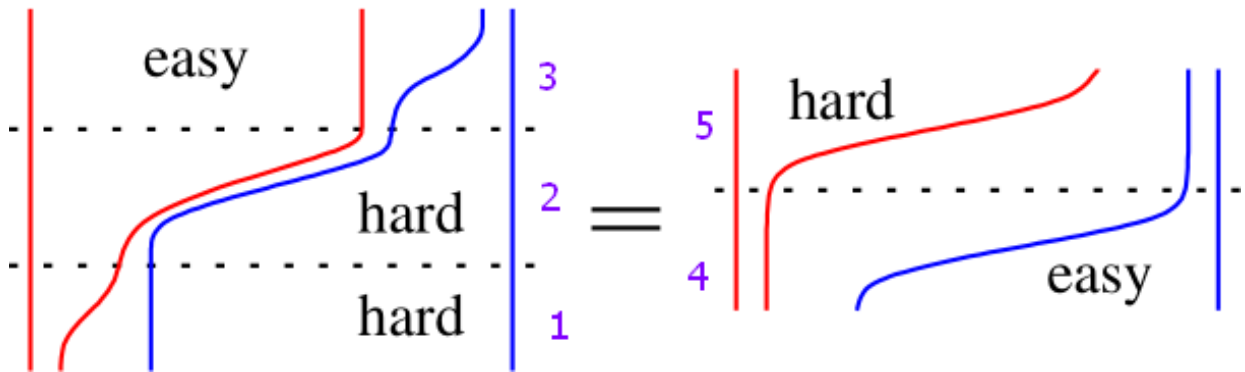
$$\begin{aligned}
 & \frac{7 AW_1[y, x, y] AW_2[] AW_1[] AW_2[]}{1920} + \frac{7 AW_1[y, y, x] AW_2[] AW_1[] AW_2[]}{5760} - \\
 & \frac{1}{576} AW_1[x] AW_2[x, y] AW_1[] AW_2[] + \frac{1}{576} AW_1[x] AW_2[y, x] AW_1[] AW_2[] + \\
 & \frac{1}{24} AW_1[] AW_2[] AW_1[x] AW_2[] - \frac{1}{480} AW_1[x, x] AW_2[] AW_1[x] AW_2[] - \\
 & \frac{1}{640} AW_1[x, y] AW_2[] AW_1[x] AW_2[] - \frac{AW_1[y, x] AW_2[] AW_1[x] AW_2[]}{1152} - \\
 & \frac{7 AW_1[y, y] AW_2[] AW_1[x] AW_2[]}{5760} + \frac{1}{576} AW_1[] AW_2[x, y] AW_1[x] AW_2[] - \\
 & \frac{1}{576} AW_1[] AW_2[y, x] AW_1[x] AW_2[] - \frac{7 AW_1[x, x] AW_2[] AW_1[y] AW_2[]}{5760} - \\
 & \frac{7 AW_1[x, y] AW_2[] AW_1[y] AW_2[]}{1920} + \frac{7 AW_1[y, x] AW_2[] AW_1[y] AW_2[]}{5760} + \\
 & \frac{1}{480} AW_1[x] AW_2[] AW_1[x, x] AW_2[] + \frac{7 AW_1[y] AW_2[] AW_1[x, x] AW_2[]}{5760} + \\
 & \frac{AW_1[x] AW_2[] AW_1[x, y] AW_2[]}{1152} - \frac{7 AW_1[y] AW_2[] AW_1[x, y] AW_2[]}{5760} + \\
 & \frac{1}{640} AW_1[x] AW_2[] AW_1[y, x] AW_2[] + \frac{7 AW_1[y] AW_2[] AW_1[y, x] AW_2[]}{1920} + \\
 & \frac{7 AW_1[x] AW_2[] AW_1[y, y] AW_2[]}{5760} - \frac{AW_1[] AW_2[] AW_1[x, x, x] AW_2[]}{1440} + \\
 & \frac{7 AW_1[] AW_2[] AW_1[x, x, y] AW_2[]}{5760} - \frac{19 AW_1[] AW_2[] AW_1[x, y, x] AW_2[]}{5760} - \\
 & \frac{7 AW_1[] AW_2[] AW_1[x, y, y] AW_2[]}{5760} + \frac{AW_1[] AW_2[] AW_1[y, x, x] AW_2[]}{1152} + \\
 & \left. \frac{7 AW_1[] AW_2[] AW_1[y, x, y] AW_2[]}{1920} - \frac{7 AW_1[] AW_2[] AW_1[y, y, x] AW_2[]}{1920} \right]
 \end{aligned}$$



In[*]:= Pent3 = $\bar{\varphi}_{em}$ // $s\sigma_{1 \rightarrow 2}$ // $p2s_{y \rightarrow 1}$ // $p\sigma_{x \rightarrow y}$ // $p\eta_x$

Out[*]=

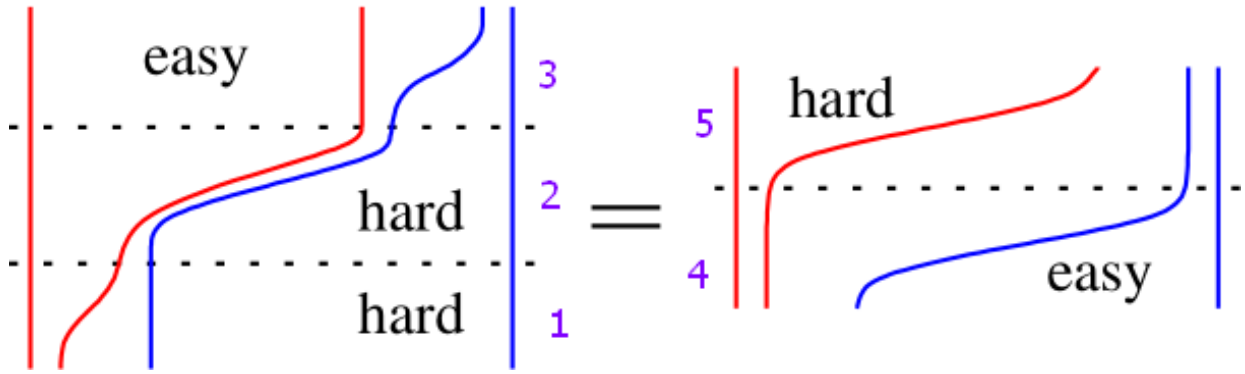
$$\mathcal{O}_{AR, \{x, y\}, \{1, 2\}} \left[\mathcal{A}_\theta [AW_1[] AW_2[]] + \mathcal{A}_c [1, 2] \left[-\frac{1}{24} AW_1[y] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[] + \frac{AW_1[y, y, y] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{1440} + \frac{1}{24} AW_1[] AW_2[] AW_{\bar{1}}[y] AW_{\bar{2}}[] - \frac{1}{480} AW_1[y, y] AW_2[] AW_{\bar{1}}[y] AW_{\bar{2}}[] + \frac{1}{480} AW_1[y] AW_2[] AW_{\bar{1}}[y, y] AW_{\bar{2}}[] - \frac{AW_1[] AW_2[] AW_{\bar{1}}[y, y, y] AW_{\bar{2}}[]}{1440} \right] \right]$$



In[*]:= Pent4 = $\bar{\Phi}_{em}$ // $\sigma_{1 \rightarrow 2}$ // $p2s_{y \rightarrow 1}$ // $p\Delta_{x \rightarrow x, y}$

Out[*]=

$$\begin{aligned} & \circlearrowleft_{AR, \{x, y\}, \{1, 2\}} \left[\right. \\ & \mathcal{A}_0 [AW_1[] AW_2[]] + \mathcal{A}_{c[1,2]} \left[-\frac{1}{24} AW_1[x] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[] - \frac{1}{24} AW_1[y] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[] + \right. \\ & \frac{AW_1[x, x, x] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{1440} + \frac{AW_1[x, x, y] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{1440} + \\ & \frac{AW_1[x, y, x] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{1440} + \frac{AW_1[x, y, y] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{1440} + \\ & \frac{AW_1[y, x, x] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{1440} + \frac{AW_1[y, x, y] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{1440} + \\ & \left. \frac{AW_1[y, y, x] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{1440} + \frac{AW_1[y, y, y] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{1440} + \right. \\ & \frac{1}{24} AW_1[] AW_2[] AW_{\bar{1}}[x] AW_{\bar{2}}[] - \frac{1}{480} AW_1[x, x] AW_2[] AW_{\bar{1}}[x] AW_{\bar{2}}[] - \\ & \frac{1}{480} AW_1[x, y] AW_2[] AW_{\bar{1}}[x] AW_{\bar{2}}[] - \frac{1}{480} AW_1[y, x] AW_2[] AW_{\bar{1}}[x] AW_{\bar{2}}[] - \\ & \frac{1}{480} AW_1[y, y] AW_2[] AW_{\bar{1}}[x] AW_{\bar{2}}[] + \frac{1}{24} AW_1[] AW_2[] AW_{\bar{1}}[y] AW_{\bar{2}}[] - \\ & \frac{1}{480} AW_1[x, x] AW_2[] AW_{\bar{1}}[y] AW_{\bar{2}}[] - \frac{1}{480} AW_1[x, y] AW_2[] AW_{\bar{1}}[y] AW_{\bar{2}}[] - \\ & \frac{1}{480} AW_1[y, x] AW_2[] AW_{\bar{1}}[y] AW_{\bar{2}}[] - \frac{1}{480} AW_1[y, y] AW_2[] AW_{\bar{1}}[y] AW_{\bar{2}}[] + \\ & \frac{1}{480} AW_1[x] AW_2[] AW_{\bar{1}}[x, x] AW_{\bar{2}}[] + \frac{1}{480} AW_1[y] AW_2[] AW_{\bar{1}}[x, x] AW_{\bar{2}}[] + \\ & \frac{1}{480} AW_1[x] AW_2[] AW_{\bar{1}}[x, y] AW_{\bar{2}}[] + \frac{1}{480} AW_1[y] AW_2[] AW_{\bar{1}}[x, y] AW_{\bar{2}}[] + \\ & \frac{1}{480} AW_1[x] AW_2[] AW_{\bar{1}}[y, x] AW_{\bar{2}}[] + \frac{1}{480} AW_1[y] AW_2[] AW_{\bar{1}}[y, x] AW_{\bar{2}}[] + \\ & \frac{1}{480} AW_1[x] AW_2[] AW_{\bar{1}}[y, y] AW_{\bar{2}}[] + \frac{1}{480} AW_1[y] AW_2[] AW_{\bar{1}}[y, y] AW_{\bar{2}}[] - \\ & \frac{AW_1[] AW_2[] AW_{\bar{1}}[x, x, x] AW_{\bar{2}}[]}{1440} - \frac{AW_1[] AW_2[] AW_{\bar{1}}[x, x, y] AW_{\bar{2}}[]}{1440} - \\ & \frac{AW_1[] AW_2[] AW_{\bar{1}}[x, y, x] AW_{\bar{2}}[]}{1440} - \frac{AW_1[] AW_2[] AW_{\bar{1}}[x, y, y] AW_{\bar{2}}[]}{1440} - \\ & \frac{AW_1[] AW_2[] AW_{\bar{1}}[y, x, x] AW_{\bar{2}}[]}{1440} - \frac{AW_1[] AW_2[] AW_{\bar{1}}[y, x, y] AW_{\bar{2}}[]}{1440} - \\ & \left. \frac{AW_1[] AW_2[] AW_{\bar{1}}[y, y, x] AW_{\bar{2}}[]}{1440} - \frac{AW_1[] AW_2[] AW_{\bar{1}}[y, y, y] AW_{\bar{2}}[]}{1440} \right] \end{aligned}$$

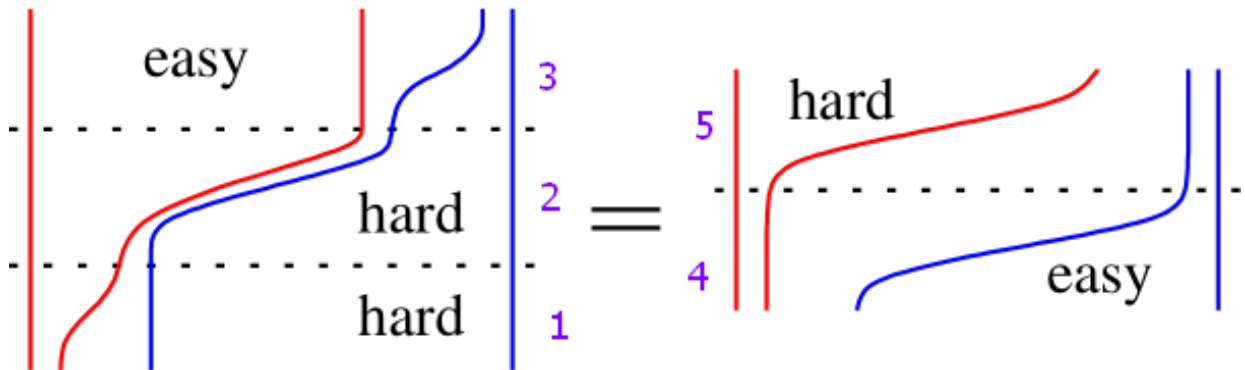


In[*]:= Pent5 = Φ_{em} // $S\Delta_{1 \rightarrow 1,2}$

Out[*]=

$$\begin{aligned} & \mathcal{O}_{AR, \{x,y\}, \{1,2\}} \left[\mathcal{A}_0 \left[AW_1[] AW_2[] + \frac{1}{24} AW_1[x, y] AW_2[] - \frac{1}{24} AW_1[y, x] AW_2[] - \frac{AW_1[x, x, x, y] AW_2[]}{1440} + \right. \right. \\ & \frac{1}{480} AW_1[x, x, y, x] AW_2[] + \frac{7 AW_1[x, x, y, y] AW_2[]}{5760} - \frac{1}{480} AW_1[x, y, x, x] AW_2[] - \\ & \frac{1}{640} AW_1[x, y, x, y] AW_2[] - \frac{AW_1[x, y, y, x] AW_2[]}{1152} - \frac{7 AW_1[x, y, y, y] AW_2[]}{5760} + \\ & \frac{AW_1[y, x, x, x] AW_2[]}{1440} - \frac{AW_1[y, x, x, y] AW_2[]}{1152} + \frac{19 AW_1[y, x, y, x] AW_2[]}{5760} + \\ & \frac{7 AW_1[y, x, y, y] AW_2[]}{1920} - \frac{7 AW_1[y, y, x, x] AW_2[]}{5760} - \frac{7 AW_1[y, y, x, y] AW_2[]}{1920} + \\ & \frac{7 AW_1[y, y, y, x] AW_2[]}{5760} + \frac{1}{24} AW_1[] AW_2[x, y] + \frac{1}{576} AW_1[x, y] AW_2[x, y] - \\ & \frac{1}{576} AW_1[y, x] AW_2[x, y] - \frac{1}{24} AW_1[] AW_2[y, x] - \frac{1}{576} AW_1[x, y] AW_2[y, x] + \\ & \frac{1}{576} AW_1[y, x] AW_2[y, x] - \frac{AW_1[] AW_2[x, x, x, y]}{1440} + \frac{1}{480} AW_1[] AW_2[x, x, y, x] + \\ & \frac{7 AW_1[] AW_2[x, x, y, y]}{5760} - \frac{1}{480} AW_1[] AW_2[x, y, x, x] - \frac{1}{640} AW_1[] AW_2[x, y, x, y] - \\ & \frac{AW_1[] AW_2[x, y, y, x]}{1152} - \frac{7 AW_1[] AW_2[x, y, y, y]}{5760} + \frac{AW_1[] AW_2[y, x, x, x]}{1440} - \\ & \frac{AW_1[] AW_2[y, x, x, y]}{1152} + \frac{19 AW_1[] AW_2[y, x, y, x]}{5760} + \frac{7 AW_1[] AW_2[y, x, y, y]}{1920} - \\ & \left. \frac{7 AW_1[] AW_2[y, y, x, x]}{5760} - \frac{7 AW_1[] AW_2[y, y, x, y]}{1920} + \frac{7 AW_1[] AW_2[y, y, y, x]}{5760} \right] + \\ & \mathcal{A}_{C[1,2]} \left[\frac{AW_1[x, x, y] AW_2[] AW_1[] AW_2[]}{5760} - \frac{AW_1[x, y, x] AW_2[] AW_1[] AW_2[]}{1152} + \right. \\ & \frac{7 AW_1[x, y, y] AW_2[] AW_1[] AW_2[]}{5760} - \frac{AW_1[y, x, x] AW_2[] AW_1[] AW_2[]}{5760} - \\ & \frac{AW_1[y, x, y] AW_2[] AW_1[] AW_2[]}{1152} - \frac{7 AW_1[y, y, x] AW_2[] AW_1[] AW_2[]}{5760} + \\ & \left. \frac{AW_1[x, y] AW_2[] AW_1[x] AW_2[]}{1920} + \frac{7 AW_1[y, x] AW_2[] AW_1[x] AW_2[]}{5760} + \right. \end{aligned}$$

$$\begin{aligned}
 & \frac{AW_1[y, y] AW_2[] AW_{\bar{1}}[x] AW_{\bar{2}}[]}{1152} + \frac{AW_1[x, x] AW_2[] AW_{\bar{1}}[y] AW_{\bar{2}}[]}{1152} - \\
 & \frac{1}{640} AW_1[x, y] AW_2[] AW_{\bar{1}}[y] AW_{\bar{2}}[] + \frac{19 AW_1[y, x] AW_2[] AW_{\bar{1}}[y] AW_{\bar{2}}[]}{5760} - \\
 & \frac{AW_1[y] AW_2[] AW_{\bar{1}}[x, x] AW_{\bar{2}}[]}{1152} - \frac{7 AW_1[x] AW_2[] AW_{\bar{1}}[x, y] AW_{\bar{2}}[]}{5760} - \\
 & \frac{19 AW_1[y] AW_2[] AW_{\bar{1}}[x, y] AW_{\bar{2}}[]}{5760} - \frac{AW_1[x] AW_2[] AW_{\bar{1}}[y, x] AW_{\bar{2}}[]}{1920} + \\
 & \frac{1}{640} AW_1[y] AW_2[] AW_{\bar{1}}[y, x] AW_{\bar{2}}[] - \frac{AW_1[x] AW_2[] AW_{\bar{1}}[y, y] AW_{\bar{2}}[]}{1152} + \\
 & \frac{AW_1[] AW_2[] AW_{\bar{1}}[x, x, y] AW_{\bar{2}}[]}{5760} + \frac{AW_1[] AW_2[] AW_{\bar{1}}[x, y, x] AW_{\bar{2}}[]}{1152} + \\
 & \frac{7 AW_1[] AW_2[] AW_{\bar{1}}[x, y, y] AW_{\bar{2}}[]}{5760} - \frac{AW_1[] AW_2[] AW_{\bar{1}}[y, x, x] AW_{\bar{2}}[]}{5760} + \\
 & \left. \left. \frac{AW_1[] AW_2[] AW_{\bar{1}}[y, x, y] AW_{\bar{2}}[]}{1152} - \frac{7 AW_1[] AW_2[] AW_{\bar{1}}[y, y, x] AW_{\bar{2}}[]}{5760} \right] \right]
 \end{aligned}$$



In[*]:= Pent12 = IM4[Pent1, Pent2]

Out[*]=

$$\begin{aligned}
 & \circ_{AR, \{x, y\}, \{1, 2\}} \left[\mathcal{F}_0 \left[AW_1[] AW_2[] + \frac{1}{24} AW_1[x, y] AW_2[] - \frac{1}{24} AW_1[y, x] AW_2[] - \frac{AW_1[x, x, x, y] AW_2[]}{1440} + \right. \right. \\
 & \frac{1}{480} AW_1[x, x, y, x] AW_2[] + \frac{7 AW_1[x, x, y, y] AW_2[]}{5760} - \frac{1}{480} AW_1[x, y, x, x] AW_2[] - \\
 & \frac{1}{640} AW_1[x, y, x, y] AW_2[] - \frac{AW_1[x, y, y, x] AW_2[]}{1152} - \frac{7 AW_1[x, y, y, y] AW_2[]}{5760} + \\
 & \frac{AW_1[y, x, x, x] AW_2[]}{1440} - \frac{AW_1[y, x, x, y] AW_2[]}{1152} + \frac{19 AW_1[y, x, y, x] AW_2[]}{5760} + \\
 & \frac{7 AW_1[y, x, y, y] AW_2[]}{1920} - \frac{7 AW_1[y, y, x, x] AW_2[]}{5760} - \frac{7 AW_1[y, y, x, y] AW_2[]}{1920} + \\
 & \frac{7 AW_1[y, y, y, x] AW_2[]}{5760} + \frac{1}{24} AW_1[] AW_2[x, y] + \frac{1}{576} AW_1[x, y] AW_2[x, y] - \\
 & \frac{1}{576} AW_1[y, x] AW_2[x, y] - \frac{1}{24} AW_1[] AW_2[y, x] - \frac{1}{576} AW_1[x, y] AW_2[y, x] + \\
 & \left. \left. \frac{1}{576} AW_1[y, x] AW_2[y, x] - \frac{AW_1[] AW_2[x, x, x, y]}{1440} + \frac{1}{480} AW_1[] AW_2[x, x, y, x] + \right. \right]
 \end{aligned}$$

$$\begin{aligned}
 & \left[\frac{7 AW_1[] AW_2[x, x, y, y]}{5760} - \frac{1}{480} AW_1[] AW_2[x, y, x, x] - \frac{1}{640} AW_1[] AW_2[x, y, x, y] - \right. \\
 & \frac{AW_1[] AW_2[x, y, y, x]}{1152} - \frac{7 AW_1[] AW_2[x, y, y, y]}{5760} + \frac{AW_1[] AW_2[y, x, x, x]}{1440} - \\
 & \frac{AW_1[] AW_2[y, x, x, y]}{1152} + \frac{19 AW_1[] AW_2[y, x, y, x]}{5760} + \frac{7 AW_1[] AW_2[y, x, y, y]}{1920} - \\
 & \left. \frac{7 AW_1[] AW_2[y, y, x, x]}{5760} - \frac{7 AW_1[] AW_2[y, y, x, y]}{1920} + \frac{7 AW_1[] AW_2[y, y, y, x]}{5760} \right] + \\
 & \mathcal{A}_{c[1,2]} \left[-\frac{1}{24} AW_1[x] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[] + \frac{AW_1[x, x, x] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{1440} - \right. \\
 & \frac{AW_1[x, x, y] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{1152} + \frac{1}{640} AW_1[x, y, x] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[] + \\
 & \frac{7 AW_1[x, y, y] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{1920} + \frac{AW_1[y, x, x] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{1920} - \\
 & \frac{7 AW_1[y, x, y] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{1920} + \frac{7 AW_1[y, y, x] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{5760} - \\
 & \frac{1}{576} AW_1[x] AW_2[x, y] AW_{\bar{1}}[] AW_{\bar{2}}[] + \frac{1}{576} AW_1[x] AW_2[y, x] AW_{\bar{1}}[] AW_{\bar{2}}[] + \\
 & \frac{1}{24} AW_1[] AW_2[] AW_{\bar{1}}[x] AW_{\bar{2}}[] - \frac{1}{480} AW_1[x, x] AW_2[] AW_{\bar{1}}[x] AW_{\bar{2}}[] + \\
 & \frac{AW_1[x, y] AW_2[] AW_{\bar{1}}[x] AW_{\bar{2}}[]}{5760} - \frac{1}{384} AW_1[y, x] AW_2[] AW_{\bar{1}}[x] AW_{\bar{2}}[] - \\
 & \frac{7 AW_1[y, y] AW_2[] AW_{\bar{1}}[x] AW_{\bar{2}}[]}{5760} + \frac{1}{576} AW_1[] AW_2[x, y] AW_{\bar{1}}[x] AW_{\bar{2}}[] - \\
 & \frac{1}{576} AW_1[] AW_2[y, x] AW_{\bar{1}}[x] AW_{\bar{2}}[] - \frac{7 AW_1[x, x] AW_2[] AW_{\bar{1}}[y] AW_{\bar{2}}[]}{5760} - \\
 & \frac{7 AW_1[x, y] AW_2[] AW_{\bar{1}}[y] AW_{\bar{2}}[]}{1920} + \frac{7 AW_1[y, x] AW_2[] AW_{\bar{1}}[y] AW_{\bar{2}}[]}{5760} + \\
 & \frac{1}{480} AW_1[x] AW_2[] AW_{\bar{1}}[x, x] AW_{\bar{2}}[] + \frac{7 AW_1[y] AW_2[] AW_{\bar{1}}[x, x] AW_{\bar{2}}[]}{5760} + \\
 & \frac{AW_1[x] AW_2[] AW_{\bar{1}}[x, y] AW_{\bar{2}}[]}{1152} - \frac{7 AW_1[y] AW_2[] AW_{\bar{1}}[x, y] AW_{\bar{2}}[]}{5760} + \\
 & \frac{1}{640} AW_1[x] AW_2[] AW_{\bar{1}}[y, x] AW_{\bar{2}}[] + \frac{7 AW_1[y] AW_2[] AW_{\bar{1}}[y, x] AW_{\bar{2}}[]}{1920} + \\
 & \frac{7 AW_1[x] AW_2[] AW_{\bar{1}}[y, y] AW_{\bar{2}}[]}{5760} - \frac{AW_1[] AW_2[] AW_{\bar{1}}[x, x, x] AW_{\bar{2}}[]}{1440} + \\
 & \frac{7 AW_1[] AW_2[] AW_{\bar{1}}[x, x, y] AW_{\bar{2}}[]}{5760} - \frac{19 AW_1[] AW_2[] AW_{\bar{1}}[x, y, x] AW_{\bar{2}}[]}{5760} - \\
 & \frac{7 AW_1[] AW_2[] AW_{\bar{1}}[x, y, y] AW_{\bar{2}}[]}{5760} + \frac{AW_1[] AW_2[] AW_{\bar{1}}[y, x, x] AW_{\bar{2}}[]}{1152} + \\
 & \left. \frac{7 AW_1[] AW_2[] AW_{\bar{1}}[y, x, y] AW_{\bar{2}}[]}{1920} - \frac{7 AW_1[] AW_2[] AW_{\bar{1}}[y, y, x] AW_{\bar{2}}[]}{1920} \right]
 \end{aligned}$$

In[*]:= Pent12 === (EM4[Pent1, Pent2 // sσ_{1→3} // sσ_{2→4}] // sm_{1,3→1} // sm_{2,4→2})

Out[*]=

True

In[*]:= **Pent123 = IM₄[Pent1, Pent2, Pent3]**

Out[*]=

$$\begin{aligned}
 & \circlearrowleft_{AR, \{x,y\}, \{1,2\}} \left[\mathcal{A}_0 \left[AW_1[] AW_2[] + \frac{1}{24} AW_1[x, y] AW_2[] - \frac{1}{24} AW_1[y, x] AW_2[] - \frac{AW_1[x, x, x, y] AW_2[]}{1440} + \right. \right. \\
 & \quad \frac{1}{480} AW_1[x, x, y, x] AW_2[] + \frac{7 AW_1[x, x, y, y] AW_2[]}{5760} - \frac{1}{480} AW_1[x, y, x, x] AW_2[] - \\
 & \quad \frac{1}{640} AW_1[x, y, x, y] AW_2[] - \frac{AW_1[x, y, y, x] AW_2[]}{1152} - \frac{7 AW_1[x, y, y, y] AW_2[]}{5760} + \\
 & \quad \frac{AW_1[y, x, x, x] AW_2[]}{1440} - \frac{AW_1[y, x, x, y] AW_2[]}{1152} + \frac{19 AW_1[y, x, y, x] AW_2[]}{5760} + \\
 & \quad \frac{7 AW_1[y, x, y, y] AW_2[]}{1920} - \frac{7 AW_1[y, y, x, x] AW_2[]}{5760} - \frac{7 AW_1[y, y, x, y] AW_2[]}{1920} + \\
 & \quad \frac{7 AW_1[y, y, y, x] AW_2[]}{5760} + \frac{1}{24} AW_1[] AW_2[x, y] + \frac{1}{576} AW_1[x, y] AW_2[x, y] - \\
 & \quad \frac{1}{576} AW_1[y, x] AW_2[x, y] - \frac{1}{24} AW_1[] AW_2[y, x] - \frac{1}{576} AW_1[x, y] AW_2[y, x] + \\
 & \quad \frac{1}{576} AW_1[y, x] AW_2[y, x] - \frac{AW_1[] AW_2[x, x, x, y]}{1440} + \frac{1}{480} AW_1[] AW_2[x, x, y, x] + \\
 & \quad \frac{7 AW_1[] AW_2[x, x, y, y]}{5760} - \frac{1}{480} AW_1[] AW_2[x, y, x, x] - \frac{1}{640} AW_1[] AW_2[x, y, x, y] - \\
 & \quad \frac{AW_1[] AW_2[x, y, y, x]}{1152} - \frac{7 AW_1[] AW_2[x, y, y, y]}{5760} + \frac{AW_1[] AW_2[y, x, x, x]}{1440} - \\
 & \quad \frac{AW_1[] AW_2[y, x, x, y]}{1152} + \frac{19 AW_1[] AW_2[y, x, y, x]}{5760} + \frac{7 AW_1[] AW_2[y, x, y, y]}{1920} - \\
 & \quad \left. \frac{7 AW_1[] AW_2[y, y, x, x]}{5760} - \frac{7 AW_1[] AW_2[y, y, x, y]}{1920} + \frac{7 AW_1[] AW_2[y, y, y, x]}{5760} \right] + \\
 & \mathcal{A}_C[1,2] \left[-\frac{1}{24} AW_1[x] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[] - \frac{1}{24} AW_1[y] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[] + \right. \\
 & \quad \frac{AW_1[x, x, x] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{1440} - \frac{AW_1[x, x, y] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{1152} + \\
 & \quad \frac{1}{640} AW_1[x, y, x] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[] + \frac{11 AW_1[x, y, y] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{5760} + \\
 & \quad \frac{AW_1[y, x, x] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{1920} - \frac{11 AW_1[y, x, y] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{5760} + \\
 & \quad \frac{7 AW_1[y, y, x] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{5760} + \frac{AW_1[y, y, y] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{1440} - \\
 & \quad \frac{1}{576} AW_1[x] AW_2[x, y] AW_{\bar{1}}[] AW_{\bar{2}}[] - \frac{1}{576} AW_1[y] AW_2[x, y] AW_{\bar{1}}[] AW_{\bar{2}}[] + \\
 & \quad \frac{1}{576} AW_1[x] AW_2[y, x] AW_{\bar{1}}[] AW_{\bar{2}}[] + \frac{1}{576} AW_1[y] AW_2[y, x] AW_{\bar{1}}[] AW_{\bar{2}}[] + \\
 & \quad \left. \frac{1}{24} AW_1[] AW_2[] AW_{\bar{1}}[x] AW_{\bar{2}}[] - \frac{1}{480} AW_1[x, x] AW_2[] AW_{\bar{1}}[x] AW_{\bar{2}}[] + \right.
 \end{aligned}$$

$$\begin{aligned}
 & \frac{AW_1[x, y] AW_2[] AW_{\bar{1}}[x] AW_{\bar{2}}[]}{5760} - \frac{1}{384} AW_1[y, x] AW_2[] AW_{\bar{1}}[x] AW_{\bar{2}}[] - \\
 & \frac{7 AW_1[y, y] AW_2[] AW_{\bar{1}}[x] AW_{\bar{2}}[]}{5760} + \frac{1}{576} AW_1[] AW_2[x, y] AW_{\bar{1}}[x] AW_{\bar{2}}[] - \\
 & \frac{1}{576} AW_1[] AW_2[y, x] AW_{\bar{1}}[x] AW_{\bar{2}}[] + \frac{1}{24} AW_1[] AW_2[] AW_{\bar{1}}[y] AW_{\bar{2}}[] - \\
 & \frac{7 AW_1[x, x] AW_2[] AW_{\bar{1}}[y] AW_{\bar{2}}[]}{5760} - \frac{11 AW_1[x, y] AW_2[] AW_{\bar{1}}[y] AW_{\bar{2}}[]}{5760} - \\
 & \frac{AW_1[y, x] AW_2[] AW_{\bar{1}}[y] AW_{\bar{2}}[]}{1920} - \frac{1}{480} AW_1[y, y] AW_2[] AW_{\bar{1}}[y] AW_{\bar{2}}[] + \\
 & \frac{1}{576} AW_1[] AW_2[x, y] AW_{\bar{1}}[y] AW_{\bar{2}}[] - \frac{1}{576} AW_1[] AW_2[y, x] AW_{\bar{1}}[y] AW_{\bar{2}}[] + \\
 & \frac{1}{480} AW_1[x] AW_2[] AW_{\bar{1}}[x, x] AW_{\bar{2}}[] + \frac{7 AW_1[y] AW_2[] AW_{\bar{1}}[x, x] AW_{\bar{2}}[]}{5760} + \\
 & \frac{AW_1[x] AW_2[] AW_{\bar{1}}[x, y] AW_{\bar{2}}[]}{1152} - \frac{7 AW_1[y] AW_2[] AW_{\bar{1}}[x, y] AW_{\bar{2}}[]}{5760} + \\
 & \frac{1}{640} AW_1[x] AW_2[] AW_{\bar{1}}[y, x] AW_{\bar{2}}[] + \frac{7 AW_1[y] AW_2[] AW_{\bar{1}}[y, x] AW_{\bar{2}}[]}{1920} + \\
 & \frac{7 AW_1[x] AW_2[] AW_{\bar{1}}[y, y] AW_{\bar{2}}[]}{5760} + \frac{1}{480} AW_1[y] AW_2[] AW_{\bar{1}}[y, y] AW_{\bar{2}}[] - \\
 & \frac{AW_1[] AW_2[] AW_{\bar{1}}[x, x, x] AW_{\bar{2}}[]}{1440} + \frac{7 AW_1[] AW_2[] AW_{\bar{1}}[x, x, y] AW_{\bar{2}}[]}{5760} - \\
 & \frac{19 AW_1[] AW_2[] AW_{\bar{1}}[x, y, x] AW_{\bar{2}}[]}{5760} - \frac{7 AW_1[] AW_2[] AW_{\bar{1}}[x, y, y] AW_{\bar{2}}[]}{5760} + \\
 & \frac{AW_1[] AW_2[] AW_{\bar{1}}[y, x, x] AW_{\bar{2}}[]}{1152} + \frac{7 AW_1[] AW_2[] AW_{\bar{1}}[y, x, y] AW_{\bar{2}}[]}{1920} - \\
 & \left. \frac{7 AW_1[] AW_2[] AW_{\bar{1}}[y, y, x] AW_{\bar{2}}[]}{1920} - \frac{AW_1[] AW_2[] AW_{\bar{1}}[y, y, y] AW_{\bar{2}}[]}{1440} \right]]
 \end{aligned}$$

In[]:= Pent45 = IM4[Pent4, Pent5]

Out[]:=

$$\begin{aligned}
 & \circlearrowleft_{AR, \{x, y\}, \{1, 2\}} \left[\mathcal{P}_0 \left[AW_1[] AW_2[] + \frac{1}{24} AW_1[x, y] AW_2[] - \frac{1}{24} AW_1[y, x] AW_2[] - \frac{AW_1[x, x, x, y] AW_2[]}{1440} + \right. \right. \\
 & \frac{1}{480} AW_1[x, x, y, x] AW_2[] + \frac{7 AW_1[x, x, y, y] AW_2[]}{5760} - \frac{1}{480} AW_1[x, y, x, x] AW_2[] - \\
 & \frac{1}{640} AW_1[x, y, x, y] AW_2[] - \frac{AW_1[x, y, y, x] AW_2[]}{1152} - \frac{7 AW_1[x, y, y, y] AW_2[]}{5760} + \\
 & \frac{AW_1[y, x, x, x] AW_2[]}{1440} - \frac{AW_1[y, x, x, y] AW_2[]}{1152} + \frac{19 AW_1[y, x, y, x] AW_2[]}{5760} + \\
 & \frac{7 AW_1[y, x, y, y] AW_2[]}{1920} - \frac{7 AW_1[y, y, x, x] AW_2[]}{5760} - \frac{7 AW_1[y, y, x, y] AW_2[]}{1920} + \\
 & \frac{7 AW_1[y, y, y, x] AW_2[]}{5760} + \frac{1}{24} AW_1[] AW_2[x, y] + \frac{1}{576} AW_1[x, y] AW_2[x, y] - \\
 & \left. \frac{1}{576} AW_1[y, x] AW_2[x, y] - \frac{1}{24} AW_1[] AW_2[y, x] - \frac{1}{576} AW_1[x, y] AW_2[y, x] + \right.
 \end{aligned}$$

$$\begin{aligned}
 & \frac{1}{576} AW_1[y, x] AW_2[y, x] - \frac{AW_1[] AW_2[x, x, x, y]}{1440} + \frac{1}{480} AW_1[] AW_2[x, x, y, x] + \\
 & \frac{7 AW_1[] AW_2[x, x, y, y]}{5760} - \frac{1}{480} AW_1[] AW_2[x, y, x, x] - \frac{1}{640} AW_1[] AW_2[x, y, x, y] - \\
 & \frac{AW_1[] AW_2[x, y, y, x]}{1152} - \frac{7 AW_1[] AW_2[x, y, y, y]}{5760} + \frac{AW_1[] AW_2[y, x, x, x]}{1440} - \\
 & \frac{AW_1[] AW_2[y, x, x, y]}{1152} + \frac{19 AW_1[] AW_2[y, x, y, x]}{5760} + \frac{7 AW_1[] AW_2[y, x, y, y]}{1920} - \\
 & \left. \frac{7 AW_1[] AW_2[y, y, x, x]}{5760} - \frac{7 AW_1[] AW_2[y, y, x, y]}{1920} + \frac{7 AW_1[] AW_2[y, y, y, x]}{5760} \right] + \\
 & \mathcal{A}_{C[1,2]} \left[-\frac{1}{24} AW_1[x] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[] - \frac{1}{24} AW_1[y] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[] + \right. \\
 & \frac{AW_1[x, x, x] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{1440} - \frac{AW_1[x, x, y] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{1152} + \\
 & \frac{1}{640} AW_1[x, y, x] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[] + \frac{11 AW_1[x, y, y] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{5760} + \\
 & \frac{AW_1[y, x, x] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{1920} - \frac{11 AW_1[y, x, y] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{5760} + \\
 & \frac{7 AW_1[y, y, x] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{5760} + \frac{AW_1[y, y, y] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{1440} - \\
 & \frac{1}{576} AW_1[x] AW_2[x, y] AW_{\bar{1}}[] AW_{\bar{2}}[] - \frac{1}{576} AW_1[y] AW_2[x, y] AW_{\bar{1}}[] AW_{\bar{2}}[] + \\
 & \frac{1}{576} AW_1[x] AW_2[y, x] AW_{\bar{1}}[] AW_{\bar{2}}[] + \frac{1}{576} AW_1[y] AW_2[y, x] AW_{\bar{1}}[] AW_{\bar{2}}[] + \\
 & \frac{1}{24} AW_1[] AW_2[] AW_{\bar{1}}[x] AW_{\bar{2}}[] - \frac{1}{480} AW_1[x, x] AW_2[] AW_{\bar{1}}[x] AW_{\bar{2}}[] + \\
 & \frac{AW_1[x, y] AW_2[] AW_{\bar{1}}[x] AW_{\bar{2}}[]}{5760} - \frac{1}{384} AW_1[y, x] AW_2[] AW_{\bar{1}}[x] AW_{\bar{2}}[] - \\
 & \frac{7 AW_1[y, y] AW_2[] AW_{\bar{1}}[x] AW_{\bar{2}}[]}{5760} + \frac{1}{576} AW_1[] AW_2[x, y] AW_{\bar{1}}[x] AW_{\bar{2}}[] - \\
 & \frac{1}{576} AW_1[] AW_2[y, x] AW_{\bar{1}}[x] AW_{\bar{2}}[] + \frac{1}{24} AW_1[] AW_2[] AW_{\bar{1}}[y] AW_{\bar{2}}[] - \\
 & \frac{7 AW_1[x, x] AW_2[] AW_{\bar{1}}[y] AW_{\bar{2}}[]}{5760} - \frac{11 AW_1[x, y] AW_2[] AW_{\bar{1}}[y] AW_{\bar{2}}[]}{5760} - \\
 & \frac{AW_1[y, x] AW_2[] AW_{\bar{1}}[y] AW_{\bar{2}}[]}{1920} - \frac{1}{480} AW_1[y, y] AW_2[] AW_{\bar{1}}[y] AW_{\bar{2}}[] + \\
 & \frac{1}{576} AW_1[] AW_2[x, y] AW_{\bar{1}}[y] AW_{\bar{2}}[] - \frac{1}{576} AW_1[] AW_2[y, x] AW_{\bar{1}}[y] AW_{\bar{2}}[] + \\
 & \frac{1}{480} AW_1[x] AW_2[] AW_{\bar{1}}[x, x] AW_{\bar{2}}[] + \frac{7 AW_1[y] AW_2[] AW_{\bar{1}}[x, x] AW_{\bar{2}}[]}{5760} + \\
 & \frac{AW_1[x] AW_2[] AW_{\bar{1}}[x, y] AW_{\bar{2}}[]}{1152} - \frac{7 AW_1[y] AW_2[] AW_{\bar{1}}[x, y] AW_{\bar{2}}[]}{5760} + \\
 & \frac{1}{640} AW_1[x] AW_2[] AW_{\bar{1}}[y, x] AW_{\bar{2}}[] + \frac{7 AW_1[y] AW_2[] AW_{\bar{1}}[y, x] AW_{\bar{2}}[]}{1920} +
 \end{aligned}$$

$$\frac{7 AW_1[x] AW_2[] AW_{\bar{1}}[y, y] AW_{\bar{2}}[]}{5760} + \frac{1}{480} AW_1[y] AW_2[] AW_{\bar{1}}[y, y] AW_{\bar{2}}[] - \frac{AW_1[] AW_2[] AW_{\bar{1}}[x, x, x] AW_{\bar{2}}[]}{1440} + \frac{7 AW_1[] AW_2[] AW_{\bar{1}}[x, x, y] AW_{\bar{2}}[]}{5760} - \frac{19 AW_1[] AW_2[] AW_{\bar{1}}[x, y, x] AW_{\bar{2}}[]}{5760} - \frac{7 AW_1[] AW_2[] AW_{\bar{1}}[x, y, y] AW_{\bar{2}}[]}{5760} + \frac{AW_1[] AW_2[] AW_{\bar{1}}[y, x, x] AW_{\bar{2}}[]}{1152} + \frac{7 AW_1[] AW_2[] AW_{\bar{1}}[y, x, y] AW_{\bar{2}}[]}{1920} - \frac{7 AW_1[] AW_2[] AW_{\bar{1}}[y, y, x] AW_{\bar{2}}[]}{1920} - \frac{AW_1[] AW_2[] AW_{\bar{1}}[y, y, y] AW_{\bar{2}}[]}{1440}]]$$

In[*]:= Pent123 === Pent45

Out[*]=

True

In[*]:= $\bar{\Phi}_{em,d} := \bar{\Phi}_{em,d} = \mathcal{O}_{AR, \{x,y\}, \{1\}} [\mathcal{A}_0 [AWExp_d [L[\varphi[d]]] /. aw_AW \Rightarrow AW_1 @@ (aw /. \{1 \rightarrow x, 2 \rightarrow y\})]]]$

In[*]:= $Pent[d_] := IM_d [\bar{\Phi}_{em,d} // s\eta_2, \bar{\Phi}_{em,d} // s\sigma_{1 \rightarrow 2} // p\Delta_{y \rightarrow y,z} // p2s_{z \rightarrow 1}, \bar{\Phi}_{em,d} // s\sigma_{1 \rightarrow 2} // p2s_{y \rightarrow 1} // p\sigma_{x \rightarrow y} // p\eta_x] === IM_d [\bar{\Phi}_{em,d} // s\sigma_{1 \rightarrow 2} // p2s_{y \rightarrow 1} // p\Delta_{x \rightarrow x,y}, \bar{\Phi}_{em,d} // s\Delta_{1 \rightarrow 1,2}]$

In[*]:= Temp1 = With[{d = 4},

$IM_d [\bar{\Phi}_{em,d} // s\eta_2, \bar{\Phi}_{em,d} // s\sigma_{1 \rightarrow 2} // p\Delta_{y \rightarrow y,z} // p2s_{z \rightarrow 1}, \bar{\Phi}_{em,d} // s\sigma_{1 \rightarrow 2} // p2s_{y \rightarrow 1} // p\sigma_{x \rightarrow y} // p\eta_x]]]$

Out[*]=

$$\mathcal{O}_{AR, \{x,y\}, \{1,2\}} [\mathcal{A}_0 [AW_1[] AW_2[] + \frac{1}{24} AW_1[x, y] AW_2[] - \frac{1}{24} AW_1[y, x] AW_2[] - \frac{AW_1[x, x, x, y] AW_2[]}{1440} + \frac{1}{480} AW_1[x, x, y, x] AW_2[] + \frac{7 AW_1[x, x, y, y] AW_2[]}{5760} - \frac{1}{480} AW_1[x, y, x, x] AW_2[] - \frac{1}{640} AW_1[x, y, x, y] AW_2[] - \frac{AW_1[x, y, y, x] AW_2[]}{1152} - \frac{7 AW_1[x, y, y, y] AW_2[]}{5760} + \frac{AW_1[y, x, x, x] AW_2[]}{1440} - \frac{AW_1[y, x, x, y] AW_2[]}{1152} + \frac{19 AW_1[y, x, y, x] AW_2[]}{5760} + \frac{7 AW_1[y, x, y, y] AW_2[]}{1920} - \frac{7 AW_1[y, y, x, x] AW_2[]}{5760} - \frac{7 AW_1[y, y, x, y] AW_2[]}{1920} + \frac{7 AW_1[y, y, y, x] AW_2[]}{5760} + \frac{1}{24} AW_1[] AW_2[x, y] + \frac{1}{576} AW_1[x, y] AW_2[x, y] - \frac{1}{576} AW_1[y, x] AW_2[x, y] - \frac{1}{24} AW_1[] AW_2[y, x] - \frac{1}{576} AW_1[x, y] AW_2[y, x] + \frac{1}{576} AW_1[y, x] AW_2[y, x] - \frac{AW_1[] AW_2[x, x, x, y]}{1440} + \frac{1}{480} AW_1[] AW_2[x, x, y, x] + \frac{7 AW_1[] AW_2[x, x, y, y]}{5760} - \frac{1}{480} AW_1[] AW_2[x, y, x, x] - \frac{1}{640} AW_1[] AW_2[x, y, x, y] - \frac{AW_1[] AW_2[x, y, y, x]}{1152} - \frac{7 AW_1[] AW_2[x, y, y, y]}{5760} + \frac{AW_1[] AW_2[y, x, x, x]}{1440}]]$$

$$\begin{aligned}
 & \left. \begin{aligned}
 & \frac{AW_1[] AW_2[y, x, x, y]}{1152} + \frac{19 AW_1[] AW_2[y, x, y, x]}{5760} + \frac{7 AW_1[] AW_2[y, x, y, y]}{1920} - \\
 & \frac{7 AW_1[] AW_2[y, y, x, x]}{5760} - \frac{7 AW_1[] AW_2[y, y, x, y]}{1920} + \frac{7 AW_1[] AW_2[y, y, y, x]}{5760} \right] + \\
 & \mathcal{A}_{C[1,2]} \left[-\frac{1}{24} AW_1[x] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[] - \frac{1}{24} AW_1[y] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[] + \right. \\
 & \left. \frac{AW_1[x, x, x] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{1440} - \frac{AW_1[x, x, y] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{1152} + \right. \\
 & \left. \frac{1}{640} AW_1[x, y, x] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[] + \frac{11 AW_1[x, y, y] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{5760} + \right. \\
 & \left. \frac{AW_1[y, x, x] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{1920} - \frac{11 AW_1[y, x, y] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{5760} + \right. \\
 & \left. \frac{7 AW_1[y, y, x] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{5760} + \frac{AW_1[y, y, y] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{1440} - \right. \\
 & \left. \frac{1}{576} AW_1[x] AW_2[x, y] AW_{\bar{1}}[] AW_{\bar{2}}[] - \frac{1}{576} AW_1[y] AW_2[x, y] AW_{\bar{1}}[] AW_{\bar{2}}[] + \right. \\
 & \left. \frac{1}{576} AW_1[x] AW_2[y, x] AW_{\bar{1}}[] AW_{\bar{2}}[] + \frac{1}{576} AW_1[y] AW_2[y, x] AW_{\bar{1}}[] AW_{\bar{2}}[] + \right. \\
 & \left. \frac{1}{24} AW_1[] AW_2[] AW_{\bar{1}}[x] AW_{\bar{2}}[] - \frac{1}{480} AW_1[x, x] AW_2[] AW_{\bar{1}}[x] AW_{\bar{2}}[] + \right. \\
 & \left. \frac{AW_1[x, y] AW_2[] AW_{\bar{1}}[x] AW_{\bar{2}}[]}{5760} - \frac{1}{384} AW_1[y, x] AW_2[] AW_{\bar{1}}[x] AW_{\bar{2}}[] - \right. \\
 & \left. \frac{7 AW_1[y, y] AW_2[] AW_{\bar{1}}[x] AW_{\bar{2}}[]}{5760} + \frac{1}{576} AW_1[] AW_2[x, y] AW_{\bar{1}}[x] AW_{\bar{2}}[] - \right. \\
 & \left. \frac{1}{576} AW_1[] AW_2[y, x] AW_{\bar{1}}[x] AW_{\bar{2}}[] + \frac{1}{24} AW_1[] AW_2[] AW_{\bar{1}}[y] AW_{\bar{2}}[] - \right. \\
 & \left. \frac{7 AW_1[x, x] AW_2[] AW_{\bar{1}}[y] AW_{\bar{2}}[]}{5760} - \frac{11 AW_1[x, y] AW_2[] AW_{\bar{1}}[y] AW_{\bar{2}}[]}{5760} - \right. \\
 & \left. \frac{AW_1[y, x] AW_2[] AW_{\bar{1}}[y] AW_{\bar{2}}[]}{1920} - \frac{1}{480} AW_1[y, y] AW_2[] AW_{\bar{1}}[y] AW_{\bar{2}}[] + \right. \\
 & \left. \frac{1}{576} AW_1[] AW_2[x, y] AW_{\bar{1}}[y] AW_{\bar{2}}[] - \frac{1}{576} AW_1[] AW_2[y, x] AW_{\bar{1}}[y] AW_{\bar{2}}[] + \right. \\
 & \left. \frac{1}{480} AW_1[x] AW_2[] AW_{\bar{1}}[x, x] AW_{\bar{2}}[] + \frac{7 AW_1[y] AW_2[] AW_{\bar{1}}[x, x] AW_{\bar{2}}[]}{5760} + \right. \\
 & \left. \frac{AW_1[x] AW_2[] AW_{\bar{1}}[x, y] AW_{\bar{2}}[]}{1152} - \frac{7 AW_1[y] AW_2[] AW_{\bar{1}}[x, y] AW_{\bar{2}}[]}{5760} + \right. \\
 & \left. \frac{1}{640} AW_1[x] AW_2[] AW_{\bar{1}}[y, x] AW_{\bar{2}}[] + \frac{7 AW_1[y] AW_2[] AW_{\bar{1}}[y, x] AW_{\bar{2}}[]}{1920} + \right. \\
 & \left. \frac{7 AW_1[x] AW_2[] AW_{\bar{1}}[y, y] AW_{\bar{2}}[]}{5760} + \frac{1}{480} AW_1[y] AW_2[] AW_{\bar{1}}[y, y] AW_{\bar{2}}[] - \right. \\
 & \left. \frac{AW_1[] AW_2[] AW_{\bar{1}}[x, x, x] AW_{\bar{2}}[]}{1440} + \frac{7 AW_1[] AW_2[] AW_{\bar{1}}[x, x, y] AW_{\bar{2}}[]}{5760} - \right. \\
 & \left. \frac{19 AW_1[] AW_2[] AW_{\bar{1}}[x, y, x] AW_{\bar{2}}[]}{5760} - \frac{7 AW_1[] AW_2[] AW_{\bar{1}}[x, y, y] AW_{\bar{2}}[]}{5760} + \right.
 \end{aligned}
 \end{aligned}$$

$$\frac{AW_1[] AW_2[] AW_1[y, x, x] AW_2[]}{1152} + \frac{7 AW_1[] AW_2[] AW_1[y, x, y] AW_2[]}{1920} - \left[\frac{7 AW_1[] AW_2[] AW_1[y, y, x] AW_2[]}{1920} - \frac{AW_1[] AW_2[] AW_1[y, y, y] AW_2[]}{1440} \right]$$

In[]:= Pent [4]

Out[]:= True

In[]:= $\Phi_{em,6}$

Out[]:=

$$\begin{aligned} & \circlearrowleft_{AR, \{x, y\}, \{1\}} \left[\mathcal{A}_0 \left[AW_1[] + \frac{1}{24} AW_1[x, y] - \frac{1}{24} AW_1[y, x] - \frac{AW_1[x, x, x, y]}{1440} + \frac{1}{480} AW_1[x, x, y, x] + \right. \right. \\ & \frac{7 AW_1[x, x, y, y]}{5760} - \frac{1}{480} AW_1[x, y, x, x] - \frac{1}{640} AW_1[x, y, x, y] - \frac{AW_1[x, y, y, x]}{1152} - \\ & \frac{7 AW_1[x, y, y, y]}{5760} + \frac{AW_1[y, x, x, x]}{1440} - \frac{AW_1[y, x, x, y]}{1152} + \frac{19 AW_1[y, x, y, x]}{5760} + \\ & \frac{7 AW_1[y, x, y, y]}{1920} - \frac{7 AW_1[y, y, x, x]}{5760} - \frac{7 AW_1[y, y, x, y]}{1920} + \frac{7 AW_1[y, y, y, x]}{5760} + \\ & \frac{AW_1[x, x, x, x, x, y]}{60480} - \frac{AW_1[x, x, x, x, y, x]}{12096} - \frac{13 AW_1[x, x, x, x, y, y]}{241920} + \\ & \frac{AW_1[x, x, x, y, x, x]}{6048} + \frac{19 AW_1[x, x, x, y, x, y]}{145152} + \frac{61 AW_1[x, x, x, y, y, x]}{725760} + \\ & \frac{83 AW_1[x, x, x, y, y, y]}{967680} - \frac{AW_1[x, x, y, x, x, x]}{6048} - \frac{17 AW_1[x, x, y, x, x, y]}{241920} - \\ & \frac{61 AW_1[x, x, y, x, y, x]}{241920} - \frac{89 AW_1[x, x, y, x, y, y]}{414720} + \frac{71 AW_1[x, x, y, y, x, y]}{967680} - \\ & \frac{337 AW_1[x, x, y, y, y, x]}{2903040} - \frac{31 AW_1[x, x, y, y, y, y]}{483840} + \frac{AW_1[x, y, x, x, x, x]}{12096} + \\ & \frac{13 AW_1[x, y, x, x, x, y]}{725760} + \frac{AW_1[x, y, x, x, y, x]}{11520} + \frac{37 AW_1[x, y, x, x, y, y]}{580608} + \\ & \frac{AW_1[x, y, x, y, x, x]}{6048} + \frac{79 AW_1[x, y, x, y, x, y]}{967680} + \frac{71 AW_1[x, y, x, y, y, x]}{322560} + \\ & \frac{73 AW_1[x, y, x, y, y, y]}{483840} - \frac{AW_1[x, y, y, x, x, x]}{18144} - \frac{53 AW_1[x, y, y, x, x, y]}{967680} - \\ & \frac{23 AW_1[x, y, y, x, y, x]}{193536} - \frac{11 AW_1[x, y, y, x, y, y]}{161280} + \frac{19 AW_1[x, y, y, y, x, x]}{290304} - \\ & \frac{AW_1[x, y, y, y, x, y]}{193536} + \frac{7 AW_1[x, y, y, y, y, x]}{138240} + \frac{31 AW_1[x, y, y, y, y, y]}{967680} - \\ & \frac{AW_1[y, x, x, x, x, x]}{60480} + \frac{AW_1[y, x, x, x, x, y]}{34560} - \frac{97 AW_1[y, x, x, x, y, x]}{725760} - \\ & \frac{103 AW_1[y, x, x, x, y, y]}{967680} + \frac{19 AW_1[y, x, x, y, x, x]}{120960} + \frac{583 AW_1[y, x, x, y, x, y]}{2903040} + \\ & \frac{53 AW_1[y, x, x, y, y, x]}{967680} + \frac{17 AW_1[y, x, x, y, y, y]}{161280} - \frac{29 AW_1[y, x, y, x, x, x]}{181440} - \end{aligned}$$

$$\begin{aligned}
 & \frac{289 \text{ AW}_1[y, x, y, x, x, y]}{2903040} - \frac{55 \text{ AW}_1[y, x, y, x, y, x]}{193536} - \frac{17 \text{ AW}_1[y, x, y, x, y, y]}{53760} - \\
 & \frac{11 \text{ AW}_1[y, x, y, y, x, x]}{483840} + \frac{7 \text{ AW}_1[y, x, y, y, x, y]}{46080} - \frac{191 \text{ AW}_1[y, x, y, y, y, x]}{967680} - \\
 & \frac{31 \text{ AW}_1[y, x, y, y, y, y]}{193536} + \frac{13 \text{ AW}_1[y, y, x, x, x, x]}{241920} + \frac{\text{ AW}_1[y, y, x, x, x, y]}{17920} - \\
 & \frac{19 \text{ AW}_1[y, y, x, x, y, x]}{1451520} + \frac{89 \text{ AW}_1[y, y, x, y, x, x]}{414720} + \frac{53 \text{ AW}_1[y, y, x, y, x, y]}{322560} + \\
 & \frac{71 \text{ AW}_1[y, y, x, y, y, x]}{322560} + \frac{31 \text{ AW}_1[y, y, x, y, y, y]}{96768} - \frac{83 \text{ AW}_1[y, y, y, x, x, x]}{967680} - \\
 & \frac{53 \text{ AW}_1[y, y, y, x, x, y]}{967680} - \frac{13 \text{ AW}_1[y, y, y, x, y, x]}{64512} - \frac{31 \text{ AW}_1[y, y, y, x, y, y]}{96768} + \\
 & \frac{31 \text{ AW}_1[y, y, y, y, x, x]}{483840} + \frac{31 \text{ AW}_1[y, y, y, y, x, y]}{193536} - \frac{31 \text{ AW}_1[y, y, y, y, y, x]}{967680} \Big]]
 \end{aligned}$$

In[*]:= Pent [6]
 Out[*]= True

Now some computations in HR

In[*]:= $\Phi_{HR,d} := \Phi_{HR,d} = \mathcal{O}_{HR,\{x,y\},\{1\}}[\mathcal{A}_0[\text{AWExp}_d[\mathcal{L}[\varphi[d]]]] /. \text{aw_AW} \rightarrow \text{AW}_1 @@ (\text{aw} /. \{1 \rightarrow x, 2 \rightarrow y\})]$

In[*]:= $\text{Pent}_{HR}[d_] := \text{IM}_d[\Phi_{HR,d} // s\eta_2, \Phi_{HR,d} // s\sigma_{1 \rightarrow 2} // p\Delta_{y \rightarrow y,z} // p2s_{z \rightarrow 1}, \Phi_{HR,d} // s\sigma_{1 \rightarrow 2} // p2s_{y \rightarrow 1} // p\sigma_{x \rightarrow y} // p\eta_x] === \text{IM}_d[\Phi_{HR,d} // s\sigma_{1 \rightarrow 2} // p2s_{y \rightarrow 1} // p\Delta_{x \rightarrow x,y}, \Phi_{HR,d} // s\Delta_{1 \rightarrow 1,2}]$

In[*]:= Pent_{HR} [2]
 Out[*]= True

In[*]:= Temp2 = With[{d = 4},
 $\text{IM}_d[\Phi_{HR,d} // s\eta_2, \Phi_{HR,d} // s\sigma_{1 \rightarrow 2} // p\Delta_{y \rightarrow y,z} // p2s_{z \rightarrow 1}, \Phi_{HR,d} // s\sigma_{1 \rightarrow 2} // p2s_{y \rightarrow 1} // p\sigma_{x \rightarrow y} // p\eta_x]$

Out[*]=

$$\begin{aligned}
 & \mathcal{O}_{HR,\{x,y\},\{1,2\}} \left[\mathcal{A}_0 \left[\text{AW}_1[] \text{AW}_2[] + \frac{1}{24} \text{AW}_1[x, y] \text{AW}_2[] - \frac{1}{24} \text{AW}_1[y, x] \text{AW}_2[] - \frac{\text{AW}_1[x, x, x, y] \text{AW}_2[]}{1440} + \right. \right. \\
 & \frac{1}{480} \text{AW}_1[x, x, y, x] \text{AW}_2[] + \frac{7 \text{AW}_1[x, x, y, y] \text{AW}_2[]}{5760} - \frac{1}{480} \text{AW}_1[x, y, x, x] \text{AW}_2[] - \\
 & \frac{1}{640} \text{AW}_1[x, y, x, y] \text{AW}_2[] - \frac{\text{AW}_1[x, y, y, x] \text{AW}_2[]}{1152} - \frac{7 \text{AW}_1[x, y, y, y] \text{AW}_2[]}{5760} + \\
 & \frac{\text{AW}_1[y, x, x, x] \text{AW}_2[]}{1440} - \frac{\text{AW}_1[y, x, x, y] \text{AW}_2[]}{1152} + \frac{19 \text{AW}_1[y, x, y, x] \text{AW}_2[]}{5760} + \\
 & \left. \frac{7 \text{AW}_1[y, x, y, y] \text{AW}_2[]}{1920} - \frac{7 \text{AW}_1[y, y, x, x] \text{AW}_2[]}{5760} - \frac{7 \text{AW}_1[y, y, x, y] \text{AW}_2[]}{1920} + \right]
 \end{aligned}$$

$$\begin{aligned}
 & \frac{7 AW_1[y, y, y, x] AW_2[]}{5760} + \frac{1}{24} AW_1[] AW_2[x, y] + \frac{1}{576} AW_1[x, y] AW_2[x, y] - \\
 & \frac{1}{576} AW_1[y, x] AW_2[x, y] - \frac{1}{24} AW_1[] AW_2[y, x] - \frac{1}{576} AW_1[x, y] AW_2[y, x] + \\
 & \frac{1}{576} AW_1[y, x] AW_2[y, x] - \frac{AW_1[] AW_2[x, x, x, y]}{1440} + \frac{1}{480} AW_1[] AW_2[x, x, y, x] + \\
 & \frac{7 AW_1[] AW_2[x, x, y, y]}{5760} - \frac{1}{480} AW_1[] AW_2[x, y, x, x] - \frac{1}{640} AW_1[] AW_2[x, y, x, y] - \\
 & \frac{AW_1[] AW_2[x, y, y, x]}{1152} - \frac{7 AW_1[] AW_2[x, y, y, y]}{5760} + \frac{AW_1[] AW_2[y, x, x, x]}{1440} - \\
 & \frac{AW_1[] AW_2[y, x, x, y]}{1152} + \frac{19 AW_1[] AW_2[y, x, y, x]}{5760} + \frac{7 AW_1[] AW_2[y, x, y, y]}{1920} - \\
 & \left. \frac{7 AW_1[] AW_2[y, y, x, x]}{5760} - \frac{7 AW_1[] AW_2[y, y, x, y]}{1920} + \frac{7 AW_1[] AW_2[y, y, y, x]}{5760} \right] + \\
 & \mathcal{A}_{C[1,2]} \left[-\frac{1}{24} AW_1[x] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[] - \frac{1}{24} AW_1[y] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[] + \right. \\
 & \frac{AW_1[x, x, x] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{1440} - \frac{AW_1[x, x, y] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{1152} + \\
 & \frac{1}{640} AW_1[x, y, x] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[] + \frac{11 AW_1[x, y, y] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{5760} + \\
 & \frac{AW_1[y, x, x] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{1920} - \frac{11 AW_1[y, x, y] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{5760} + \\
 & \frac{7 AW_1[y, y, x] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{5760} + \frac{AW_1[y, y, y] AW_2[] AW_{\bar{1}}[] AW_{\bar{2}}[]}{1440} + \\
 & \frac{1}{24} AW_1[] AW_2[x] AW_{\bar{1}}[] AW_{\bar{2}}[] - \frac{1}{480} AW_1[x, x] AW_2[x] AW_{\bar{1}}[] AW_{\bar{2}}[] + \\
 & \frac{AW_1[x, y] AW_2[x] AW_{\bar{1}}[] AW_{\bar{2}}[]}{5760} - \frac{1}{384} AW_1[y, x] AW_2[x] AW_{\bar{1}}[] AW_{\bar{2}}[] - \\
 & \frac{7 AW_1[y, y] AW_2[x] AW_{\bar{1}}[] AW_{\bar{2}}[]}{5760} + \frac{1}{24} AW_1[] AW_2[y] AW_{\bar{1}}[] AW_{\bar{2}}[] - \\
 & \frac{7 AW_1[x, x] AW_2[y] AW_{\bar{1}}[] AW_{\bar{2}}[]}{5760} - \frac{11 AW_1[x, y] AW_2[y] AW_{\bar{1}}[] AW_{\bar{2}}[]}{5760} - \\
 & \frac{AW_1[y, x] AW_2[y] AW_{\bar{1}}[] AW_{\bar{2}}[]}{1920} - \frac{1}{480} AW_1[y, y] AW_2[y] AW_{\bar{1}}[] AW_{\bar{2}}[] + \\
 & \frac{1}{480} AW_1[x] AW_2[x, x] AW_{\bar{1}}[] AW_{\bar{2}}[] + \frac{7 AW_1[y] AW_2[x, x] AW_{\bar{1}}[] AW_{\bar{2}}[]}{5760} - \\
 & \frac{AW_1[x] AW_2[x, y] AW_{\bar{1}}[] AW_{\bar{2}}[]}{1152} - \frac{17 AW_1[y] AW_2[x, y] AW_{\bar{1}}[] AW_{\bar{2}}[]}{5760} + \\
 & \frac{19 AW_1[x] AW_2[y, x] AW_{\bar{1}}[] AW_{\bar{2}}[]}{5760} + \frac{31 AW_1[y] AW_2[y, x] AW_{\bar{1}}[] AW_{\bar{2}}[]}{5760} + \\
 & \frac{7 AW_1[x] AW_2[y, y] AW_{\bar{1}}[] AW_{\bar{2}}[]}{5760} + \frac{1}{480} AW_1[y] AW_2[y, y] AW_{\bar{1}}[] AW_{\bar{2}}[] - \\
 & \frac{AW_1[] AW_2[x, x, x] AW_{\bar{1}}[] AW_{\bar{2}}[]}{1440} + \frac{7 AW_1[] AW_2[x, x, y] AW_{\bar{1}}[] AW_{\bar{2}}[]}{5760} -
 \end{aligned}$$

$$\left[\frac{1}{640} AW_1[] AW_2[x, y, x] AW_{\bar{1}}[] AW_{\bar{2}}[] + \frac{AW_1[] AW_2[x, y, y] AW_{\bar{1}}[] AW_{\bar{2}}[]}{1920} - \frac{AW_1[] AW_2[y, x, x] AW_{\bar{1}}[] AW_{\bar{2}}[]}{1152} + \frac{11 AW_1[] AW_2[y, x, y] AW_{\bar{1}}[] AW_{\bar{2}}[]}{5760} - \frac{7 AW_1[] AW_2[y, y, x] AW_{\bar{1}}[] AW_{\bar{2}}[]}{1920} - \frac{AW_1[] AW_2[y, y, y] AW_{\bar{1}}[] AW_{\bar{2}}[]}{1440} \right]$$

In[*]:= **Pent_{HR}[4]**

Out[*]=

True

In[*]:= **Pent_{HR}[6]**

Out[*]=

True