

Pensieve Header: Solving for a PPS associator degree by degree.

Initialization

(Alt) In[]:=

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\People\\Kuno"];
<< FreeLie.m
<< AwCalculus.m
<< FAA.m
<< EmergentChordDiagrams.m
BeginProfile[]
```

FreeLie` implements / extends

{*, +, **, \$SeriesShowDegree, ⟨⟩, ∫, ≡, ad, Ad, adSeries, AllCyclicWords, AllLyndonWords, AllWords, Arbitrator, AS, ASeries, AW, b, BCH, BooleanSequence, BracketForm, BS, CC, Crop, cw, CW, CWS, CWSeries, D, Deg, DegreeScale, DerivationSeries, div, DK, DKS, DKSeries, EulerE, Exp, Inverse, j, J, JA, LieDerivation, LieMorphism, LieSeries, LS, LW, LyndonFactorization, Morphism, New, RandomCWSeries, Randomizer, RandomLieSeries, RC, SeriesSolve, Support, t, tb, TopBracketForm, tr, UndeterminedCoefficients, α Map, Γ , ι , Δ , σ , \hbar , \mapsto , \curvearrowright }.

FreeLie` is in the public domain. Dror Bar-Natan is committed to support it within reason until July 15, 2022. This is version 150814.

AwCalculus` implements / extends {*, **, ≡, dA, dc, deg, dm, dS, d Δ , d η , d σ , El, Es, hA, hm, hS, h Δ , h η , h σ , RandomElSeries, RandomEsSeries, tA, tha, tm, tS, t Δ , t η , t σ , Γ , Δ }.

AwCalculus` is in the public domain. Dror Bar-Natan is committed to support it within reason until July 15, 2022. This is version 150909.

This is Profile.m of <http://www.drorbn.net/AcademicPensieve/Projects/Profile/>.

This version: April 2020. Original version: July 1994.

FreeLie` implements / extends

{*, +, **, \$SeriesShowDegree, ⟨⟩, ∫, ≡, ad, Ad, adSeries, AllCyclicWords, AllLyndonWords, AllWords, Arbitrator, AS, ASeries, AW, b, BCH, BooleanSequence, BracketForm, BS, CC, Crop, cw, CW, CWS, CWSeries, D, Deg, DegreeScale, DerivationSeries, div, DK, DKS, DKSeries, EulerE, Exp, Inverse, j, J, JA, LieDerivation, LieMorphism, LieSeries, LS, LW, LyndonFactorization, Morphism, New, RandomCWSeries, Randomizer, RandomLieSeries, RC, SeriesSolve, Support, t, tb, TopBracketForm, tr, UndeterminedCoefficients, α Map, Γ , ι , Δ , σ , \hbar , \mapsto , \curvearrowright }.

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AwCalculus` implements / extends {*, **, ≡, dA, dc, deg, dm, dS, d Δ , d η , d σ , El, Es, hA, hm, hS, h Δ , h η , h σ , RandomElSeries, RandomEsSeries, tA, tha, tm, tS, t Δ , t η , t σ , Γ , Δ }.

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(Alt) Out[]=

ProfileRoot

(Alt) In[]:=

$$\mathfrak{E}[2] = \mathbb{O}_{AR, \{x,y\}, \{1\}} \left[\mathcal{A}_0 \left[AW_1[] + \frac{1}{24} AW_1[x, y] - \frac{1}{24} AW_1[y, x] \right] \right]$$

(Alt) Out[]:=

$$\mathbb{O}_{AR, \{x,y\}, \{1\}} \left[\mathcal{A}_0 \left[AW_1[] + \frac{1}{24} AW_1[x, y] - \frac{1}{24} AW_1[y, x] \right] \right]$$

(Alt) In[]:=

Pentagon_d[\mathfrak{E}] :=

$$\mathbf{IM}_d \left[\mathfrak{E} // s\eta_2, \mathfrak{E} // s\sigma_{1 \rightarrow 2} // p\Delta_{y \rightarrow y, z} // p2s_{z \rightarrow 1}, \mathfrak{E} // s\sigma_{1 \rightarrow 2} // p2s_{y \rightarrow 1} // p\sigma_{x \rightarrow y} // p\eta_x \right] - \mathbf{IM}_d \left[\mathfrak{E} // s\sigma_{1 \rightarrow 2} // p2s_{y \rightarrow 1} // p\Delta_{x \rightarrow x, y}, \mathfrak{E} // s\Delta_{1 \rightarrow 1, 2} \right]$$

Solving to Degree 3

(Alt) In[]:=

Select[Basis₃[$\mathbb{O}_{AR, \{x,y\}, \{1\}}$], FreeQ[#, $\mathcal{A}_c[1]$] &]

(Alt) Out[]:=

$$\left\{ \mathbb{O}_{AR, \{x,y\}, \{1\}} \left[\mathcal{A}_0 \left[AW_1[x, x, x] \right] \right], \mathbb{O}_{AR, \{x,y\}, \{1\}} \left[\mathcal{A}_0 \left[AW_1[x, x, y] \right] \right], \right. \\ \left. \mathbb{O}_{AR, \{x,y\}, \{1\}} \left[\mathcal{A}_0 \left[AW_1[x, y, x] \right] \right], \mathbb{O}_{AR, \{x,y\}, \{1\}} \left[\mathcal{A}_0 \left[AW_1[x, y, y] \right] \right], \mathbb{O}_{AR, \{x,y\}, \{1\}} \left[\mathcal{A}_0 \left[AW_1[y, x, x] \right] \right], \right. \\ \left. \mathbb{O}_{AR, \{x,y\}, \{1\}} \left[\mathcal{A}_0 \left[AW_1[y, x, y] \right] \right], \mathbb{O}_{AR, \{x,y\}, \{1\}} \left[\mathcal{A}_0 \left[AW_1[y, y, x] \right] \right], \mathbb{O}_{AR, \{x,y\}, \{1\}} \left[\mathcal{A}_0 \left[AW_1[y, y, y] \right] \right] \right\}$$

(Alt) In[]:=

d = 3; i = 0;

$\mathfrak{E}[d] = \mathfrak{E}[d - 1] + \text{Sum}[c_{d,++i} B, \{B, \text{Select}[Basis_d[$\mathbb{O}_{AR, \{x,y\}, \{1\}}$], FreeQ[#, $\mathcal{A}_c[1]$] &]]]$

(Alt) Out[]:=

$$\mathbb{O}_{AR, \{x,y\}, \{1\}} \left[\mathcal{A}_0 \left[AW_1[] + \frac{1}{24} AW_1[x, y] - \frac{1}{24} AW_1[y, x] + c_{3,1} AW_1[x, x, x] + c_{3,2} AW_1[x, x, y] + c_{3,3} AW_1[x, y, x] + \right. \right. \\ \left. \left. c_{3,4} AW_1[x, y, y] + c_{3,5} AW_1[y, x, x] + c_{3,6} AW_1[y, x, y] + c_{3,7} AW_1[y, y, x] + c_{3,8} AW_1[y, y, y] \right] \right]$$

(Alt) In[]:=

rels = Union@@(List@@Pentagon_d[$\mathfrak{E}[d]$][[1]] /. {
 $\mathcal{A}_0[A_] \Rightarrow \text{Table}[\text{Coefficient}[A, B], \{B, \text{Basis}_{d, \{x,y\}}[AW_1 AW_2]\}$],
 $\mathcal{A}_c[1,2][A_] \Rightarrow \text{Table}[\text{Coefficient}[A, B], \{B, AW_2[] \text{Basis}_{d-1, \{x,y\}}[AW_1 AW_2 AW_1]\}$
})

(Alt) Out[]:=

$$\{0, -4 c_{3,1}, -3 c_{3,1}, -c_{3,1}, 3 c_{3,1}, 5 c_{3,1}, -2 c_{3,2} - c_{3,3}, -c_{3,3} - 2 c_{3,5}, \\ -c_{3,2} - c_{3,3} - c_{3,5}, -c_{3,2} - c_{3,3} - c_{3,4} - c_{3,5}, c_{3,3} + 2 c_{3,5}, -2 c_{3,4} - c_{3,6}, \\ -c_{3,3} - 2 c_{3,5} - c_{3,6}, c_{3,2} - c_{3,5} - c_{3,6}, -c_{3,2} - c_{3,3} + 2 c_{3,4} - c_{3,5} + c_{3,6}, \\ c_{3,3} + c_{3,4} + 2 c_{3,5} + c_{3,6}, -c_{3,6} - 2 c_{3,7}, c_{3,2} + 2 c_{3,3} + 3 c_{3,5} - c_{3,6} - 2 c_{3,7}, \\ c_{3,2} + 2 c_{3,3} + 3 c_{3,5} - c_{3,7}, -c_{3,4} - c_{3,6} - c_{3,7}, -c_{3,2} - c_{3,3} - c_{3,5} + c_{3,6} + c_{3,7}, \\ -c_{3,2} - 2 c_{3,3} - 3 c_{3,5} + c_{3,6} + 2 c_{3,7}, -c_{3,2} - c_{3,3} - c_{3,5} + c_{3,6} + 2 c_{3,7}, -3 c_{3,8}, 3 c_{3,8}\}$$

(Alt) In[]:=

eqns = # == 0 & /@ rels

(Alt) Out[]:=

{True, -4 c_{3,1} == 0, -3 c_{3,1} == 0, -c_{3,1} == 0, 3 c_{3,1} == 0, 5 c_{3,1} == 0, -2 c_{3,2} - c_{3,3} == 0,
 -c_{3,3} - 2 c_{3,5} == 0, -c_{3,2} - c_{3,3} - c_{3,5} == 0, -c_{3,2} - c_{3,3} - c_{3,4} - c_{3,5} == 0, c_{3,3} + 2 c_{3,5} == 0,
 -2 c_{3,4} - c_{3,6} == 0, -c_{3,3} - 2 c_{3,5} - c_{3,6} == 0, c_{3,2} - c_{3,5} - c_{3,6} == 0, -c_{3,2} - c_{3,3} + 2 c_{3,4} - c_{3,5} + c_{3,6} == 0,
 c_{3,3} + c_{3,4} + 2 c_{3,5} + c_{3,6} == 0, -c_{3,6} - 2 c_{3,7} == 0, c_{3,2} + 2 c_{3,3} + 3 c_{3,5} - c_{3,6} - 2 c_{3,7} == 0,
 c_{3,2} + 2 c_{3,3} + 3 c_{3,5} - c_{3,7} == 0, -c_{3,4} - c_{3,6} - c_{3,7} == 0, -c_{3,2} - c_{3,3} - c_{3,5} + c_{3,6} + c_{3,7} == 0,
 -c_{3,2} - 2 c_{3,3} - 3 c_{3,5} + c_{3,6} + 2 c_{3,7} == 0, -c_{3,2} - c_{3,3} - c_{3,5} + c_{3,6} + 2 c_{3,7} == 0, -3 c_{3,8} == 0, 3 c_{3,8} == 0}

(Alt) In[]:=

vars = Union[Cases[eqns, c_{d,_,} ∞]]

(Alt) Out[]:=

{c_{3,1}, c_{3,2}, c_{3,3}, c_{3,4}, c_{3,5}, c_{3,6}, c_{3,7}, c_{3,8}}

(Alt) In[]:=

sol = Solve[eqns, vars] [[1]]

Solve: Equations may not give solutions for all "solve" variables.

(Alt) Out[]:=

{c_{3,1} → 0, c_{3,3} → -2 c_{3,2}, c_{3,4} → 0, c_{3,5} → c_{3,2}, c_{3,6} → 0, c_{3,7} → 0, c_{3,8} → 0}

(Alt) In[]:=

sol /. Rule → Set

(Alt) Out[]:=

{0, -2 c_{3,2}, 0, c_{3,2}, 0, 0, 0}

(Alt) In[]:=

ϕ[3]

(Alt) Out[]:=

$\mathbb{O}_{AR, \{x, y\}, \{1\}} \left[\mathcal{A}_0 \left[AW_1[] + \frac{1}{24} AW_1[x, y] - \frac{1}{24} AW_1[y, x] + c_{3,2} AW_1[x, x, y] - 2 c_{3,2} AW_1[x, y, x] + c_{3,2} AW_1[y, x, x] \right] \right]$

(Alt) In[]:=

c_{3,2} = 0

(Alt) Out[]:=

0

(Alt) In[]:=

ϕ[3]

(Alt) Out[]:=

$\mathbb{O}_{AR, \{x, y\}, \{1\}} \left[\mathcal{A}_0 \left[AW_1[] + \frac{1}{24} AW_1[x, y] - \frac{1}{24} AW_1[y, x] \right] \right]$

Solving to Degree 4

(Alt) In[]:=

d = 4; i = 0;

ϕ[d] = ϕ[d - 1] + Sum[C_{d,++i} B, {B, Select[Basis_d[O_{AR,{x,y},{1}}], FreeQ[#, ϑ_{c[1]}] &]}]

(Alt) Out[]:=

$$\begin{aligned} & \text{O}_{AR, \{x,y\}, \{1\}} \left[\mathcal{A}_0 \left[AW_1[] + \frac{1}{24} AW_1[x, y] - \frac{1}{24} AW_1[y, x] + c_{4,1} AW_1[x, x, x, x] + \right. \right. \\ & c_{4,2} AW_1[x, x, x, y] + c_{4,3} AW_1[x, x, y, x] + c_{4,4} AW_1[x, x, y, y] + c_{4,5} AW_1[x, y, x, x] + \\ & c_{4,6} AW_1[x, y, x, y] + c_{4,7} AW_1[x, y, y, x] + c_{4,8} AW_1[x, y, y, y] + c_{4,9} AW_1[y, x, x, x] + \\ & c_{4,10} AW_1[y, x, x, y] + c_{4,11} AW_1[y, x, y, x] + c_{4,12} AW_1[y, x, y, y] + c_{4,13} AW_1[y, y, x, x] + \\ & \left. c_{4,14} AW_1[y, y, x, y] + c_{4,15} AW_1[y, y, y, x] + c_{4,16} AW_1[y, y, y, y] \right] \end{aligned}$$

(Alt) In[]:=

rels = Union@@(List@@Pentagon_d[ϕ[d]] [[1]] /. {
ϑ₀[A_] => Table[Coefficient[A, B], {B, Basis_{d,{x,y}}[AW₁ AW₂]}],
ϑ_{c[1,2]}[A_] => Table[Coefficient[A, B], {B, AW₂[] Basis_{d-1,{x,y}}[AW₁ AW₂ AW₁]}]
})

(Alt) Out[]:=

$$\begin{aligned} & \left\{ 0, -16 c_{4,1}, -11 c_{4,1}, -7 c_{4,1}, -6 c_{4,1}, -4 c_{4,1}, -c_{4,1}, c_{4,1}, 6 c_{4,1}, 17 c_{4,1}, 20 c_{4,1}, \right. \\ & -3 c_{4,2} - c_{4,3}, -2 c_{4,3} - 2 c_{4,5}, -3 c_{4,2} - 2 c_{4,3} - c_{4,5}, \frac{1}{576} - 4 c_{4,4} - 2 c_{4,6}, -2 c_{4,4} - c_{4,6} - c_{4,7}, \\ & -c_{4,3} - 2 c_{4,5} - 3 c_{4,9}, -c_{4,5} - 3 c_{4,9}, -3 c_{4,2} - 3 c_{4,3} - 2 c_{4,4} - 3 c_{4,5} - c_{4,6} - c_{4,7} - 3 c_{4,9}, \\ & -c_{4,2} - c_{4,3} - c_{4,5} - c_{4,9}, \frac{1}{576} - 4 c_{4,2} - 3 c_{4,3} - c_{4,4} - c_{4,5} - c_{4,6} - c_{4,9}, \\ & c_{4,3} + 2 c_{4,5} + 3 c_{4,9}, -\frac{1}{288} - 3 c_{4,3} - 3 c_{4,5} - c_{4,6} - c_{4,7} - c_{4,9} - 2 c_{4,10}, \\ & -2 c_{4,4} - c_{4,6} - c_{4,10}, -c_{4,2} - c_{4,3} + 2 c_{4,4} - c_{4,5} + c_{4,6} - c_{4,9} + c_{4,10}, \\ & 5 c_{4,2} + c_{4,3} - 2 c_{4,5} - c_{4,6} - c_{4,7} - 3 c_{4,9} - 4 c_{4,10} - 2 c_{4,11}, -c_{4,6} - 2 c_{4,7} - c_{4,11}, \\ & 5 c_{4,2} + 6 c_{4,3} + 3 c_{4,5} + c_{4,6} + 3 c_{4,9} - c_{4,11}, -c_{4,6} - 2 c_{4,10} - c_{4,11}, -\frac{1}{576} - c_{4,6} - 2 c_{4,7} - 2 c_{4,10} - c_{4,11}, \\ & -2 c_{4,3} - 4 c_{4,5} - c_{4,6} - 6 c_{4,9} - 2 c_{4,10} - c_{4,11}, 3 c_{4,2} + c_{4,3} - c_{4,5} - c_{4,6} - 3 c_{4,9} - 2 c_{4,10} - c_{4,11}, \\ & -\frac{1}{576} + 3 c_{4,2} + 3 c_{4,3} + 3 c_{4,5} - c_{4,6} - 2 c_{4,7} + 3 c_{4,9} - 2 c_{4,10} - c_{4,11}, \\ & \frac{1}{576} - c_{4,3} - 3 c_{4,5} - 5 c_{4,9} - c_{4,10} - c_{4,11}, -c_{4,2} - c_{4,3} - c_{4,5} + c_{4,6} + 2 c_{4,7} - c_{4,9} + c_{4,11}, \\ & -\frac{1}{576} - c_{4,2} - c_{4,3} + c_{4,9} + c_{4,10} + c_{4,11}, \frac{1}{576} - 3 c_{4,2} - 3 c_{4,3} - 3 c_{4,5} + c_{4,6} + 2 c_{4,7} - 3 c_{4,9} + 2 c_{4,10} + c_{4,11}, \\ & 2 c_{4,3} + 2 c_{4,4} + 4 c_{4,5} + 2 c_{4,6} + c_{4,7} + 6 c_{4,9} + 2 c_{4,10} + c_{4,11}, \\ & 3 c_{4,3} + c_{4,4} + 3 c_{4,5} + 2 c_{4,6} + c_{4,7} + 3 c_{4,9} + 3 c_{4,10} + c_{4,11}, -3 c_{4,8} - c_{4,12}, \\ & \frac{1}{576} + c_{4,4} + c_{4,9} - c_{4,12}, -2 c_{4,4} - c_{4,5} - 2 c_{4,6} - 2 c_{4,7} - 3 c_{4,8} - 3 c_{4,9} - c_{4,10} - c_{4,11} - c_{4,12}, \\ & -c_{4,2} - c_{4,3} - c_{4,5} + 3 c_{4,8} - c_{4,9} + c_{4,12}, \frac{1}{576} - 2 c_{4,11} - 4 c_{4,13}, \end{aligned}$$

$$\begin{aligned}
 & \frac{1}{576} + 2 c_{4,3} + 4 c_{4,5} + 6 c_{4,9} - 2 c_{4,11} - 4 c_{4,13}, 5 c_{4,2} + 5 c_{4,3} + 7 c_{4,5} + 11 c_{4,9} - 2 c_{4,10} - 2 c_{4,11} - 4 c_{4,13}, \\
 & 3 c_{4,3} + 6 c_{4,5} + 8 c_{4,9} + 2 c_{4,10} - 2 c_{4,13}, \frac{1}{288} + 2 c_{4,3} + 2 c_{4,5} + c_{4,9} - 2 c_{4,11} - 2 c_{4,13}, \\
 & -c_{4,7} - c_{4,11} - 2 c_{4,13}, 3 c_{4,2} + 5 c_{4,3} + 7 c_{4,5} - c_{4,7} + 9 c_{4,9} - c_{4,11} - 2 c_{4,13}, -c_{4,10} - c_{4,11} - 2 c_{4,13}, \\
 & -c_{4,4} - c_{4,6} - c_{4,7} - c_{4,10} - c_{4,11} - c_{4,13}, -\frac{1}{576} + c_{4,2} - c_{4,5} + c_{4,9} - c_{4,10} + c_{4,13}, \\
 & -c_{4,2} - 2 c_{4,3} - 2 c_{4,5} - 4 c_{4,9} + c_{4,11} + c_{4,13}, -4 c_{4,2} - 4 c_{4,3} - 5 c_{4,5} - 7 c_{4,9} + c_{4,10} + c_{4,11} + 2 c_{4,13}, \\
 & -c_{4,2} - c_{4,3} - 2 c_{4,5} - 4 c_{4,9} + c_{4,10} + c_{4,11} + 2 c_{4,13}, -c_{4,2} - c_{4,3} - c_{4,5} - c_{4,9} + c_{4,10} + c_{4,11} + 2 c_{4,13}, \\
 & -3 c_{4,2} - 3 c_{4,3} - 3 c_{4,5} + c_{4,6} + c_{4,7} - 3 c_{4,9} + 2 c_{4,10} + 2 c_{4,11} + 2 c_{4,13}, \\
 & -5 c_{4,2} - 5 c_{4,3} - 2 c_{4,5} + c_{4,7} - 3 c_{4,9} + 2 c_{4,10} + 3 c_{4,11} + 2 c_{4,13}, \\
 & -\frac{1}{576} - 2 c_{4,3} - 4 c_{4,5} - 6 c_{4,9} + 2 c_{4,11} + 4 c_{4,13}, \frac{1}{288} - c_{4,6} - 2 c_{4,7} - c_{4,9} - c_{4,10} - c_{4,11} - 3 c_{4,12} - 2 c_{4,14}, \\
 & -2 c_{4,12} - 2 c_{4,14}, 2 c_{4,4} - c_{4,5} + c_{4,6} - 3 c_{4,9} + c_{4,10} - 2 c_{4,12} - 2 c_{4,14}, \\
 & -c_{4,5} - c_{4,6} - 2 c_{4,7} - 3 c_{4,9} - c_{4,10} - 2 c_{4,11} - 2 c_{4,12} - 2 c_{4,13} - 2 c_{4,14}, \\
 & -3 c_{4,8} - 2 c_{4,12} - c_{4,14}, -\frac{1}{576} - c_{4,4} - c_{4,6} - c_{4,7} - 4 c_{4,8} - c_{4,9} - 2 c_{4,12} - c_{4,14}, \\
 & -c_{4,7} - c_{4,8} - 3 c_{4,9} - c_{4,12} - c_{4,14}, -\frac{1}{288} - 2 c_{4,4} - c_{4,6} + c_{4,9} + 2 c_{4,12} + c_{4,14}, \\
 & -c_{4,2} - c_{4,3} - c_{4,5} - c_{4,9} + 2 c_{4,12} + 2 c_{4,14}, 2 c_{4,5} + c_{4,6} + 2 c_{4,7} + 3 c_{4,8} + 6 c_{4,9} + c_{4,11} + 3 c_{4,12} + 2 c_{4,14}, \\
 & c_{4,6} + 2 c_{4,7} + 5 c_{4,8} + 3 c_{4,9} + 4 c_{4,12} + 2 c_{4,14}, 2 c_{4,4} + 2 c_{4,6} - 3 c_{4,9} + c_{4,10} - 2 c_{4,12} - 4 c_{4,14} - 3 c_{4,15}, \\
 & -c_{4,5} - 3 c_{4,8} - 3 c_{4,9} - 3 c_{4,12} - 3 c_{4,14} - 3 c_{4,15}, \\
 & -\frac{1}{576} - c_{4,7} - c_{4,9} - c_{4,11} - c_{4,12} - c_{4,13} - 3 c_{4,14} - 3 c_{4,15}, -c_{4,12} - 2 c_{4,14} - 3 c_{4,15}, \\
 & c_{4,3} + c_{4,4} + 2 c_{4,5} + c_{4,6} + c_{4,7} + 3 c_{4,9} + c_{4,10} + c_{4,11} - c_{4,12} + c_{4,13} - 2 c_{4,14} - 3 c_{4,15}, \\
 & -c_{4,14} - 3 c_{4,15}, 2 c_{4,4} + 2 c_{4,5} + 3 c_{4,6} + 4 c_{4,7} + 6 c_{4,9} + 2 c_{4,10} + 3 c_{4,11} + 2 c_{4,13} - c_{4,14} - 3 c_{4,15}, \\
 & -c_{4,8} - c_{4,12} - c_{4,14} - c_{4,15}, \frac{1}{576} + c_{4,4} + c_{4,6} + c_{4,7} + c_{4,9} - c_{4,12} - c_{4,14} - c_{4,15}, \\
 & c_{4,4} + 2 c_{4,6} + 4 c_{4,7} + 3 c_{4,9} + c_{4,10} + 2 c_{4,11} + 2 c_{4,12} + c_{4,13} + 2 c_{4,14} - c_{4,15}, \\
 & -c_{4,2} - c_{4,3} - c_{4,5} - c_{4,9} + c_{4,14} + 3 c_{4,15}, \\
 & -c_{4,3} - c_{4,4} - 2 c_{4,5} - c_{4,6} - c_{4,7} - 3 c_{4,9} - c_{4,10} - c_{4,11} + c_{4,12} - c_{4,13} + 2 c_{4,14} + 3 c_{4,15}, \\
 & -2 c_{4,4} - c_{4,5} - 2 c_{4,6} - 2 c_{4,7} - 3 c_{4,9} - c_{4,10} - c_{4,11} + 2 c_{4,12} + 3 c_{4,14} + 3 c_{4,15}, \\
 & 2 c_{4,5} + 3 c_{4,8} + 6 c_{4,9} + 3 c_{4,12} + 3 c_{4,14} + 3 c_{4,15}, 2 c_{4,7} + 3 c_{4,9} + c_{4,11} + 3 c_{4,12} + 5 c_{4,14} + 3 c_{4,15}, \\
 & -2 c_{4,4} - 3 c_{4,6} - 4 c_{4,7} - 3 c_{4,9} - c_{4,10} - c_{4,11} + 2 c_{4,14} + 5 c_{4,15}, \\
 & -12 c_{4,16}, -10 c_{4,16}, -6 c_{4,16}, -4 c_{4,16}, 4 c_{4,16}, 12 c_{4,16}, 14 c_{4,16} \}
 \end{aligned}$$

(Alt) In[]:=

eqns = # == 0 & /@rels

(Alt) Out[]:=

{True, -16 c_{4,1} == 0, -11 c_{4,1} == 0, -7 c_{4,1} == 0, -6 c_{4,1} == 0, -4 c_{4,1} == 0, -c_{4,1} == 0, c_{4,1} == 0, 6 c_{4,1} == 0, 17 c_{4,1} == 0, 20 c_{4,1} == 0, -3 c_{4,2} - c_{4,3} == 0, -2 c_{4,3} - 2 c_{4,5} == 0, -3 c_{4,2} - 2 c_{4,3} - c_{4,5} == 0, $\frac{1}{576} - 4 c_{4,4} - 2 c_{4,6} == 0$, -2 c_{4,4} - c_{4,6} - c_{4,7} == 0, -c_{4,3} - 2 c_{4,5} - 3 c_{4,9} == 0, -c_{4,5} - 3 c_{4,9} == 0, -3 c_{4,2} - 3 c_{4,3} - 2 c_{4,4} - 3 c_{4,5} - c_{4,6} - c_{4,7} - 3 c_{4,9} == 0,

$$\begin{aligned}
& -c_{4,2} - c_{4,3} - c_{4,5} - c_{4,9} = 0, \quad \frac{1}{576} - 4c_{4,2} - 3c_{4,3} - c_{4,4} - c_{4,5} - c_{4,6} - c_{4,9} = 0, \\
& c_{4,3} + 2c_{4,5} + 3c_{4,9} = 0, \quad -\frac{1}{288} - 3c_{4,3} - 3c_{4,5} - c_{4,6} - c_{4,7} - c_{4,9} - 2c_{4,10} = 0, \\
& -2c_{4,4} - c_{4,6} - c_{4,10} = 0, \quad -c_{4,2} - c_{4,3} + 2c_{4,4} - c_{4,5} + c_{4,6} - c_{4,9} + c_{4,10} = 0, \\
& 5c_{4,2} + c_{4,3} - 2c_{4,5} - c_{4,6} - c_{4,7} - 3c_{4,9} - 4c_{4,10} - 2c_{4,11} = 0, \\
& -c_{4,6} - 2c_{4,7} - c_{4,11} = 0, \quad 5c_{4,2} + 6c_{4,3} + 3c_{4,5} + c_{4,6} + 3c_{4,9} - c_{4,11} = 0, \\
& -c_{4,6} - 2c_{4,10} - c_{4,11} = 0, \quad -\frac{1}{576} - c_{4,6} - 2c_{4,7} - 2c_{4,10} - c_{4,11} = 0, \\
& -2c_{4,3} - 4c_{4,5} - c_{4,6} - 6c_{4,9} - 2c_{4,10} - c_{4,11} = 0, \quad 3c_{4,2} + c_{4,3} - c_{4,5} - c_{4,6} - 3c_{4,9} - 2c_{4,10} - c_{4,11} = 0, \\
& -\frac{1}{576} + 3c_{4,2} + 3c_{4,3} + 3c_{4,5} - c_{4,6} - 2c_{4,7} + 3c_{4,9} - 2c_{4,10} - c_{4,11} = 0, \\
& \frac{1}{576} - c_{4,3} - 3c_{4,5} - 5c_{4,9} - c_{4,10} - c_{4,11} = 0, \\
& -c_{4,2} - c_{4,3} - c_{4,5} + c_{4,6} + 2c_{4,7} - c_{4,9} + c_{4,11} = 0, \quad -\frac{1}{576} - c_{4,2} - c_{4,3} + c_{4,9} + c_{4,10} + c_{4,11} = 0, \\
& \frac{1}{576} - 3c_{4,2} - 3c_{4,3} - 3c_{4,5} + c_{4,6} + 2c_{4,7} - 3c_{4,9} + 2c_{4,10} + c_{4,11} = 0, \\
& 2c_{4,3} + 2c_{4,4} + 4c_{4,5} + 2c_{4,6} + c_{4,7} + 6c_{4,9} + 2c_{4,10} + c_{4,11} = 0, \\
& 3c_{4,3} + c_{4,4} + 3c_{4,5} + 2c_{4,6} + c_{4,7} + 3c_{4,9} + 3c_{4,10} + c_{4,11} = 0, \quad -3c_{4,8} - c_{4,12} = 0, \\
& \frac{1}{576} + c_{4,4} + c_{4,9} - c_{4,12} = 0, \quad -2c_{4,4} - c_{4,5} - 2c_{4,6} - 2c_{4,7} - 3c_{4,8} - 3c_{4,9} - c_{4,10} - c_{4,11} - c_{4,12} = 0, \\
& -c_{4,2} - c_{4,3} - c_{4,5} + 3c_{4,8} - c_{4,9} + c_{4,12} = 0, \quad \frac{1}{576} - 2c_{4,11} - 4c_{4,13} = 0, \\
& \frac{1}{576} + 2c_{4,3} + 4c_{4,5} + 6c_{4,9} - 2c_{4,11} - 4c_{4,13} = 0, \\
& 5c_{4,2} + 5c_{4,3} + 7c_{4,5} + 11c_{4,9} - 2c_{4,10} - 2c_{4,11} - 4c_{4,13} = 0, \\
& 3c_{4,3} + 6c_{4,5} + 8c_{4,9} + 2c_{4,10} - 2c_{4,13} = 0, \quad \frac{1}{288} + 2c_{4,3} + 2c_{4,5} + c_{4,9} - 2c_{4,11} - 2c_{4,13} = 0, \\
& -c_{4,7} - c_{4,11} - 2c_{4,13} = 0, \quad 3c_{4,2} + 5c_{4,3} + 7c_{4,5} - c_{4,7} + 9c_{4,9} - c_{4,11} - 2c_{4,13} = 0, \\
& -c_{4,10} - c_{4,11} - 2c_{4,13} = 0, \quad -c_{4,4} - c_{4,6} - c_{4,7} - c_{4,10} - c_{4,11} - c_{4,13} = 0, \\
& -\frac{1}{576} + c_{4,2} - c_{4,5} + c_{4,9} - c_{4,10} + c_{4,13} = 0, \quad -c_{4,2} - 2c_{4,3} - 2c_{4,5} - 4c_{4,9} + c_{4,11} + c_{4,13} = 0, \\
& -4c_{4,2} - 4c_{4,3} - 5c_{4,5} - 7c_{4,9} + c_{4,10} + c_{4,11} + 2c_{4,13} = 0, \\
& -c_{4,2} - c_{4,3} - 2c_{4,5} - 4c_{4,9} + c_{4,10} + c_{4,11} + 2c_{4,13} = 0, \\
& -c_{4,2} - c_{4,3} - c_{4,5} - c_{4,9} + c_{4,10} + c_{4,11} + 2c_{4,13} = 0, \\
& -3c_{4,2} - 3c_{4,3} - 3c_{4,5} + c_{4,6} + c_{4,7} - 3c_{4,9} + 2c_{4,10} + 2c_{4,11} + 2c_{4,13} = 0, \\
& -5c_{4,2} - 5c_{4,3} - 2c_{4,5} + c_{4,7} - 3c_{4,9} + 2c_{4,10} + 3c_{4,11} + 2c_{4,13} = 0, \\
& -\frac{1}{576} - 2c_{4,3} - 4c_{4,5} - 6c_{4,9} + 2c_{4,11} + 4c_{4,13} = 0, \\
& \frac{1}{288} - c_{4,6} - 2c_{4,7} - c_{4,9} - c_{4,10} - c_{4,11} - 3c_{4,12} - 2c_{4,14} = 0, \\
& -2c_{4,12} - 2c_{4,14} = 0, \quad 2c_{4,4} - c_{4,5} + c_{4,6} - 3c_{4,9} + c_{4,10} - 2c_{4,12} - 2c_{4,14} = 0, \\
& -c_{4,5} - c_{4,6} - 2c_{4,7} - 3c_{4,9} - c_{4,10} - 2c_{4,11} - 2c_{4,12} - 2c_{4,13} - 2c_{4,14} = 0, \quad -3c_{4,8} - 2c_{4,12} - c_{4,14} = 0,
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{576} - c_{4,4} - c_{4,6} - c_{4,7} - 4c_{4,8} - c_{4,9} - 2c_{4,12} - c_{4,14} = 0, \quad -c_{4,7} - c_{4,8} - 3c_{4,9} - c_{4,12} - c_{4,14} = 0, \\
& -\frac{1}{288} - 2c_{4,4} - c_{4,6} + c_{4,9} + 2c_{4,12} + c_{4,14} = 0, \quad -c_{4,2} - c_{4,3} - c_{4,5} - c_{4,9} + 2c_{4,12} + 2c_{4,14} = 0, \\
& 2c_{4,5} + c_{4,6} + 2c_{4,7} + 3c_{4,8} + 6c_{4,9} + c_{4,11} + 3c_{4,12} + 2c_{4,14} = 0, \\
& c_{4,6} + 2c_{4,7} + 5c_{4,8} + 3c_{4,9} + 4c_{4,12} + 2c_{4,14} = 0, \\
& 2c_{4,4} + 2c_{4,6} - 3c_{4,9} + c_{4,10} - 2c_{4,12} - 4c_{4,14} - 3c_{4,15} = 0, \\
& -c_{4,5} - 3c_{4,8} - 3c_{4,9} - 3c_{4,12} - 3c_{4,14} - 3c_{4,15} = 0, \\
& -\frac{1}{576} - c_{4,7} - c_{4,9} - c_{4,11} - c_{4,12} - c_{4,13} - 3c_{4,14} - 3c_{4,15} = 0, \quad -c_{4,12} - 2c_{4,14} - 3c_{4,15} = 0, \\
& c_{4,3} + c_{4,4} + 2c_{4,5} + c_{4,6} + c_{4,7} + 3c_{4,9} + c_{4,10} + c_{4,11} - c_{4,12} + c_{4,13} - 2c_{4,14} - 3c_{4,15} = 0, \\
& -c_{4,14} - 3c_{4,15} = 0, \quad 2c_{4,4} + 2c_{4,5} + 3c_{4,6} + 4c_{4,7} + 6c_{4,9} + 2c_{4,10} + 3c_{4,11} + 2c_{4,13} - c_{4,14} - 3c_{4,15} = 0, \\
& -c_{4,8} - c_{4,12} - c_{4,14} - c_{4,15} = 0, \quad \frac{1}{576} + c_{4,4} + c_{4,6} + c_{4,7} + c_{4,9} - c_{4,12} - c_{4,14} - c_{4,15} = 0, \\
& c_{4,4} + 2c_{4,6} + 4c_{4,7} + 3c_{4,9} + c_{4,10} + 2c_{4,11} + 2c_{4,12} + c_{4,13} + 2c_{4,14} - c_{4,15} = 0, \\
& -c_{4,2} - c_{4,3} - c_{4,5} - c_{4,9} + c_{4,14} + 3c_{4,15} = 0, \\
& -c_{4,3} - c_{4,4} - 2c_{4,5} - c_{4,6} - c_{4,7} - 3c_{4,9} - c_{4,10} - c_{4,11} + c_{4,12} - c_{4,13} + 2c_{4,14} + 3c_{4,15} = 0, \\
& -2c_{4,4} - c_{4,5} - 2c_{4,6} - 2c_{4,7} - 3c_{4,9} - c_{4,10} - c_{4,11} + 2c_{4,12} + 3c_{4,14} + 3c_{4,15} = 0, \\
& 2c_{4,5} + 3c_{4,8} + 6c_{4,9} + 3c_{4,12} + 3c_{4,14} + 3c_{4,15} = 0, \quad 2c_{4,7} + 3c_{4,9} + c_{4,11} + 3c_{4,12} + 5c_{4,14} + 3c_{4,15} = 0, \\
& -2c_{4,4} - 3c_{4,6} - 4c_{4,7} - 3c_{4,9} - c_{4,10} - c_{4,11} + 2c_{4,14} + 5c_{4,15} = 0, \quad -12c_{4,16} = 0, \\
& -10c_{4,16} = 0, \quad -6c_{4,16} = 0, \quad -4c_{4,16} = 0, \quad 4c_{4,16} = 0, \quad 12c_{4,16} = 0, \quad 14c_{4,16} = 0 \}
\end{aligned}$$

(Alt) In[]:=

vars = Union[Cases[eqns, c_d,_, ∞]]

(Alt) Out[]:=

{c_{4,1}, c_{4,2}, c_{4,3}, c_{4,4}, c_{4,5}, c_{4,6}, c_{4,7}, c_{4,8}, c_{4,9}, c_{4,10}, c_{4,11}, c_{4,12}, c_{4,13}, c_{4,14}, c_{4,15}, c_{4,16}}

(Alt) In[]:=

sol = Solve[eqns, vars] [[1]]

(Alt) Out[]:=

$$\begin{aligned}
& \left\{ c_{4,1} \rightarrow 0, \quad c_{4,2} \rightarrow -\frac{1}{1440}, \quad c_{4,3} \rightarrow \frac{1}{480}, \quad c_{4,4} \rightarrow \frac{7}{5760}, \quad c_{4,5} \rightarrow -\frac{1}{480}, \right. \\
& c_{4,6} \rightarrow -\frac{1}{640}, \quad c_{4,7} \rightarrow -\frac{1}{1152}, \quad c_{4,8} \rightarrow -\frac{7}{5760}, \quad c_{4,9} \rightarrow \frac{1}{1440}, \quad c_{4,10} \rightarrow -\frac{1}{1152}, \\
& \left. c_{4,11} \rightarrow \frac{19}{5760}, \quad c_{4,12} \rightarrow \frac{7}{1920}, \quad c_{4,13} \rightarrow -\frac{7}{5760}, \quad c_{4,14} \rightarrow -\frac{7}{1920}, \quad c_{4,15} \rightarrow \frac{7}{5760}, \quad c_{4,16} \rightarrow 0 \right\}
\end{aligned}$$

(Alt) In[]:=

sol /. Rule → Set

(Alt) Out[]:=

$$\begin{aligned}
& \left\{ 0, -\frac{1}{1440}, \frac{1}{480}, \frac{7}{5760}, -\frac{1}{480}, -\frac{1}{640}, -\frac{1}{1152}, \right. \\
& \left. -\frac{7}{5760}, \frac{1}{1440}, -\frac{1}{1152}, \frac{19}{5760}, \frac{7}{1920}, -\frac{7}{5760}, -\frac{7}{1920}, \frac{7}{5760}, 0 \right\}
\end{aligned}$$

(Alt) In[]:=

ϕ[d]

(Alt) Out[]:=

$$\begin{aligned} & \circlearrowleft_{AR, \{x,y\}, \{1\}} \left[\mathcal{A}_0 \left[AW_1[] + \frac{1}{24} AW_1[x, y] - \frac{1}{24} AW_1[y, x] - \frac{AW_1[x, x, x, y]}{1440} + \frac{1}{480} AW_1[x, x, y, x] + \right. \right. \\ & \quad \frac{7 AW_1[x, x, y, y]}{5760} - \frac{1}{480} AW_1[x, y, x, x] - \frac{1}{640} AW_1[x, y, x, y] - \frac{AW_1[x, y, y, x]}{1152} - \\ & \quad \frac{7 AW_1[x, y, y, y]}{5760} + \frac{AW_1[y, x, x, x]}{1440} - \frac{AW_1[y, x, x, y]}{1152} + \frac{19 AW_1[y, x, y, x]}{5760} + \\ & \quad \left. \left. \frac{7 AW_1[y, x, y, y]}{1920} - \frac{7 AW_1[y, y, x, x]}{5760} - \frac{7 AW_1[y, y, x, y]}{1920} + \frac{7 AW_1[y, y, y, x]}{5760} \right] \right] \end{aligned}$$

(Alt) In[]:=

PrintProfile[]

(Alt) Out[]:=

```
ProfileRoot is root. Profiled time: 31.547
( 3)  0/ 0 above EMBasis
( 6)  0.016/ 30.450 above EMIM
( 6)  0.015/ 0.093 above EMp2s
( 4)  0.015/ 0.015 above EMpΔ
( 2)  0/ 0 above EMpσ
( 2)  0.532/ 0.985 above EMsΔ
( 6)  0/ 0 above EMsσ
EMEM: called 6 times, time in 25.812/25.812
( 6)  25.810/ 25.810 under EMIM
EMAR: called 76 times, time in 2.333/3.079
( 44) 1.376/ 1.765 under EMCF
( 32) 0.957/ 1.314 under ○
( 228) 0.746/ 0.746 above FAAm
○: called 26 times, time in 1.422/4.515
( 24) 1.422/ 4.390 under EMsm
( 2)  0/ 0.125 under EMsΔ
( 32) 0.957/ 1.314 above EMAR
( 26) 0.015/ 1.702 above EMCF
( 96) 0.077/ 0.077 above FAAm
FAAm: called 372 times, time in 1.087/1.087
( 228) 0.746/ 0.746 under EMAR
( 32) 0.046/ 0.046 under EMsm
( 16) 0.218/ 0.218 under EMsΔ
( 96) 0.077/ 0.077 under ○
EMsΔ: called 2 times, time in 0.532/0.985
( 2) 0.532/ 0.985 under ProfileRoot
( 16) 0.218/ 0.218 above FAAm
( 4) 0.110/ 0.110 above FAAσ
( 2) 0/ 0.125 above ○
FAAσ: called 114 times, time in 0.284/0.284
( 20) 0.079/ 0.079 under EMsm
```



```

( 4) 0.110/ 0.110 under EMs $\Delta$ 
( 90) 0.095/ 0.095 under EMs $\sigma$ 
EMs $\sigma$ : called 18 times, time in 0.016/0.111
( 12) 0.016/ 0.111 under EMIM
( 6) 0/ 0 under ProfileRoot
( 90) 0.095/ 0.095 above FAA $\sigma$ 
EMIM: called 6 times, time in 0.016/30.454
( 6) 0.016/ 30.450 under ProfileRoot
( 6) 25.810/ 25.810 above EMEM
( 12) 0/ 4.515 above EMsm
( 12) 0.016/ 0.111 above EMs $\sigma$ 
EMp $\Delta$ : called 4 times, time in 0.015/0.015
( 4) 0.015/ 0.015 under ProfileRoot
EMp2s: called 6 times, time in 0.015/0.093
( 6) 0.015/ 0.093 under ProfileRoot
( 6) 0/ 0.078 above EMCF
EMCF: called 32 times, time in 0.015/1.78
( 6) 0/ 0.078 under EMp2s
( 26) 0.015/ 1.702 under  $\mathbb{O}$ 
( 44) 1.376/ 1.765 above EMAR
EMsm: called 12 times, time in 0./4.515
( 12) 0/ 4.515 under EMIM
( 32) 0.046/ 0.046 above FAAm
( 20) 0.079/ 0.079 above FAA $\sigma$ 
( 24) 1.422/ 4.390 above  $\mathbb{O}$ 
EMp $\sigma$ : called 2 times, time in 0./0.
( 2) 0/ 0 under ProfileRoot
EMBasis: called 3 times, time in 0./0.
( 3) 0/ 0 under ProfileRoot

```

Solving to Degree 5

```

In[*]:= d = 5; i = 0;
 $\Phi$ [d] =  $\Phi$ [d - 1] + Sum[cd,++i B, {B, Select[Basisd[ $\mathbb{O}_{AR, \{x,y\}, \{1\}}$ ], FreeQ[#,  $\mathcal{A}_c[1]$ ] &]}]

```

Out[*]=

$$\begin{aligned} & \mathcal{O}_{AR, \{x,y\}, \{1\}} \left[\mathcal{A}_0 \left[\right. \right. \\ & \quad AW_1[] + \frac{1}{24} AW_1[x, y] - \frac{1}{24} AW_1[y, x] - \frac{AW_1[x, x, x, y]}{1440} + \frac{1}{480} AW_1[x, x, y, x] + \frac{7 AW_1[x, x, y, y]}{5760} - \\ & \quad \frac{1}{480} AW_1[x, y, x, x] - \frac{1}{640} AW_1[x, y, x, y] - \frac{AW_1[x, y, y, x]}{1152} - \frac{7 AW_1[x, y, y, y]}{5760} + \\ & \quad \frac{AW_1[y, x, x, x]}{1440} - \frac{AW_1[y, x, x, y]}{1152} + \frac{19 AW_1[y, x, y, x]}{5760} + \frac{7 AW_1[y, x, y, y]}{1920} - \frac{7 AW_1[y, y, x, x]}{5760} - \\ & \quad \frac{7 AW_1[y, y, x, y]}{1920} + \frac{7 AW_1[y, y, y, x]}{5760} + c_{5,1} AW_1[x, x, x, x, x] + c_{5,2} AW_1[x, x, x, x, y] + \\ & \quad c_{5,3} AW_1[x, x, x, y, x] + c_{5,4} AW_1[x, x, x, y, y] + c_{5,5} AW_1[x, x, y, x, x] + \\ & \quad c_{5,6} AW_1[x, x, y, x, y] + c_{5,7} AW_1[x, x, y, y, x] + c_{5,8} AW_1[x, x, y, y, y] + \\ & \quad c_{5,9} AW_1[x, y, x, x, x] + c_{5,10} AW_1[x, y, x, x, y] + c_{5,11} AW_1[x, y, x, y, x] + \\ & \quad c_{5,12} AW_1[x, y, x, y, y] + c_{5,13} AW_1[x, y, y, x, x] + c_{5,14} AW_1[x, y, y, x, y] + \\ & \quad c_{5,15} AW_1[x, y, y, y, x] + c_{5,16} AW_1[x, y, y, y, y] + c_{5,17} AW_1[y, x, x, x, x] + \\ & \quad c_{5,18} AW_1[y, x, x, x, y] + c_{5,19} AW_1[y, x, x, y, x] + c_{5,20} AW_1[y, x, x, y, y] + \\ & \quad c_{5,21} AW_1[y, x, y, x, x] + c_{5,22} AW_1[y, x, y, x, y] + c_{5,23} AW_1[y, x, y, y, x] + \\ & \quad c_{5,24} AW_1[y, x, y, y, y] + c_{5,25} AW_1[y, y, x, x, x] + c_{5,26} AW_1[y, y, x, x, y] + \\ & \quad c_{5,27} AW_1[y, y, x, y, x] + c_{5,28} AW_1[y, y, x, y, y] + c_{5,29} AW_1[y, y, y, x, x] + \\ & \quad c_{5,30} AW_1[y, y, y, x, y] + c_{5,31} AW_1[y, y, y, y, x] + c_{5,32} AW_1[y, y, y, y, y] \left. \right] \end{aligned}$$

In[*]:= Short[

```
rels = Union@@ (List@@ Pentagond[ $\mathfrak{a}$ [d]] [[1]] /. {
   $\mathcal{A}_0[A\_]$   $\Rightarrow$  Table[Coefficient[A, B], {B, Basisd, {x,y}[AW1 AW2]}},
   $\mathcal{A}_c[1,2][A\_]$   $\Rightarrow$  Table[Coefficient[A, B], {B, AW2[] Basisd-1, {x,y}[AW1 AW2 AW1]}},
}) ,
10]
```

Out[*]//Short=

$$\begin{aligned} & \{0, -55 c_{5,1}, -40 c_{5,1}, -35 c_{5,1}, -31 c_{5,1}, -26 c_{5,1}, -10 c_{5,1}, -5 c_{5,1}, -c_{5,1}, 5 c_{5,1}, 9 c_{5,1}, 10 c_{5,1}, \\ & 49 c_{5,1}, 50 c_{5,1}, \ll 251 \gg, 3 c_{5,9} + 14 c_{5,16} + 12 c_{5,17} + 14 c_{5,24} + 14 c_{5,28} + 14 c_{5,30} + 14 c_{5,31}, \\ & -3 c_{5,8} - 5 c_{5,12} - 8 c_{5,14} - 12 c_{5,15} - 6 c_{5,17} - 2 c_{5,20} - 3 c_{5,22} - 4 c_{5,23} - 6 c_{5,24} - c_{5,26} - \\ & c_{5,27} - 3 c_{5,28} + 6 c_{5,30} + 17 c_{5,31}, -6 c_{5,8} - 3 c_{5,9} - 8 c_{5,12} - 10 c_{5,14} - 12 c_{5,15} - 12 c_{5,17} - \\ & 4 c_{5,20} - 5 c_{5,22} - 6 c_{5,23} - 2 c_{5,26} - 2 c_{5,27} + 4 c_{5,28} + 11 c_{5,30} + 20 c_{5,31}, -50 c_{5,32}, \\ & -30 c_{5,32}, -25 c_{5,32}, -20 c_{5,32}, -10 c_{5,32}, -5 c_{5,32}, 5 c_{5,32}, 30 c_{5,32}, 45 c_{5,32}, 70 c_{5,32} \} \end{aligned}$$

In[*]:= eqns = # == 0 & /@ rels;

In[*]:= vars = Union[Cases[eqns, c_{d, _}, ∞]]

Out[*]=

$$\{c_{5,1}, c_{5,2}, c_{5,3}, c_{5,4}, c_{5,5}, c_{5,6}, c_{5,7}, c_{5,8}, c_{5,9}, c_{5,10}, c_{5,11}, c_{5,12}, c_{5,13}, c_{5,14}, c_{5,15}, c_{5,16}, c_{5,17}, c_{5,18}, c_{5,19}, c_{5,20}, c_{5,21}, c_{5,22}, c_{5,23}, c_{5,24}, c_{5,25}, c_{5,26}, c_{5,27}, c_{5,28}, c_{5,29}, c_{5,30}, c_{5,31}, c_{5,32}\}$$

In[*]:= **sol = Solve [eqns, vars] [[1]**

Solve: Equations may not give solutions for all "solve" variables.

Out[*]=

$$\left\{ \begin{aligned} &C_{5,1} \rightarrow 0, C_{5,2} \rightarrow C_{5,8}, C_{5,3} \rightarrow -4 C_{5,8}, C_{5,4} \rightarrow -C_{5,8}, C_{5,5} \rightarrow 6 C_{5,8}, C_{5,6} \rightarrow \frac{3 C_{5,8}}{2}, \\ &C_{5,7} \rightarrow \frac{3 C_{5,8}}{2}, C_{5,9} \rightarrow -4 C_{5,8}, C_{5,10} \rightarrow \frac{3 C_{5,8}}{2}, C_{5,11} \rightarrow -6 C_{5,8}, C_{5,12} \rightarrow -4 C_{5,8}, C_{5,13} \rightarrow \frac{3 C_{5,8}}{2}, \\ &C_{5,14} \rightarrow 6 C_{5,8}, C_{5,15} \rightarrow -4 C_{5,8}, C_{5,16} \rightarrow 0, C_{5,17} \rightarrow C_{5,8}, C_{5,18} \rightarrow -C_{5,8}, C_{5,19} \rightarrow \frac{3 C_{5,8}}{2}, \\ &C_{5,20} \rightarrow C_{5,8}, C_{5,21} \rightarrow \frac{3 C_{5,8}}{2}, C_{5,22} \rightarrow -4 C_{5,8}, C_{5,23} \rightarrow 6 C_{5,8}, C_{5,24} \rightarrow 0, C_{5,25} \rightarrow -C_{5,8}, \\ &C_{5,26} \rightarrow C_{5,8}, C_{5,27} \rightarrow -4 C_{5,8}, C_{5,28} \rightarrow 0, C_{5,29} \rightarrow C_{5,8}, C_{5,30} \rightarrow 0, C_{5,31} \rightarrow 0, C_{5,32} \rightarrow 0 \end{aligned} \right\}$$

In[*]:= **sol /. Rule -> Set**

Out[*]=

$$\left\{ \begin{aligned} &0, C_{5,8}, -4 C_{5,8}, -C_{5,8}, 6 C_{5,8}, \frac{3 C_{5,8}}{2}, \frac{3 C_{5,8}}{2}, -4 C_{5,8}, \frac{3 C_{5,8}}{2}, -6 C_{5,8}, -4 C_{5,8}, \frac{3 C_{5,8}}{2}, 6 C_{5,8}, \\ &-4 C_{5,8}, 0, C_{5,8}, -C_{5,8}, \frac{3 C_{5,8}}{2}, C_{5,8}, \frac{3 C_{5,8}}{2}, -4 C_{5,8}, 6 C_{5,8}, 0, -C_{5,8}, C_{5,8}, -4 C_{5,8}, 0, C_{5,8}, 0, 0, 0 \end{aligned} \right\}$$

In[*]:= **C_{5,8} = 0**

Out[*]=

$$0$$

In[*]:= **ϕ[d]**

Out[*]=

$$\begin{aligned} &O_{AR, \{x,y\}, \{1\}} \left[\mathcal{A}_0 \left[AW_1 [] + \frac{1}{24} AW_1 [x, y] - \frac{1}{24} AW_1 [y, x] - \frac{AW_1 [x, x, x, y]}{1440} + \frac{1}{480} AW_1 [x, x, y, x] + \right. \right. \\ &\quad \frac{7 AW_1 [x, x, y, y]}{5760} - \frac{1}{480} AW_1 [x, y, x, x] - \frac{1}{640} AW_1 [x, y, x, y] - \frac{AW_1 [x, y, y, x]}{1152} - \\ &\quad \frac{7 AW_1 [x, y, y, y]}{5760} + \frac{AW_1 [y, x, x, x]}{1440} - \frac{AW_1 [y, x, x, y]}{1152} + \frac{19 AW_1 [y, x, y, x]}{5760} + \\ &\quad \left. \left. \frac{7 AW_1 [y, x, y, y]}{1920} - \frac{7 AW_1 [y, y, x, x]}{5760} - \frac{7 AW_1 [y, y, x, y]}{1920} + \frac{7 AW_1 [y, y, y, x]}{5760} \right] \right] \end{aligned}$$

Solving to Degree 6

In[*]:= **d = 6; i = 0;**

Ⓢ[d] = Ⓢ[d - 1] + Sum[c_{d,++i} B, {B, Select[Basis_d[O_{AR,{x,y},{1}}], FreeQ[#, Ⓢ_{c[1]}] &]}]

Out[*]=

$$\begin{aligned}
 & \mathcal{O}_{AR, \{x,y\}, \{1\}} \left[\mathcal{A}_0 \left[AW_1[] + \frac{1}{24} AW_1[x, y] - \frac{1}{24} AW_1[y, x] - \frac{AW_1[x, x, x, y]}{1440} + \frac{1}{480} AW_1[x, x, y, x] + \right. \right. \\
 & \quad \frac{7 AW_1[x, x, y, y]}{5760} - \frac{1}{480} AW_1[x, y, x, x] - \frac{1}{640} AW_1[x, y, x, y] - \frac{AW_1[x, y, y, x]}{1152} - \\
 & \quad \frac{7 AW_1[x, y, y, y]}{5760} + \frac{AW_1[y, x, x, x]}{1440} - \frac{AW_1[y, x, x, y]}{1152} + \frac{19 AW_1[y, x, y, x]}{5760} + \frac{7 AW_1[y, x, y, y]}{1920} - \\
 & \quad \frac{7 AW_1[y, y, x, x]}{5760} - \frac{7 AW_1[y, y, x, y]}{1920} + \frac{7 AW_1[y, y, y, x]}{5760} + c_{6,1} AW_1[x, x, x, x, x, x] + \\
 & \quad c_{6,2} AW_1[x, x, x, x, x, y] + c_{6,3} AW_1[x, x, x, x, y, x] + c_{6,4} AW_1[x, x, x, x, y, y] + \\
 & \quad c_{6,5} AW_1[x, x, x, y, x, x] + c_{6,6} AW_1[x, x, x, y, x, y] + c_{6,7} AW_1[x, x, x, y, y, x] + \\
 & \quad c_{6,8} AW_1[x, x, x, y, y, y] + c_{6,9} AW_1[x, x, y, x, x, x] + c_{6,10} AW_1[x, x, y, x, x, y] + \\
 & \quad c_{6,11} AW_1[x, x, y, x, y, x] + c_{6,12} AW_1[x, x, y, x, y, y] + c_{6,13} AW_1[x, x, y, y, x, x] + \\
 & \quad c_{6,14} AW_1[x, x, y, y, x, y] + c_{6,15} AW_1[x, x, y, y, y, x] + c_{6,16} AW_1[x, x, y, y, y, y] + \\
 & \quad c_{6,17} AW_1[x, y, x, x, x, x] + c_{6,18} AW_1[x, y, x, x, x, y] + c_{6,19} AW_1[x, y, x, x, y, x] + \\
 & \quad c_{6,20} AW_1[x, y, x, x, y, y] + c_{6,21} AW_1[x, y, x, y, x, x] + c_{6,22} AW_1[x, y, x, y, x, y] + \\
 & \quad c_{6,23} AW_1[x, y, x, y, y, x] + c_{6,24} AW_1[x, y, x, y, y, y] + c_{6,25} AW_1[x, y, y, x, x, x] + \\
 & \quad c_{6,26} AW_1[x, y, y, x, x, y] + c_{6,27} AW_1[x, y, y, x, y, x] + c_{6,28} AW_1[x, y, y, x, y, y] + \\
 & \quad c_{6,29} AW_1[x, y, y, y, x, x] + c_{6,30} AW_1[x, y, y, y, x, y] + c_{6,31} AW_1[x, y, y, y, y, x] + \\
 & \quad c_{6,32} AW_1[x, y, y, y, y, y] + c_{6,33} AW_1[y, x, x, x, x, x] + c_{6,34} AW_1[y, x, x, x, x, y] + \\
 & \quad c_{6,35} AW_1[y, x, x, x, y, x] + c_{6,36} AW_1[y, x, x, x, y, y] + c_{6,37} AW_1[y, x, x, y, x, x] + \\
 & \quad c_{6,38} AW_1[y, x, x, y, x, y] + c_{6,39} AW_1[y, x, x, y, y, x] + c_{6,40} AW_1[y, x, x, y, y, y] + \\
 & \quad c_{6,41} AW_1[y, x, y, x, x, x] + c_{6,42} AW_1[y, x, y, x, x, y] + c_{6,43} AW_1[y, x, y, x, y, x] + \\
 & \quad c_{6,44} AW_1[y, x, y, x, y, y] + c_{6,45} AW_1[y, x, y, y, x, x] + c_{6,46} AW_1[y, x, y, y, x, y] + \\
 & \quad c_{6,47} AW_1[y, x, y, y, y, x] + c_{6,48} AW_1[y, x, y, y, y, y] + c_{6,49} AW_1[y, y, x, x, x, x] + \\
 & \quad c_{6,50} AW_1[y, y, x, x, x, y] + c_{6,51} AW_1[y, y, x, x, y, x] + c_{6,52} AW_1[y, y, x, x, y, y] + \\
 & \quad c_{6,53} AW_1[y, y, x, y, x, x] + c_{6,54} AW_1[y, y, x, y, x, y] + c_{6,55} AW_1[y, y, x, y, y, x] + \\
 & \quad c_{6,56} AW_1[y, y, x, y, y, y] + c_{6,57} AW_1[y, y, y, x, x, x] + c_{6,58} AW_1[y, y, y, x, x, y] + \\
 & \quad c_{6,59} AW_1[y, y, y, x, y, x] + c_{6,60} AW_1[y, y, y, x, y, y] + c_{6,61} AW_1[y, y, y, y, x, x] + \\
 & \quad c_{6,62} AW_1[y, y, y, y, x, y] + c_{6,63} AW_1[y, y, y, y, y, x] + c_{6,64} AW_1[y, y, y, y, y, y] \left. \right]
 \end{aligned}$$

```
In[*]:= Short[
  reIs = Union@@ (List@@Pentagond[ $\Phi$ [d]][[1]] /. {
     $\mathcal{A}_0[A\_]$   $\Rightarrow$  Table[Coefficient[A, B], {B, Basisd, {x,y} [AW1 AW2] }},
     $\mathcal{A}_{C[1,2]}[A\_]$   $\Rightarrow$  Table[Coefficient[A, B], {B, AW2[[]] Basisd-1, {x,y} [AW1 AW2 AW1] }},
  }),
  10]
```

```
Out[*]//Short=
{0, -186 c6,1, -165 c6,1, -156 c6,1, -111 c6,1, -105 c6,1, -80 c6,1, -57 c6,1,
-20 c6,1, -15 c6,1, -11 c6,1, -6 c6,1, -c6,1, c6,1, 15 c6,1, 49 c6,1, <<752>>,
-12 c6,16 - 6 c6,17 - 18 c6,24 - 26 c6,28 - 36 c6,30 - 48 c6,31 - 30 c6,33 - 9 c6,40 - 13 c6,44 - 18 c6,46 -
24 c6,47 - 24 c6,48 - 6 c6,52 - 8 c6,54 - 10 c6,55 - 21 c6,56 - 3 c6,58 - 3 c6,59 + 3 c6,60 + 41 c6,62 + 85 c6,63,
-150 c6,64, -101 c6,64, -60 c6,64, -56 c6,64, -20 c6,64, -15 c6,64, -6 c6,64,
6 c6,64, 30 c6,64, 39 c6,64, 60 c6,64, 124 c6,64, 210 c6,64, 270 c6,64}
```

```
In[*]:= eqns = # == 0 & /@ reIs;
```

```
In[*]:= vars = Union[Cases[eqns, cd, ∞]]
```

```
Out[*]=
{c6,1, c6,2, c6,3, c6,4, c6,5, c6,6, c6,7, c6,8, c6,9, c6,10, c6,11, c6,12, c6,13, c6,14, c6,15, c6,16, c6,17,
c6,18, c6,19, c6,20, c6,21, c6,22, c6,23, c6,24, c6,25, c6,26, c6,27, c6,28, c6,29, c6,30, c6,31, c6,32, c6,33,
c6,34, c6,35, c6,36, c6,37, c6,38, c6,39, c6,40, c6,41, c6,42, c6,43, c6,44, c6,45, c6,46, c6,47, c6,48, c6,49,
c6,50, c6,51, c6,52, c6,53, c6,54, c6,55, c6,56, c6,57, c6,58, c6,59, c6,60, c6,61, c6,62, c6,63, c6,64}
```

In[*]:= sol = Solve[eqns, vars] [[1]]

Out[*]=

$$\left\{ c_{6,1} \rightarrow 0, c_{6,2} \rightarrow \frac{1}{60480}, c_{6,3} \rightarrow -\frac{1}{12096}, c_{6,4} \rightarrow -\frac{13}{241920}, c_{6,5} \rightarrow \frac{1}{6048}, c_{6,6} \rightarrow \frac{19}{145152}, \right.$$

$$c_{6,7} \rightarrow \frac{61}{725760}, c_{6,8} \rightarrow \frac{83}{967680}, c_{6,9} \rightarrow -\frac{1}{6048}, c_{6,10} \rightarrow -\frac{17}{241920}, c_{6,11} \rightarrow -\frac{61}{241920},$$

$$c_{6,12} \rightarrow -\frac{89}{414720}, c_{6,13} \rightarrow 0, c_{6,14} \rightarrow \frac{71}{967680}, c_{6,15} \rightarrow -\frac{337}{2903040}, c_{6,16} \rightarrow -\frac{31}{483840},$$

$$c_{6,17} \rightarrow \frac{1}{12096}, c_{6,18} \rightarrow \frac{13}{725760}, c_{6,19} \rightarrow \frac{1}{11520}, c_{6,20} \rightarrow \frac{37}{580608}, c_{6,21} \rightarrow \frac{1}{6048}, c_{6,22} \rightarrow \frac{79}{967680},$$

$$c_{6,23} \rightarrow \frac{71}{322560}, c_{6,24} \rightarrow \frac{73}{483840}, c_{6,25} \rightarrow -\frac{1}{18144}, c_{6,26} \rightarrow -\frac{53}{967680}, c_{6,27} \rightarrow -\frac{23}{193536},$$

$$c_{6,28} \rightarrow -\frac{11}{161280}, c_{6,29} \rightarrow \frac{19}{290304}, c_{6,30} \rightarrow -\frac{1}{193536}, c_{6,31} \rightarrow \frac{7}{138240}, c_{6,32} \rightarrow \frac{31}{967680},$$

$$c_{6,33} \rightarrow -\frac{1}{60480}, c_{6,34} \rightarrow \frac{1}{34560}, c_{6,35} \rightarrow -\frac{1}{725760}, c_{6,36} \rightarrow -\frac{1}{967680}, c_{6,37} \rightarrow \frac{19}{120960},$$

$$c_{6,38} \rightarrow \frac{583}{2903040}, c_{6,39} \rightarrow \frac{53}{967680}, c_{6,40} \rightarrow \frac{17}{161280}, c_{6,41} \rightarrow -\frac{103}{181440}, c_{6,42} \rightarrow -\frac{289}{2903040},$$

$$c_{6,43} \rightarrow -\frac{55}{193536}, c_{6,44} \rightarrow -\frac{17}{53760}, c_{6,45} \rightarrow -\frac{11}{483840}, c_{6,46} \rightarrow \frac{7}{46080}, c_{6,47} \rightarrow -\frac{191}{967680},$$

$$c_{6,48} \rightarrow -\frac{31}{193536}, c_{6,49} \rightarrow \frac{13}{241920}, c_{6,50} \rightarrow \frac{1}{17920}, c_{6,51} \rightarrow -\frac{19}{1451520}, c_{6,52} \rightarrow 0, c_{6,53} \rightarrow \frac{89}{414720},$$

$$c_{6,54} \rightarrow \frac{53}{322560}, c_{6,55} \rightarrow \frac{71}{322560}, c_{6,56} \rightarrow \frac{31}{96768}, c_{6,57} \rightarrow -\frac{83}{967680}, c_{6,58} \rightarrow -\frac{53}{967680},$$

$$c_{6,59} \rightarrow -\frac{13}{64512}, c_{6,60} \rightarrow -\frac{31}{96768}, c_{6,61} \rightarrow \frac{31}{483840}, c_{6,62} \rightarrow \frac{31}{193536}, c_{6,63} \rightarrow -\frac{31}{967680}, c_{6,64} \rightarrow 0 \left. \right\}$$

In[*]:= sol /. Rule -> Set

Out[*]=

$$\left\{ 0, \frac{1}{60480}, -\frac{1}{12096}, -\frac{13}{241920}, \frac{1}{6048}, \frac{19}{145152}, \frac{61}{725760}, \frac{83}{967680}, -\frac{1}{6048}, -\frac{17}{241920}, \right.$$

$$-\frac{61}{241920}, -\frac{89}{414720}, 0, \frac{71}{967680}, -\frac{337}{2903040}, -\frac{31}{483840}, \frac{1}{12096}, \frac{13}{725760}, \frac{1}{11520},$$

$$\frac{37}{580608}, \frac{1}{6048}, \frac{79}{967680}, \frac{71}{322560}, \frac{73}{483840}, -\frac{1}{18144}, -\frac{53}{967680}, -\frac{23}{193536}, -\frac{11}{161280},$$

$$\frac{19}{290304}, -\frac{1}{193536}, \frac{7}{138240}, \frac{31}{967680}, -\frac{1}{60480}, \frac{1}{34560}, -\frac{1}{725760}, -\frac{1}{967680}, \frac{19}{120960},$$

$$\frac{583}{2903040}, \frac{53}{967680}, \frac{17}{161280}, \frac{29}{181440}, \frac{289}{2903040}, \frac{55}{193536}, \frac{17}{53760}, -\frac{11}{483840},$$

$$\frac{7}{46080}, -\frac{191}{967680}, -\frac{31}{193536}, -\frac{13}{241920}, \frac{1}{17920}, -\frac{19}{1451520}, 0, \frac{89}{414720}, \frac{53}{322560}, \frac{71}{322560},$$

$$\frac{31}{96768}, -\frac{83}{967680}, -\frac{53}{967680}, \frac{13}{64512}, -\frac{31}{96768}, \frac{31}{483840}, \frac{31}{193536}, -\frac{31}{967680}, 0 \left. \right\}$$

In[*]:= $\mathcal{A}[d]$

Out[*]=

$$\mathcal{O}_{AR, \{x, y\}, \{1\}} \left[\mathcal{A}_0 \left[AW_1[] + \frac{1}{24} AW_1[x, y] - \frac{1}{24} AW_1[y, x] - \frac{AW_1[x, x, x, y]}{1440} + \frac{1}{480} AW_1[x, x, y, x] + \right. \right.$$

$$\begin{aligned}
 & \frac{7 AW_1[x, x, y, y]}{5760} - \frac{1}{480} AW_1[x, y, x, x] - \frac{1}{640} AW_1[x, y, x, y] - \frac{AW_1[x, y, y, x]}{1152} - \\
 & \frac{7 AW_1[x, y, y, y]}{5760} + \frac{AW_1[y, x, x, x]}{1440} - \frac{AW_1[y, x, x, y]}{1152} + \frac{19 AW_1[y, x, y, x]}{5760} + \\
 & \frac{7 AW_1[y, x, y, y]}{1920} - \frac{7 AW_1[y, y, x, x]}{5760} - \frac{7 AW_1[y, y, x, y]}{1920} + \frac{7 AW_1[y, y, y, x]}{5760} + \\
 & \frac{AW_1[x, x, x, x, x, y]}{60480} - \frac{AW_1[x, x, x, x, y, x]}{12096} - \frac{13 AW_1[x, x, x, x, y, y]}{241920} + \\
 & \frac{AW_1[x, x, x, y, x, x]}{6048} + \frac{19 AW_1[x, x, x, y, x, y]}{145152} + \frac{61 AW_1[x, x, x, y, y, x]}{725760} + \\
 & \frac{83 AW_1[x, x, x, y, y, y]}{967680} - \frac{AW_1[x, x, y, x, x, x]}{6048} - \frac{17 AW_1[x, x, y, x, x, y]}{241920} - \\
 & \frac{61 AW_1[x, x, y, x, y, x]}{241920} - \frac{89 AW_1[x, x, y, x, y, y]}{414720} + \frac{71 AW_1[x, x, y, y, x, y]}{967680} - \\
 & \frac{337 AW_1[x, x, y, y, y, x]}{2903040} - \frac{31 AW_1[x, x, y, y, y, y]}{483840} + \frac{AW_1[x, y, x, x, x, x]}{12096} + \\
 & \frac{13 AW_1[x, y, x, x, x, y]}{725760} + \frac{AW_1[x, y, x, x, y, x]}{11520} + \frac{37 AW_1[x, y, x, x, y, y]}{580608} + \\
 & \frac{AW_1[x, y, x, y, x, x]}{6048} + \frac{79 AW_1[x, y, x, y, x, y]}{967680} + \frac{71 AW_1[x, y, x, y, y, x]}{322560} + \\
 & \frac{73 AW_1[x, y, x, y, y, y]}{483840} - \frac{AW_1[x, y, y, x, x, x]}{18144} - \frac{53 AW_1[x, y, y, x, x, y]}{967680} - \\
 & \frac{23 AW_1[x, y, y, x, y, x]}{193536} - \frac{11 AW_1[x, y, y, x, y, y]}{161280} + \frac{19 AW_1[x, y, y, y, x, x]}{290304} - \\
 & \frac{AW_1[x, y, y, y, x, y]}{193536} + \frac{7 AW_1[x, y, y, y, y, x]}{138240} + \frac{31 AW_1[x, y, y, y, y, y]}{967680} - \\
 & \frac{AW_1[y, x, x, x, x, x]}{60480} + \frac{AW_1[y, x, x, x, x, y]}{34560} - \frac{97 AW_1[y, x, x, x, y, x]}{725760} - \\
 & \frac{103 AW_1[y, x, x, x, y, y]}{967680} + \frac{19 AW_1[y, x, x, y, x, x]}{120960} + \frac{583 AW_1[y, x, x, y, x, y]}{2903040} + \\
 & \frac{53 AW_1[y, x, x, y, y, x]}{967680} + \frac{17 AW_1[y, x, x, y, y, y]}{161280} - \frac{29 AW_1[y, x, y, x, x, x]}{181440} - \\
 & \frac{289 AW_1[y, x, y, x, x, y]}{2903040} - \frac{55 AW_1[y, x, y, x, y, x]}{193536} - \frac{17 AW_1[y, x, y, x, y, y]}{53760} - \\
 & \frac{11 AW_1[y, x, y, y, x, x]}{483840} + \frac{7 AW_1[y, x, y, y, x, y]}{46080} - \frac{191 AW_1[y, x, y, y, y, x]}{967680} - \\
 & \frac{31 AW_1[y, x, y, y, y, y]}{193536} + \frac{13 AW_1[y, y, x, x, x, x]}{241920} + \frac{AW_1[y, y, x, x, x, y]}{17920} - \\
 & \frac{19 AW_1[y, y, x, x, y, x]}{1451520} + \frac{89 AW_1[y, y, x, y, x, x]}{414720} + \frac{53 AW_1[y, y, x, y, x, y]}{322560} + \\
 & \frac{71 AW_1[y, y, x, y, y, x]}{322560} + \frac{31 AW_1[y, y, x, y, y, y]}{96768} - \frac{83 AW_1[y, y, y, x, x, x]}{967680} - \\
 & \frac{53 AW_1[y, y, y, x, x, y]}{967680} - \frac{13 AW_1[y, y, y, x, y, x]}{64512} - \frac{31 AW_1[y, y, y, x, y, y]}{96768} +
 \end{aligned}$$

$$\left. \left. \left. \frac{31 AW_1[y, y, y, y, x, x]}{483840} + \frac{31 AW_1[y, y, y, y, x, y]}{193536} - \frac{31 AW_1[y, y, y, y, y, x]}{967680} \right] \right]$$

Solving to Degree 7

In[]:= **d = 7; i = 0;**

ϕ[d] = ϕ[d - 1] + Sum[c_{d,++i} B, {B, Select[Basis_d[O_{AR, {x,y}, {1}}], FreeQ[#, ϑ_{c[1]}] &]}]

Out[]:=

$$O_{AR, \{x,y\}, \{1\}} \left[\mathcal{A}_0 \left[\begin{aligned} & AW_1[] + \frac{1}{24} AW_1[x, y] - \frac{1}{24} AW_1[y, x] - \frac{AW_1[x, x, x, y]}{1440} + \frac{1}{480} AW_1[x, x, y, x] + \frac{7 AW_1[x, x, y, y]}{5760} - \\ & \frac{1}{480} AW_1[x, y, x, x] - \frac{1}{640} AW_1[x, y, x, y] - \frac{AW_1[x, y, y, x]}{1152} - \frac{7 AW_1[x, y, y, y]}{5760} + \\ & \frac{AW_1[y, x, x, x]}{1440} - \frac{AW_1[y, x, x, y]}{1152} + \frac{19 AW_1[y, x, y, x]}{5760} + \frac{7 AW_1[y, x, y, y]}{1920} - \frac{7 AW_1[y, y, x, x]}{5760} - \\ & \frac{7 AW_1[y, y, x, y]}{1920} + \frac{7 AW_1[y, y, y, x]}{5760} + \frac{AW_1[x, x, x, x, x, y]}{60480} - \frac{AW_1[x, x, x, x, y, x]}{12096} - \\ & \frac{13 AW_1[x, x, x, x, y, y]}{241920} + \frac{AW_1[x, x, x, y, x, x]}{6048} + \frac{19 AW_1[x, x, x, y, x, y]}{145152} + \\ & \frac{61 AW_1[x, x, x, y, y, x]}{725760} + \frac{83 AW_1[x, x, x, y, y, y]}{967680} - \frac{AW_1[x, x, y, x, x, x]}{6048} - \\ & \frac{17 AW_1[x, x, y, x, x, y]}{241920} - \frac{61 AW_1[x, x, y, x, y, x]}{241920} - \frac{89 AW_1[x, x, y, x, y, y]}{414720} + \\ & \frac{71 AW_1[x, x, y, y, x, y]}{967680} - \frac{337 AW_1[x, x, y, y, y, x]}{2903040} - \frac{31 AW_1[x, x, y, y, y, y]}{483840} + \\ & \frac{AW_1[x, y, x, x, x, x]}{12096} + \frac{13 AW_1[x, y, x, x, x, y]}{725760} + \frac{AW_1[x, y, x, x, y, x]}{11520} + \\ & \frac{37 AW_1[x, y, x, x, y, y]}{580608} + \frac{AW_1[x, y, x, y, x, x]}{6048} + \frac{79 AW_1[x, y, x, y, x, y]}{967680} + \\ & \frac{71 AW_1[x, y, x, y, y, x]}{322560} + \frac{73 AW_1[x, y, x, y, y, y]}{483840} - \frac{AW_1[x, y, y, x, x, x]}{18144} - \\ & \frac{53 AW_1[x, y, y, x, x, y]}{967680} - \frac{23 AW_1[x, y, y, x, y, x]}{193536} - \frac{11 AW_1[x, y, y, x, y, y]}{161280} + \\ & \frac{19 AW_1[x, y, y, y, x, x]}{290304} - \frac{AW_1[x, y, y, y, x, y]}{193536} + \frac{7 AW_1[x, y, y, y, y, x]}{138240} + \\ & \frac{31 AW_1[x, y, y, y, y, y]}{967680} - \frac{AW_1[y, x, x, x, x, x]}{60480} + \frac{AW_1[y, x, x, x, x, y]}{34560} - \\ & \frac{97 AW_1[y, x, x, x, y, x]}{725760} - \frac{103 AW_1[y, x, x, x, y, y]}{967680} + \frac{19 AW_1[y, x, x, y, x, x]}{120960} + \\ & \frac{583 AW_1[y, x, x, y, x, y]}{2903040} + \frac{53 AW_1[y, x, x, y, y, x]}{967680} + \frac{17 AW_1[y, x, x, y, y, y]}{161280} - \\ & \frac{29 AW_1[y, x, y, x, x, x]}{181440} - \frac{289 AW_1[y, x, y, x, x, y]}{2903040} - \frac{55 AW_1[y, x, y, x, y, x]}{193536} - \end{aligned} \right]$$

$$\begin{aligned}
 & \frac{17 AW_1[y, x, y, x, y, y]}{53760} - \frac{11 AW_1[y, x, y, y, x, x]}{483840} + \frac{7 AW_1[y, x, y, y, x, y]}{46080} - \\
 & \frac{191 AW_1[y, x, y, y, y, x]}{967680} - \frac{31 AW_1[y, x, y, y, y, y]}{193536} + \frac{13 AW_1[y, y, x, x, x, x]}{241920} + \\
 & \frac{AW_1[y, y, x, x, x, y]}{17920} - \frac{19 AW_1[y, y, x, x, y, x]}{1451520} + \frac{89 AW_1[y, y, x, y, x, x]}{414720} + \\
 & \frac{53 AW_1[y, y, x, y, x, y]}{322560} + \frac{71 AW_1[y, y, x, y, y, x]}{322560} + \frac{31 AW_1[y, y, x, y, y, y]}{96768} - \\
 & \frac{83 AW_1[y, y, y, x, x, x]}{967680} - \frac{53 AW_1[y, y, y, x, x, y]}{967680} - \frac{13 AW_1[y, y, y, x, y, x]}{64512} - \\
 & \frac{31 AW_1[y, y, y, x, y, y]}{96768} + \frac{31 AW_1[y, y, y, y, x, x]}{483840} + \frac{31 AW_1[y, y, y, y, x, y]}{193536} - \\
 & \frac{31 AW_1[y, y, y, y, y, x]}{967680} + c_{7,1} AW_1[x, x, x, x, x, x, x] + c_{7,2} AW_1[x, x, x, x, x, x, y] + \\
 & c_{7,3} AW_1[x, x, x, x, x, y, x] + c_{7,4} AW_1[x, x, x, x, x, y, y] + c_{7,5} AW_1[x, x, x, x, y, x, x] + \\
 & c_{7,6} AW_1[x, x, x, x, y, x, y] + c_{7,7} AW_1[x, x, x, x, y, y, x] + c_{7,8} AW_1[x, x, x, x, y, y, y] + \\
 & c_{7,9} AW_1[x, x, x, y, x, x, x] + c_{7,10} AW_1[x, x, x, y, x, x, y] + c_{7,11} AW_1[x, x, x, y, x, y, x] + \\
 & c_{7,12} AW_1[x, x, x, y, x, y, y] + c_{7,13} AW_1[x, x, x, y, y, x, x] + c_{7,14} AW_1[x, x, x, y, y, x, y] + \\
 & c_{7,15} AW_1[x, x, x, y, y, y, x] + c_{7,16} AW_1[x, x, x, y, y, y, y] + c_{7,17} AW_1[x, x, y, x, x, x, x] + \\
 & c_{7,18} AW_1[x, x, y, x, x, x, y] + c_{7,19} AW_1[x, x, y, x, x, y, x] + c_{7,20} AW_1[x, x, y, x, x, y, y] + \\
 & c_{7,21} AW_1[x, x, y, x, y, x, x] + c_{7,22} AW_1[x, x, y, x, y, x, y] + c_{7,23} AW_1[x, x, y, x, y, y, x] + \\
 & c_{7,24} AW_1[x, x, y, x, y, y, y] + c_{7,25} AW_1[x, x, y, y, x, x, x] + c_{7,26} AW_1[x, x, y, y, x, x, y] + \\
 & c_{7,27} AW_1[x, x, y, y, x, y, x] + c_{7,28} AW_1[x, x, y, y, x, y, y] + c_{7,29} AW_1[x, x, y, y, y, x, x] + \\
 & c_{7,30} AW_1[x, x, y, y, y, x, y] + c_{7,31} AW_1[x, x, y, y, y, y, x] + c_{7,32} AW_1[x, x, y, y, y, y, y] + \\
 & c_{7,33} AW_1[x, y, x, x, x, x, x] + c_{7,34} AW_1[x, y, x, x, x, x, y] + c_{7,35} AW_1[x, y, x, x, x, y, x] + \\
 & c_{7,36} AW_1[x, y, x, x, x, y, y] + c_{7,37} AW_1[x, y, x, x, y, x, x] + c_{7,38} AW_1[x, y, x, x, y, x, y] + \\
 & c_{7,39} AW_1[x, y, x, x, y, y, x] + c_{7,40} AW_1[x, y, x, x, y, y, y] + c_{7,41} AW_1[x, y, x, y, x, x, x] + \\
 & c_{7,42} AW_1[x, y, x, y, x, x, y] + c_{7,43} AW_1[x, y, x, y, x, y, x] + c_{7,44} AW_1[x, y, x, y, x, y, y] + \\
 & c_{7,45} AW_1[x, y, x, y, y, x, x] + c_{7,46} AW_1[x, y, x, y, y, x, y] + c_{7,47} AW_1[x, y, x, y, y, y, x] + \\
 & c_{7,48} AW_1[x, y, x, y, y, y, y] + c_{7,49} AW_1[x, y, y, x, x, x, x] + c_{7,50} AW_1[x, y, y, x, x, x, y] + \\
 & c_{7,51} AW_1[x, y, y, x, x, y, x] + c_{7,52} AW_1[x, y, y, x, x, y, y] + c_{7,53} AW_1[x, y, y, x, y, x, x] + \\
 & c_{7,54} AW_1[x, y, y, x, y, x, y] + c_{7,55} AW_1[x, y, y, x, y, y, x] + c_{7,56} AW_1[x, y, y, x, y, y, y] + \\
 & c_{7,57} AW_1[x, y, y, y, x, x, x] + c_{7,58} AW_1[x, y, y, y, x, x, y] + c_{7,59} AW_1[x, y, y, y, x, y, x] + \\
 & c_{7,60} AW_1[x, y, y, y, x, y, y] + c_{7,61} AW_1[x, y, y, y, y, x, x] + c_{7,62} AW_1[x, y, y, y, y, x, y] + \\
 & c_{7,63} AW_1[x, y, y, y, y, y, x] + c_{7,64} AW_1[x, y, y, y, y, y, y] + c_{7,65} AW_1[y, x, x, x, x, x, x] + \\
 & c_{7,66} AW_1[y, x, x, x, x, x, y] + c_{7,67} AW_1[y, x, x, x, x, y, x] + c_{7,68} AW_1[y, x, x, x, x, y, y] + \\
 & c_{7,69} AW_1[y, x, x, x, y, x, x] + c_{7,70} AW_1[y, x, x, x, y, x, y] + c_{7,71} AW_1[y, x, x, x, y, y, x] + \\
 & c_{7,72} AW_1[y, x, x, x, y, y, y] + c_{7,73} AW_1[y, x, x, y, x, x, x] + c_{7,74} AW_1[y, x, x, y, x, x, y] + \\
 & c_{7,75} AW_1[y, x, x, y, x, y, x] + c_{7,76} AW_1[y, x, x, y, x, y, y] + c_{7,77} AW_1[y, x, x, y, y, x, x] + \\
 & c_{7,78} AW_1[y, x, x, y, y, x, y] + c_{7,79} AW_1[y, x, x, y, y, y, x] + c_{7,80} AW_1[y, x, x, y, y, y, y] + \\
 & c_{7,81} AW_1[y, x, y, x, x, x, x] + c_{7,82} AW_1[y, x, y, x, x, x, y] + c_{7,83} AW_1[y, x, y, x, x, y, x] + \\
 & c_{7,84} AW_1[y, x, y, x, x, y, y] + c_{7,85} AW_1[y, x, y, x, y, x, x] + c_{7,86} AW_1[y, x, y, x, y, x, y] + \\
 & c_{7,87} AW_1[y, x, y, x, y, y, x] + c_{7,88} AW_1[y, x, y, x, y, y, y] + c_{7,89} AW_1[y, x, y, y, x, x, x] + \\
 & c_{7,90} AW_1[y, x, y, y, x, x, y] + c_{7,91} AW_1[y, x, y, y, x, y, x] + c_{7,92} AW_1[y, x, y, y, x, y, y] + \\
 & c_{7,93} AW_1[y, x, y, y, y, x, x] + c_{7,94} AW_1[y, x, y, y, y, x, y] + c_{7,95} AW_1[y, x, y, y, y, y, x] +
 \end{aligned}$$

$$\begin{aligned}
 & C_{7,96} AW_1[y, x, y, y, y, y, y] + C_{7,97} AW_1[y, y, x, x, x, x, x] + C_{7,98} AW_1[y, y, x, x, x, x, y] + \\
 & C_{7,99} AW_1[y, y, x, x, x, y, x] + C_{7,100} AW_1[y, y, x, x, x, y, y] + C_{7,101} AW_1[y, y, x, x, y, x, x] + \\
 & C_{7,102} AW_1[y, y, x, x, y, x, y] + C_{7,103} AW_1[y, y, x, x, y, y, x] + C_{7,104} AW_1[y, y, x, x, y, y, y] + \\
 & C_{7,105} AW_1[y, y, x, y, x, x, x] + C_{7,106} AW_1[y, y, x, y, x, x, y] + C_{7,107} AW_1[y, y, x, y, x, y, x] + \\
 & C_{7,108} AW_1[y, y, x, y, x, y, y] + C_{7,109} AW_1[y, y, x, y, y, x, x] + C_{7,110} AW_1[y, y, x, y, y, x, y] + \\
 & C_{7,111} AW_1[y, y, x, y, y, y, x] + C_{7,112} AW_1[y, y, x, y, y, y, y] + C_{7,113} AW_1[y, y, y, x, x, x, x] + \\
 & C_{7,114} AW_1[y, y, y, x, x, x, y] + C_{7,115} AW_1[y, y, y, x, x, y, x] + C_{7,116} AW_1[y, y, y, x, x, y, y] + \\
 & C_{7,117} AW_1[y, y, y, x, y, x, x] + C_{7,118} AW_1[y, y, y, x, y, x, y] + C_{7,119} AW_1[y, y, y, x, y, y, x] + \\
 & C_{7,120} AW_1[y, y, y, x, y, y, y] + C_{7,121} AW_1[y, y, y, y, x, x, x] + C_{7,122} AW_1[y, y, y, y, x, x, y] + \\
 & C_{7,123} AW_1[y, y, y, y, x, y, x] + C_{7,124} AW_1[y, y, y, y, x, y, y] + C_{7,125} AW_1[y, y, y, y, y, x, x] + \\
 & C_{7,126} AW_1[y, y, y, y, y, x, y] + C_{7,127} AW_1[y, y, y, y, y, y, x] + C_{7,128} AW_1[y, y, y, y, y, y, y] \Big]
 \end{aligned}$$

```

In[*]:= Short[
  rels = Union@@(List@@Pentagon_d[ $\emptyset$ ][d]][[1]] /. {
     $\mathcal{A}_0[A_] \Rightarrow$  Table[Coefficient[A, B], {B, Basis_d,{x,y}[AW_1 AW_2]}},
     $\mathcal{A}_c[1,2][A_] \Rightarrow$  Table[Coefficient[A, B], {B, AW_2[] Basis_d-1,{x,y}[AW_1 AW_2 AW_1]}},
  }),
  10]

```

```

Out[*]//Short=
{0, -777 c7,1, -651 c7,1, -546 c7,1, -399 c7,1, -385 c7,1, -351 c7,1, -245 c7,1, -140 c7,1, -120 c7,1,
-77 c7,1, -71 c7,1, -35 c7,1, -21 c7,1, -7 c7,1, -c7,1, 7 c7,1, 13 c7,1, 21 c7,1, 35 c7,1,
175 c7,1, 189 c7,1, <<2034>>, -707 c7,128, -525 c7,128, -392 c7,128, -350 c7,128, -336 c7,128,
-140 c7,128, -119 c7,128, -105 c7,128, -56 c7,128, -42 c7,128, -35 c7,128, -21 c7,128, -7 c7,128,
7 c7,128, 105 c7,128, 189 c7,128, 273 c7,128, 315 c7,128, 490 c7,128, 868 c7,128, 945 c7,128}

```

```

In[*]:= eqns = # == 0 & /@ rels;
In[*]:= vars = Union[Cases[eqns, c_d, _],  $\infty$ ]

```

```

Out[*]=
{c7,1, c7,2, c7,3, c7,4, c7,5, c7,6, c7,7, c7,8, c7,9, c7,10, c7,11, c7,12, c7,13, c7,14, c7,15, c7,16,
c7,17, c7,18, c7,19, c7,20, c7,21, c7,22, c7,23, c7,24, c7,25, c7,26, c7,27, c7,28, c7,29, c7,30, c7,31,
c7,32, c7,33, c7,34, c7,35, c7,36, c7,37, c7,38, c7,39, c7,40, c7,41, c7,42, c7,43, c7,44, c7,45, c7,46,
c7,47, c7,48, c7,49, c7,50, c7,51, c7,52, c7,53, c7,54, c7,55, c7,56, c7,57, c7,58, c7,59, c7,60,
c7,61, c7,62, c7,63, c7,64, c7,65, c7,66, c7,67, c7,68, c7,69, c7,70, c7,71, c7,72, c7,73, c7,74,
c7,75, c7,76, c7,77, c7,78, c7,79, c7,80, c7,81, c7,82, c7,83, c7,84, c7,85, c7,86, c7,87, c7,88,
c7,89, c7,90, c7,91, c7,92, c7,93, c7,94, c7,95, c7,96, c7,97, c7,98, c7,99, c7,100, c7,101, c7,102,
c7,103, c7,104, c7,105, c7,106, c7,107, c7,108, c7,109, c7,110, c7,111, c7,112, c7,113, c7,114, c7,115,
c7,116, c7,117, c7,118, c7,119, c7,120, c7,121, c7,122, c7,123, c7,124, c7,125, c7,126, c7,127, c7,128}

```

```

In[*]:= sol = Solve[eqns, vars][[1]]
Solve: Equations may not give solutions for all "solve" variables.

```

Out[]=

$$\begin{aligned}
& \{ c_{7,1} \rightarrow 0, c_{7,2} \rightarrow c_{7,32}, c_{7,3} \rightarrow -6 c_{7,32}, c_{7,4} \rightarrow -2 c_{7,32}, c_{7,5} \rightarrow 15 c_{7,32}, c_{7,6} \rightarrow 5 c_{7,32}, c_{7,7} \rightarrow 5 c_{7,32}, \\
& c_{7,8} \rightarrow 3 c_{7,32}, c_{7,9} \rightarrow -20 c_{7,32}, c_{7,10} \rightarrow -2 c_{7,32}, c_{7,11} \rightarrow -16 c_{7,32}, c_{7,12} \rightarrow -\frac{141 c_{7,32}}{16}, \\
& c_{7,13} \rightarrow -2 c_{7,32}, c_{7,14} \rightarrow \frac{45 c_{7,32}}{8}, c_{7,15} \rightarrow -\frac{141 c_{7,32}}{16}, c_{7,16} \rightarrow -2 c_{7,32}, c_{7,17} \rightarrow 15 c_{7,32}, \\
& c_{7,18} \rightarrow -2 c_{7,32}, c_{7,19} \rightarrow 12 c_{7,32}, c_{7,20} \rightarrow \frac{45 c_{7,32}}{8}, c_{7,21} \rightarrow 12 c_{7,32}, c_{7,22} \rightarrow -\frac{99 c_{7,32}}{16}, \\
& c_{7,23} \rightarrow \frac{171 c_{7,32}}{8}, c_{7,24} \rightarrow 5 c_{7,32}, c_{7,25} \rightarrow -2 c_{7,32}, c_{7,26} \rightarrow -\frac{9 c_{7,32}}{4}, c_{7,27} \rightarrow -\frac{99 c_{7,32}}{16}, \\
& c_{7,28} \rightarrow -2 c_{7,32}, c_{7,29} \rightarrow \frac{45 c_{7,32}}{8}, c_{7,30} \rightarrow -2 c_{7,32}, c_{7,31} \rightarrow 5 c_{7,32}, c_{7,33} \rightarrow -6 c_{7,32}, c_{7,34} \rightarrow 5 c_{7,32}, \\
& c_{7,35} \rightarrow -16 c_{7,32}, c_{7,36} \rightarrow -\frac{141 c_{7,32}}{16}, c_{7,37} \rightarrow 12 c_{7,32}, c_{7,38} \rightarrow \frac{171 c_{7,32}}{8}, c_{7,39} \rightarrow -\frac{99 c_{7,32}}{16}, \\
& c_{7,40} \rightarrow 5 c_{7,32}, c_{7,41} \rightarrow -16 c_{7,32}, c_{7,42} \rightarrow -\frac{99 c_{7,32}}{16}, c_{7,43} \rightarrow -18 c_{7,32}, c_{7,44} \rightarrow -16 c_{7,32}, \\
& c_{7,45} \rightarrow -\frac{99 c_{7,32}}{16}, c_{7,46} \rightarrow 12 c_{7,32}, c_{7,47} \rightarrow -16 c_{7,32}, c_{7,48} \rightarrow -6 c_{7,32}, c_{7,49} \rightarrow 5 c_{7,32}, \\
& c_{7,50} \rightarrow \frac{45 c_{7,32}}{8}, c_{7,51} \rightarrow -\frac{99 c_{7,32}}{16}, c_{7,52} \rightarrow -2 c_{7,32}, c_{7,53} \rightarrow \frac{171 c_{7,32}}{8}, c_{7,54} \rightarrow 12 c_{7,32}, \\
& c_{7,55} \rightarrow 12 c_{7,32}, c_{7,56} \rightarrow 15 c_{7,32}, c_{7,57} \rightarrow -\frac{141 c_{7,32}}{16}, c_{7,58} \rightarrow -2 c_{7,32}, c_{7,59} \rightarrow -16 c_{7,32}, \\
& c_{7,60} \rightarrow -20 c_{7,32}, c_{7,61} \rightarrow 5 c_{7,32}, c_{7,62} \rightarrow 15 c_{7,32}, c_{7,63} \rightarrow -6 c_{7,32}, c_{7,64} \rightarrow 0, c_{7,65} \rightarrow c_{7,32}, \\
& c_{7,66} \rightarrow -2 c_{7,32}, c_{7,67} \rightarrow 5 c_{7,32}, c_{7,68} \rightarrow 3 c_{7,32}, c_{7,69} \rightarrow -2 c_{7,32}, c_{7,70} \rightarrow -\frac{141 c_{7,32}}{16}, c_{7,71} \rightarrow \frac{45 c_{7,32}}{8}, \\
& c_{7,72} \rightarrow -2 c_{7,32}, c_{7,73} \rightarrow -2 c_{7,32}, c_{7,74} \rightarrow \frac{45 c_{7,32}}{8}, c_{7,75} \rightarrow -\frac{99 c_{7,32}}{16}, c_{7,76} \rightarrow 5 c_{7,32}, \\
& c_{7,77} \rightarrow -\frac{9 c_{7,32}}{4}, c_{7,78} \rightarrow -2 c_{7,32}, c_{7,79} \rightarrow -2 c_{7,32}, c_{7,80} \rightarrow c_{7,32}, c_{7,81} \rightarrow 5 c_{7,32}, c_{7,82} \rightarrow -\frac{141 c_{7,32}}{16}, \\
& c_{7,83} \rightarrow \frac{171 c_{7,32}}{8}, c_{7,84} \rightarrow 5 c_{7,32}, c_{7,85} \rightarrow -\frac{99 c_{7,32}}{16}, c_{7,86} \rightarrow -16 c_{7,32}, c_{7,87} \rightarrow 12 c_{7,32}, \\
& c_{7,88} \rightarrow -6 c_{7,32}, c_{7,89} \rightarrow \frac{45 c_{7,32}}{8}, c_{7,90} \rightarrow -2 c_{7,32}, c_{7,91} \rightarrow 12 c_{7,32}, c_{7,92} \rightarrow 15 c_{7,32}, c_{7,93} \rightarrow -2 c_{7,32}, \\
& c_{7,94} \rightarrow -20 c_{7,32}, c_{7,95} \rightarrow 15 c_{7,32}, c_{7,96} \rightarrow 0, c_{7,97} \rightarrow -2 c_{7,32}, c_{7,98} \rightarrow 3 c_{7,32}, c_{7,99} \rightarrow -\frac{141 c_{7,32}}{16}, \\
& c_{7,100} \rightarrow -2 c_{7,32}, c_{7,101} \rightarrow \frac{45 c_{7,32}}{8}, c_{7,102} \rightarrow 5 c_{7,32}, c_{7,103} \rightarrow -2 c_{7,32}, c_{7,104} \rightarrow c_{7,32}, \\
& c_{7,105} \rightarrow -\frac{141 c_{7,32}}{16}, c_{7,106} \rightarrow 5 c_{7,32}, c_{7,107} \rightarrow -16 c_{7,32}, c_{7,108} \rightarrow -6 c_{7,32}, c_{7,109} \rightarrow -2 c_{7,32}, \\
& c_{7,110} \rightarrow 15 c_{7,32}, c_{7,111} \rightarrow -20 c_{7,32}, c_{7,112} \rightarrow 0, c_{7,113} \rightarrow 3 c_{7,32}, c_{7,114} \rightarrow -2 c_{7,32}, c_{7,115} \rightarrow 5 c_{7,32}, \\
& c_{7,116} \rightarrow c_{7,32}, c_{7,117} \rightarrow 5 c_{7,32}, c_{7,118} \rightarrow -6 c_{7,32}, c_{7,119} \rightarrow 15 c_{7,32}, c_{7,120} \rightarrow 0, c_{7,121} \rightarrow -2 c_{7,32}, \\
& c_{7,122} \rightarrow c_{7,32}, c_{7,123} \rightarrow -6 c_{7,32}, c_{7,124} \rightarrow 0, c_{7,125} \rightarrow c_{7,32}, c_{7,126} \rightarrow 0, c_{7,127} \rightarrow 0, c_{7,128} \rightarrow 0 \}
\end{aligned}$$

In[*]:= sol /. Rule -> Set

Out[*]=

$$\left\{ 0, c_{7,32}, -6 c_{7,32}, -2 c_{7,32}, 15 c_{7,32}, 5 c_{7,32}, 5 c_{7,32}, 3 c_{7,32}, -20 c_{7,32}, -2 c_{7,32}, -16 c_{7,32}, \right. \\ \left. -\frac{141 c_{7,32}}{16}, -2 c_{7,32}, \frac{45 c_{7,32}}{8}, -\frac{141 c_{7,32}}{16}, -2 c_{7,32}, 15 c_{7,32}, -2 c_{7,32}, 12 c_{7,32}, \frac{45 c_{7,32}}{8}, \right. \\ \left. 12 c_{7,32}, -\frac{99 c_{7,32}}{16}, \frac{171 c_{7,32}}{8}, 5 c_{7,32}, -2 c_{7,32}, -\frac{9 c_{7,32}}{4}, -\frac{99 c_{7,32}}{16}, -2 c_{7,32}, \frac{45 c_{7,32}}{8}, \right. \\ \left. -2 c_{7,32}, 5 c_{7,32}, -6 c_{7,32}, 5 c_{7,32}, -16 c_{7,32}, -\frac{141 c_{7,32}}{16}, 12 c_{7,32}, \frac{171 c_{7,32}}{8}, -\frac{99 c_{7,32}}{16}, \right. \\ \left. 5 c_{7,32}, -16 c_{7,32}, -\frac{99 c_{7,32}}{16}, -18 c_{7,32}, -16 c_{7,32}, -\frac{99 c_{7,32}}{16}, 12 c_{7,32}, -16 c_{7,32}, -6 c_{7,32}, \right. \\ \left. 5 c_{7,32}, \frac{45 c_{7,32}}{8}, -\frac{99 c_{7,32}}{16}, -2 c_{7,32}, \frac{171 c_{7,32}}{8}, 12 c_{7,32}, 12 c_{7,32}, 15 c_{7,32}, -\frac{141 c_{7,32}}{16}, \right. \\ \left. -2 c_{7,32}, -16 c_{7,32}, -20 c_{7,32}, 5 c_{7,32}, 15 c_{7,32}, -6 c_{7,32}, 0, c_{7,32}, -2 c_{7,32}, 5 c_{7,32}, \right. \\ \left. 3 c_{7,32}, -2 c_{7,32}, -\frac{141 c_{7,32}}{16}, \frac{45 c_{7,32}}{8}, -2 c_{7,32}, -2 c_{7,32}, \frac{45 c_{7,32}}{8}, -\frac{99 c_{7,32}}{16}, 5 c_{7,32}, \right. \\ \left. -\frac{9 c_{7,32}}{4}, -2 c_{7,32}, -2 c_{7,32}, c_{7,32}, 5 c_{7,32}, -\frac{141 c_{7,32}}{16}, \frac{171 c_{7,32}}{8}, 5 c_{7,32}, -\frac{99 c_{7,32}}{16}, \right. \\ \left. -16 c_{7,32}, 12 c_{7,32}, -6 c_{7,32}, \frac{45 c_{7,32}}{8}, -2 c_{7,32}, 12 c_{7,32}, 15 c_{7,32}, -2 c_{7,32}, -20 c_{7,32}, \right. \\ \left. 15 c_{7,32}, 0, -2 c_{7,32}, 3 c_{7,32}, -\frac{141 c_{7,32}}{16}, -2 c_{7,32}, \frac{45 c_{7,32}}{8}, 5 c_{7,32}, -2 c_{7,32}, c_{7,32}, \right. \\ \left. -\frac{141 c_{7,32}}{16}, 5 c_{7,32}, -16 c_{7,32}, -6 c_{7,32}, -2 c_{7,32}, 15 c_{7,32}, -20 c_{7,32}, 0, 3 c_{7,32}, -2 c_{7,32}, \right. \\ \left. 5 c_{7,32}, c_{7,32}, 5 c_{7,32}, -6 c_{7,32}, 15 c_{7,32}, 0, -2 c_{7,32}, c_{7,32}, -6 c_{7,32}, 0, c_{7,32}, 0, 0, 0 \right\}$$

In[*]:= c_{7,32} = 0

Out[*]=

$$0$$

In[*]:= d

Out[*]=

$$\begin{aligned} & \mathcal{O}_{AR, \{x,y\}, \{1\}} \left[\mathcal{A}_0 \left[AW_1[] + \frac{1}{24} AW_1[x, y] - \frac{1}{24} AW_1[y, x] - \frac{AW_1[x, x, x, y]}{1440} + \frac{1}{480} AW_1[x, x, y, x] + \right. \right. \\ & \frac{7 AW_1[x, x, y, y]}{5760} - \frac{1}{480} AW_1[x, y, x, x] - \frac{1}{640} AW_1[x, y, x, y] - \frac{AW_1[x, y, y, x]}{1152} - \\ & \frac{7 AW_1[x, y, y, y]}{5760} + \frac{AW_1[y, x, x, x]}{1440} - \frac{AW_1[y, x, x, y]}{1152} + \frac{19 AW_1[y, x, y, x]}{5760} + \\ & \frac{7 AW_1[y, x, y, y]}{1920} - \frac{7 AW_1[y, y, x, x]}{5760} - \frac{7 AW_1[y, y, x, y]}{1920} + \frac{7 AW_1[y, y, y, x]}{5760} + \\ & \frac{AW_1[x, x, x, x, x, y]}{60480} - \frac{AW_1[x, x, x, x, y, x]}{12096} - \frac{13 AW_1[x, x, x, x, y, y]}{241920} + \\ & \left. \frac{AW_1[x, x, x, y, x, x]}{6048} + \frac{19 AW_1[x, x, x, y, x, y]}{145152} + \frac{61 AW_1[x, x, x, y, y, x]}{725760} + \right. \end{aligned}$$

$$\begin{aligned}
 & \frac{83 \text{ AW}_1[x, x, x, y, y, y]}{967680} - \frac{\text{ AW}_1[x, x, y, x, x, x]}{6048} - \frac{17 \text{ AW}_1[x, x, y, x, x, y]}{241920} - \\
 & \frac{61 \text{ AW}_1[x, x, y, x, y, x]}{241920} - \frac{89 \text{ AW}_1[x, x, y, x, y, y]}{414720} + \frac{71 \text{ AW}_1[x, x, y, y, x, y]}{967680} - \\
 & \frac{337 \text{ AW}_1[x, x, y, y, y, x]}{2903040} - \frac{31 \text{ AW}_1[x, x, y, y, y, y]}{483840} + \frac{\text{ AW}_1[x, y, x, x, x, x]}{12096} + \\
 & \frac{13 \text{ AW}_1[x, y, x, x, x, y]}{725760} + \frac{\text{ AW}_1[x, y, x, x, y, x]}{11520} + \frac{37 \text{ AW}_1[x, y, x, x, y, y]}{580608} + \\
 & \frac{\text{ AW}_1[x, y, x, y, x, x]}{6048} + \frac{79 \text{ AW}_1[x, y, x, y, x, y]}{967680} + \frac{71 \text{ AW}_1[x, y, x, y, y, x]}{322560} + \\
 & \frac{73 \text{ AW}_1[x, y, x, y, y, y]}{483840} - \frac{\text{ AW}_1[x, y, y, x, x, x]}{18144} - \frac{53 \text{ AW}_1[x, y, y, x, x, y]}{967680} - \\
 & \frac{23 \text{ AW}_1[x, y, y, x, y, x]}{193536} - \frac{11 \text{ AW}_1[x, y, y, x, y, y]}{161280} + \frac{19 \text{ AW}_1[x, y, y, y, x, x]}{290304} - \\
 & \frac{\text{ AW}_1[x, y, y, y, x, y]}{193536} + \frac{7 \text{ AW}_1[x, y, y, y, y, x]}{138240} + \frac{31 \text{ AW}_1[x, y, y, y, y, y]}{967680} - \\
 & \frac{\text{ AW}_1[y, x, x, x, x, x]}{60480} + \frac{\text{ AW}_1[y, x, x, x, x, y]}{34560} - \frac{97 \text{ AW}_1[y, x, x, x, y, x]}{725760} - \\
 & \frac{103 \text{ AW}_1[y, x, x, x, y, y]}{967680} + \frac{19 \text{ AW}_1[y, x, x, y, x, x]}{120960} + \frac{583 \text{ AW}_1[y, x, x, y, x, y]}{2903040} + \\
 & \frac{53 \text{ AW}_1[y, x, x, y, y, x]}{967680} + \frac{17 \text{ AW}_1[y, x, x, y, y, y]}{161280} - \frac{29 \text{ AW}_1[y, x, y, x, x, x]}{181440} - \\
 & \frac{289 \text{ AW}_1[y, x, y, x, x, y]}{2903040} - \frac{55 \text{ AW}_1[y, x, y, x, y, x]}{193536} - \frac{17 \text{ AW}_1[y, x, y, x, y, y]}{53760} - \\
 & \frac{11 \text{ AW}_1[y, x, y, y, x, x]}{483840} + \frac{7 \text{ AW}_1[y, x, y, y, x, y]}{46080} - \frac{191 \text{ AW}_1[y, x, y, y, y, x]}{967680} - \\
 & \frac{31 \text{ AW}_1[y, x, y, y, y, y]}{193536} + \frac{13 \text{ AW}_1[y, y, x, x, x, x]}{241920} + \frac{\text{ AW}_1[y, y, x, x, x, y]}{17920} - \\
 & \frac{19 \text{ AW}_1[y, y, x, x, y, x]}{1451520} + \frac{89 \text{ AW}_1[y, y, x, y, x, x]}{414720} + \frac{53 \text{ AW}_1[y, y, x, y, x, y]}{322560} + \\
 & \frac{71 \text{ AW}_1[y, y, x, y, y, x]}{322560} + \frac{31 \text{ AW}_1[y, y, x, y, y, y]}{96768} - \frac{83 \text{ AW}_1[y, y, y, x, x, x]}{967680} - \\
 & \frac{53 \text{ AW}_1[y, y, y, x, x, y]}{967680} - \frac{13 \text{ AW}_1[y, y, y, x, y, x]}{64512} - \frac{31 \text{ AW}_1[y, y, y, x, y, y]}{96768} + \\
 & \frac{31 \text{ AW}_1[y, y, y, y, x, x]}{483840} + \frac{31 \text{ AW}_1[y, y, y, y, x, y]}{193536} - \frac{31 \text{ AW}_1[y, y, y, y, y, x]}{967680} \Big]
 \end{aligned}$$

Solving to Degree 8

In[]:= **d = 8; i = 0;**

ϕ[d] = ϕ[d - 1] + Sum[c_{d,++i} B, {B, Select[Basis_d[O_{AR, {x,y}, {1}}], FreeQ[#, ϑ_{c[1]}] &]}]

Out[]:=

O_{AR, {x,y}, {1}} [ϑ₀ [

$$\begin{aligned}
 & AW_1[] + \frac{1}{24} AW_1[x, y] - \frac{1}{24} AW_1[y, x] - \frac{AW_1[x, x, x, y]}{1440} + \frac{1}{480} AW_1[x, x, y, x] + \frac{7 AW_1[x, x, y, y]}{5760} - \\
 & \frac{1}{480} AW_1[x, y, x, x] - \frac{1}{640} AW_1[x, y, x, y] - \frac{AW_1[x, y, y, x]}{1152} - \frac{7 AW_1[x, y, y, y]}{5760} + \\
 & \frac{AW_1[y, x, x, x]}{1440} - \frac{AW_1[y, x, x, y]}{1152} + \frac{19 AW_1[y, x, y, x]}{5760} + \frac{7 AW_1[y, x, y, y]}{1920} - \frac{7 AW_1[y, y, x, x]}{5760} - \\
 & \frac{7 AW_1[y, y, x, y]}{1920} + \frac{7 AW_1[y, y, y, x]}{5760} + \frac{AW_1[x, x, x, x, x, y]}{60480} - \frac{AW_1[x, x, x, x, y, x]}{12096} - \\
 & \frac{13 AW_1[x, x, x, x, y, y]}{241920} + \frac{AW_1[x, x, x, y, x, x]}{6048} + \frac{19 AW_1[x, x, x, y, x, y]}{145152} + \\
 & \frac{61 AW_1[x, x, x, y, y, x]}{725760} + \frac{83 AW_1[x, x, x, y, y, y]}{967680} - \frac{AW_1[x, x, y, x, x, x]}{6048} - \\
 & \frac{17 AW_1[x, x, y, x, x, y]}{725760} - \frac{61 AW_1[x, x, y, x, y, x]}{967680} - \frac{89 AW_1[x, x, y, x, y, y]}{6048} + \\
 & \frac{71 AW_1[x, x, y, y, x, y]}{241920} - \frac{337 AW_1[x, x, y, y, y, x]}{241920} - \frac{31 AW_1[x, x, y, y, y, y]}{414720} + \\
 & \frac{AW_1[x, y, x, x, x, x]}{967680} + \frac{13 AW_1[x, y, x, x, x, y]}{2903040} + \frac{AW_1[x, y, x, x, y, x]}{483840} + \\
 & \frac{37 AW_1[x, y, x, x, y, y]}{12096} + \frac{AW_1[x, y, x, y, x, x]}{725760} + \frac{79 AW_1[x, y, x, y, x, y]}{11520} + \\
 & \frac{71 AW_1[x, y, x, y, y, x]}{580608} + \frac{73 AW_1[x, y, x, y, y, y]}{6048} - \frac{AW_1[x, y, y, x, x, x]}{967680} - \\
 & \frac{53 AW_1[x, y, y, x, x, y]}{322560} - \frac{23 AW_1[x, y, y, x, y, x]}{483840} - \frac{11 AW_1[x, y, y, x, y, y]}{18144} + \\
 & \frac{19 AW_1[x, y, y, y, x, x]}{967680} - \frac{AW_1[x, y, y, y, x, y]}{193536} + \frac{7 AW_1[x, y, y, y, y, x]}{161280} + \\
 & \frac{31 AW_1[x, y, y, y, y, y]}{290304} - \frac{AW_1[y, x, x, x, x, x]}{193536} + \frac{AW_1[y, x, x, x, x, y]}{138240} - \\
 & \frac{97 AW_1[y, x, x, x, y, x]}{967680} - \frac{103 AW_1[y, x, x, x, y, y]}{60480} + \frac{19 AW_1[y, x, x, y, x, x]}{34560} + \\
 & \frac{583 AW_1[y, x, x, y, x, y]}{725760} + \frac{53 AW_1[y, x, x, y, y, x]}{967680} + \frac{17 AW_1[y, x, x, y, y, y]}{120960} - \\
 & \frac{29 AW_1[y, x, y, x, x, x]}{2903040} - \frac{289 AW_1[y, x, y, x, x, y]}{967680} - \frac{55 AW_1[y, x, y, x, y, x]}{161280} - \\
 & \frac{17 AW_1[y, x, y, x, y, y]}{181440} - \frac{11 AW_1[y, x, y, y, x, x]}{2903040} + \frac{7 AW_1[y, x, y, y, x, y]}{193536} - \\
 & \frac{191 AW_1[y, x, y, y, y, x]}{53760} - \frac{31 AW_1[y, x, y, y, y, y]}{483840} + \frac{13 AW_1[y, y, x, x, x, x]}{46080} + \\
 & \frac{AW_1[y, y, x, x, x, y]}{967680} - \frac{19 AW_1[y, y, x, x, y, x]}{193536} + \frac{89 AW_1[y, y, x, y, x, x]}{241920} + \\
 & \frac{53 AW_1[y, y, x, y, x, y]}{17920} + \frac{71 AW_1[y, y, x, y, y, x]}{1451520} + \frac{31 AW_1[y, y, x, y, y, y]}{414720} - \\
 & \frac{83 AW_1[y, y, y, x, x, x]}{322560} - \frac{53 AW_1[y, y, y, x, x, y]}{322560} - \frac{13 AW_1[y, y, y, x, y, x]}{96768} - \\
 & \frac{967680}{967680} - \frac{967680}{967680} - \frac{64512}{64512}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{31 AW_1[y, y, y, x, y, y]}{96768} + \frac{31 AW_1[y, y, y, y, x, x]}{483840} + \frac{31 AW_1[y, y, y, y, x, y]}{193536} - \\
 & \frac{31 AW_1[y, y, y, y, y, x]}{967680} + c_{8,1} AW_1[x, x, x, x, x, x, x, x] + c_{8,2} AW_1[x, x, x, x, x, x, x, y] + \\
 & c_{8,3} AW_1[x, x, x, x, x, x, y, x] + c_{8,4} AW_1[x, x, x, x, x, x, y, y] + \\
 & c_{8,5} AW_1[x, x, x, x, x, y, x, x] + c_{8,6} AW_1[x, x, x, x, x, y, x, y] + \\
 & c_{8,7} AW_1[x, x, x, x, x, y, y, x] + c_{8,8} AW_1[x, x, x, x, x, y, y, y] + \\
 & c_{8,9} AW_1[x, x, x, x, y, x, x, x] + c_{8,10} AW_1[x, x, x, x, y, x, x, y] + \\
 & c_{8,11} AW_1[x, x, x, x, y, x, y, x] + c_{8,12} AW_1[x, x, x, x, y, x, y, y] + \\
 & c_{8,13} AW_1[x, x, x, x, y, y, x, x] + c_{8,14} AW_1[x, x, x, x, y, y, x, y] + \\
 & c_{8,15} AW_1[x, x, x, x, y, y, y, x] + c_{8,16} AW_1[x, x, x, x, y, y, y, y] + \\
 & c_{8,17} AW_1[x, x, x, y, x, x, x, x] + c_{8,18} AW_1[x, x, x, y, x, x, x, y] + \\
 & c_{8,19} AW_1[x, x, x, y, x, x, y, x] + c_{8,20} AW_1[x, x, x, y, x, x, y, y] + \\
 & c_{8,21} AW_1[x, x, x, y, x, y, x, x] + c_{8,22} AW_1[x, x, x, y, x, y, x, y] + \\
 & c_{8,23} AW_1[x, x, x, y, x, y, y, x] + c_{8,24} AW_1[x, x, x, y, x, y, y, y] + \\
 & c_{8,25} AW_1[x, x, x, y, y, x, x, x] + c_{8,26} AW_1[x, x, x, y, y, x, x, y] + \\
 & c_{8,27} AW_1[x, x, x, y, y, x, y, x] + c_{8,28} AW_1[x, x, x, y, y, x, y, y] + \\
 & c_{8,29} AW_1[x, x, x, y, y, y, x, x] + c_{8,30} AW_1[x, x, x, y, y, y, x, y] + \\
 & c_{8,31} AW_1[x, x, x, y, y, y, y, x] + c_{8,32} AW_1[x, x, x, y, y, y, y, y] + \\
 & c_{8,33} AW_1[x, x, y, x, x, x, x, x] + c_{8,34} AW_1[x, x, y, x, x, x, x, y] + \\
 & c_{8,35} AW_1[x, x, y, x, x, x, y, x] + c_{8,36} AW_1[x, x, y, x, x, x, y, y] + \\
 & c_{8,37} AW_1[x, x, y, x, x, y, x, x] + c_{8,38} AW_1[x, x, y, x, x, y, x, y] + \\
 & c_{8,39} AW_1[x, x, y, x, x, y, y, x] + c_{8,40} AW_1[x, x, y, x, x, y, y, y] + \\
 & c_{8,41} AW_1[x, x, y, x, y, x, x, x] + c_{8,42} AW_1[x, x, y, x, y, x, x, y] + \\
 & c_{8,43} AW_1[x, x, y, x, y, x, y, x] + c_{8,44} AW_1[x, x, y, x, y, x, y, y] + \\
 & c_{8,45} AW_1[x, x, y, x, y, y, x, x] + c_{8,46} AW_1[x, x, y, x, y, y, x, y] + \\
 & c_{8,47} AW_1[x, x, y, x, y, y, y, x] + c_{8,48} AW_1[x, x, y, x, y, y, y, y] + \\
 & c_{8,49} AW_1[x, x, y, y, x, x, x, x] + c_{8,50} AW_1[x, x, y, y, x, x, x, y] + \\
 & c_{8,51} AW_1[x, x, y, y, x, x, y, x] + c_{8,52} AW_1[x, x, y, y, x, x, y, y] + \\
 & c_{8,53} AW_1[x, x, y, y, x, y, x, x] + c_{8,54} AW_1[x, x, y, y, x, y, x, y] + \\
 & c_{8,55} AW_1[x, x, y, y, x, y, y, x] + c_{8,56} AW_1[x, x, y, y, x, y, y, y] + \\
 & c_{8,57} AW_1[x, x, y, y, y, x, x, x] + c_{8,58} AW_1[x, x, y, y, y, x, x, y] + \\
 & c_{8,59} AW_1[x, x, y, y, y, x, y, x] + c_{8,60} AW_1[x, x, y, y, y, x, y, y] + \\
 & c_{8,61} AW_1[x, x, y, y, y, y, x, x] + c_{8,62} AW_1[x, x, y, y, y, y, x, y] + \\
 & c_{8,63} AW_1[x, x, y, y, y, y, y, x] + c_{8,64} AW_1[x, x, y, y, y, y, y, y] + \\
 & c_{8,65} AW_1[x, y, x, x, x, x, x, x] + c_{8,66} AW_1[x, y, x, x, x, x, x, y] + \\
 & c_{8,67} AW_1[x, y, x, x, x, x, y, x] + c_{8,68} AW_1[x, y, x, x, x, x, y, y] + \\
 & c_{8,69} AW_1[x, y, x, x, x, y, x, x] + c_{8,70} AW_1[x, y, x, x, x, y, x, y] + \\
 & c_{8,71} AW_1[x, y, x, x, x, y, y, x] + c_{8,72} AW_1[x, y, x, x, x, y, y, y] + \\
 & c_{8,73} AW_1[x, y, x, x, y, x, x, x] + c_{8,74} AW_1[x, y, x, x, y, x, x, y] + \\
 & c_{8,75} AW_1[x, y, x, x, y, x, y, x] + c_{8,76} AW_1[x, y, x, x, y, x, y, y] + \\
 & c_{8,77} AW_1[x, y, x, x, y, y, x, x] + c_{8,78} AW_1[x, y, x, x, y, y, x, y] + \\
 & c_{8,79} AW_1[x, y, x, x, y, y, y, x] + c_{8,80} AW_1[x, y, x, x, y, y, y, y] + \\
 & c_{8,81} AW_1[x, y, x, y, x, x, x, x] + c_{8,82} AW_1[x, y, x, y, x, x, x, y] + \\
 & c_{8,83} AW_1[x, y, x, y, x, x, y, x] + c_{8,84} AW_1[x, y, x, y, x, x, y, y] +
 \end{aligned}$$

$$\begin{aligned}
 & C_{8,85} AW_1[x, y, x, y, x, y, x, x] + C_{8,86} AW_1[x, y, x, y, x, y, x, y] + \\
 & C_{8,87} AW_1[x, y, x, y, x, y, y, x] + C_{8,88} AW_1[x, y, x, y, x, y, y, y] + \\
 & C_{8,89} AW_1[x, y, x, y, y, x, x, x] + C_{8,90} AW_1[x, y, x, y, y, x, x, y] + \\
 & C_{8,91} AW_1[x, y, x, y, y, x, y, x] + C_{8,92} AW_1[x, y, x, y, y, x, y, y] + \\
 & C_{8,93} AW_1[x, y, x, y, y, y, x, x] + C_{8,94} AW_1[x, y, x, y, y, y, x, y] + \\
 & C_{8,95} AW_1[x, y, x, y, y, y, y, x] + C_{8,96} AW_1[x, y, x, y, y, y, y, y] + \\
 & C_{8,97} AW_1[x, y, y, x, x, x, x, x] + C_{8,98} AW_1[x, y, y, x, x, x, x, y] + \\
 & C_{8,99} AW_1[x, y, y, x, x, x, y, x] + C_{8,100} AW_1[x, y, y, x, x, x, y, y] + \\
 & C_{8,101} AW_1[x, y, y, x, x, y, x, x] + C_{8,102} AW_1[x, y, y, x, x, y, x, y] + \\
 & C_{8,103} AW_1[x, y, y, x, x, y, y, x] + C_{8,104} AW_1[x, y, y, x, x, y, y, y] + \\
 & C_{8,105} AW_1[x, y, y, x, y, x, x, x] + C_{8,106} AW_1[x, y, y, x, y, x, x, y] + \\
 & C_{8,107} AW_1[x, y, y, x, y, x, y, x] + C_{8,108} AW_1[x, y, y, x, y, x, y, y] + \\
 & C_{8,109} AW_1[x, y, y, x, y, y, x, x] + C_{8,110} AW_1[x, y, y, x, y, y, x, y] + \\
 & C_{8,111} AW_1[x, y, y, x, y, y, y, x] + C_{8,112} AW_1[x, y, y, x, y, y, y, y] + \\
 & C_{8,113} AW_1[x, y, y, y, x, x, x, x] + C_{8,114} AW_1[x, y, y, y, x, x, x, y] + \\
 & C_{8,115} AW_1[x, y, y, y, x, x, y, x] + C_{8,116} AW_1[x, y, y, y, x, x, y, y] + \\
 & C_{8,117} AW_1[x, y, y, y, x, y, x, x] + C_{8,118} AW_1[x, y, y, y, x, y, x, y] + \\
 & C_{8,119} AW_1[x, y, y, y, x, y, y, x] + C_{8,120} AW_1[x, y, y, y, x, y, y, y] + \\
 & C_{8,121} AW_1[x, y, y, y, y, x, x, x] + C_{8,122} AW_1[x, y, y, y, y, x, x, y] + \\
 & C_{8,123} AW_1[x, y, y, y, y, x, y, x] + C_{8,124} AW_1[x, y, y, y, y, x, y, y] + \\
 & C_{8,125} AW_1[x, y, y, y, y, y, x, x] + C_{8,126} AW_1[x, y, y, y, y, y, x, y] + \\
 & C_{8,127} AW_1[x, y, y, y, y, y, y, x] + C_{8,128} AW_1[x, y, y, y, y, y, y, y] + \\
 & C_{8,129} AW_1[y, x, x, x, x, x, x, x] + C_{8,130} AW_1[y, x, x, x, x, x, x, y] + \\
 & C_{8,131} AW_1[y, x, x, x, x, x, y, x] + C_{8,132} AW_1[y, x, x, x, x, x, y, y] + \\
 & C_{8,133} AW_1[y, x, x, x, x, y, x, x] + C_{8,134} AW_1[y, x, x, x, x, y, x, y] + \\
 & C_{8,135} AW_1[y, x, x, x, x, y, y, x] + C_{8,136} AW_1[y, x, x, x, x, y, y, y] + \\
 & C_{8,137} AW_1[y, x, x, x, y, x, x, x] + C_{8,138} AW_1[y, x, x, x, y, x, x, y] + \\
 & C_{8,139} AW_1[y, x, x, x, y, x, y, x] + C_{8,140} AW_1[y, x, x, x, y, x, y, y] + \\
 & C_{8,141} AW_1[y, x, x, x, y, y, x, x] + C_{8,142} AW_1[y, x, x, x, y, y, x, y] + \\
 & C_{8,143} AW_1[y, x, x, x, y, y, y, x] + C_{8,144} AW_1[y, x, x, x, y, y, y, y] + \\
 & C_{8,145} AW_1[y, x, x, y, x, x, x, x] + C_{8,146} AW_1[y, x, x, y, x, x, x, y] + \\
 & C_{8,147} AW_1[y, x, x, y, x, x, y, x] + C_{8,148} AW_1[y, x, x, y, x, x, y, y] + \\
 & C_{8,149} AW_1[y, x, x, y, x, y, x, x] + C_{8,150} AW_1[y, x, x, y, x, y, x, y] + \\
 & C_{8,151} AW_1[y, x, x, y, x, y, y, x] + C_{8,152} AW_1[y, x, x, y, x, y, y, y] + \\
 & C_{8,153} AW_1[y, x, x, y, y, x, x, x] + C_{8,154} AW_1[y, x, x, y, y, x, x, y] + \\
 & C_{8,155} AW_1[y, x, x, y, y, x, y, x] + C_{8,156} AW_1[y, x, x, y, y, x, y, y] + \\
 & C_{8,157} AW_1[y, x, x, y, y, y, x, x] + C_{8,158} AW_1[y, x, x, y, y, y, x, y] + \\
 & C_{8,159} AW_1[y, x, x, y, y, y, y, x] + C_{8,160} AW_1[y, x, x, y, y, y, y, y] + \\
 & C_{8,161} AW_1[y, x, y, x, x, x, x, x] + C_{8,162} AW_1[y, x, y, x, x, x, x, y] + \\
 & C_{8,163} AW_1[y, x, y, x, x, x, y, x] + C_{8,164} AW_1[y, x, y, x, x, x, y, y] + \\
 & C_{8,165} AW_1[y, x, y, x, x, y, x, x] + C_{8,166} AW_1[y, x, y, x, x, y, x, y] + \\
 & C_{8,167} AW_1[y, x, y, x, x, y, y, x] + C_{8,168} AW_1[y, x, y, x, x, y, y, y] + \\
 & C_{8,169} AW_1[y, x, y, x, y, x, x, x] + C_{8,170} AW_1[y, x, y, x, y, x, x, y] + \\
 & C_{8,171} AW_1[y, x, y, x, y, x, y, x] + C_{8,172} AW_1[y, x, y, x, y, x, y, y] + \\
 & C_{8,173} AW_1[y, x, y, x, y, y, x, x] + C_{8,174} AW_1[y, x, y, x, y, y, x, y] +
 \end{aligned}$$

$$\begin{aligned}
 & C_{8,175} AW_1 [y, x, y, x, y, y, y, x] + C_{8,176} AW_1 [y, x, y, x, y, y, y, y] + \\
 & C_{8,177} AW_1 [y, x, y, y, x, x, x, x] + C_{8,178} AW_1 [y, x, y, y, x, x, x, y] + \\
 & C_{8,179} AW_1 [y, x, y, y, x, x, y, x] + C_{8,180} AW_1 [y, x, y, y, x, x, y, y] + \\
 & C_{8,181} AW_1 [y, x, y, y, x, y, x, x] + C_{8,182} AW_1 [y, x, y, y, x, y, x, y] + \\
 & C_{8,183} AW_1 [y, x, y, y, x, y, y, x] + C_{8,184} AW_1 [y, x, y, y, x, y, y, y] + \\
 & C_{8,185} AW_1 [y, x, y, y, y, x, x, x] + C_{8,186} AW_1 [y, x, y, y, y, x, x, y] + \\
 & C_{8,187} AW_1 [y, x, y, y, y, x, y, x] + C_{8,188} AW_1 [y, x, y, y, y, x, y, y] + \\
 & C_{8,189} AW_1 [y, x, y, y, y, y, x, x] + C_{8,190} AW_1 [y, x, y, y, y, y, x, y] + \\
 & C_{8,191} AW_1 [y, x, y, y, y, y, y, x] + C_{8,192} AW_1 [y, x, y, y, y, y, y, y] + \\
 & C_{8,193} AW_1 [y, y, x, x, x, x, x, x] + C_{8,194} AW_1 [y, y, x, x, x, x, x, y] + \\
 & C_{8,195} AW_1 [y, y, x, x, x, x, y, x] + C_{8,196} AW_1 [y, y, x, x, x, x, y, y] + \\
 & C_{8,197} AW_1 [y, y, x, x, x, y, x, x] + C_{8,198} AW_1 [y, y, x, x, x, y, x, y] + \\
 & C_{8,199} AW_1 [y, y, x, x, x, y, y, x] + C_{8,200} AW_1 [y, y, x, x, x, y, y, y] + \\
 & C_{8,201} AW_1 [y, y, x, x, y, x, x, x] + C_{8,202} AW_1 [y, y, x, x, y, x, x, y] + \\
 & C_{8,203} AW_1 [y, y, x, x, y, x, y, x] + C_{8,204} AW_1 [y, y, x, x, y, x, y, y] + \\
 & C_{8,205} AW_1 [y, y, x, x, y, y, x, x] + C_{8,206} AW_1 [y, y, x, x, y, y, x, y] + \\
 & C_{8,207} AW_1 [y, y, x, x, y, y, y, x] + C_{8,208} AW_1 [y, y, x, x, y, y, y, y] + \\
 & C_{8,209} AW_1 [y, y, x, y, x, x, x, x] + C_{8,210} AW_1 [y, y, x, y, x, x, x, y] + \\
 & C_{8,211} AW_1 [y, y, x, y, x, x, y, x] + C_{8,212} AW_1 [y, y, x, y, x, x, y, y] + \\
 & C_{8,213} AW_1 [y, y, x, y, x, y, x, x] + C_{8,214} AW_1 [y, y, x, y, x, y, x, y] + \\
 & C_{8,215} AW_1 [y, y, x, y, x, y, y, x] + C_{8,216} AW_1 [y, y, x, y, x, y, y, y] + \\
 & C_{8,217} AW_1 [y, y, x, y, y, x, x, x] + C_{8,218} AW_1 [y, y, x, y, y, x, x, y] + \\
 & C_{8,219} AW_1 [y, y, x, y, y, x, y, x] + C_{8,220} AW_1 [y, y, x, y, y, x, y, y] + \\
 & C_{8,221} AW_1 [y, y, x, y, y, y, x, x] + C_{8,222} AW_1 [y, y, x, y, y, y, x, y] + \\
 & C_{8,223} AW_1 [y, y, x, y, y, y, y, x] + C_{8,224} AW_1 [y, y, x, y, y, y, y, y] + \\
 & C_{8,225} AW_1 [y, y, y, x, x, x, x, x] + C_{8,226} AW_1 [y, y, y, x, x, x, x, y] + \\
 & C_{8,227} AW_1 [y, y, y, x, x, x, y, x] + C_{8,228} AW_1 [y, y, y, x, x, x, y, y] + \\
 & C_{8,229} AW_1 [y, y, y, x, x, y, x, x] + C_{8,230} AW_1 [y, y, y, x, x, y, x, y] + \\
 & C_{8,231} AW_1 [y, y, y, x, x, y, y, x] + C_{8,232} AW_1 [y, y, y, x, x, y, y, y] + \\
 & C_{8,233} AW_1 [y, y, y, x, y, x, x, x] + C_{8,234} AW_1 [y, y, y, x, y, x, x, y] + \\
 & C_{8,235} AW_1 [y, y, y, x, y, x, y, x] + C_{8,236} AW_1 [y, y, y, x, y, x, y, y] + \\
 & C_{8,237} AW_1 [y, y, y, x, y, y, x, x] + C_{8,238} AW_1 [y, y, y, x, y, y, x, y] + \\
 & C_{8,239} AW_1 [y, y, y, x, y, y, y, x] + C_{8,240} AW_1 [y, y, y, x, y, y, y, y] + \\
 & C_{8,241} AW_1 [y, y, y, y, x, x, x, x] + C_{8,242} AW_1 [y, y, y, y, x, x, x, y] + \\
 & C_{8,243} AW_1 [y, y, y, y, x, x, y, x] + C_{8,244} AW_1 [y, y, y, y, x, x, y, y] + \\
 & C_{8,245} AW_1 [y, y, y, y, x, y, x, x] + C_{8,246} AW_1 [y, y, y, y, x, y, x, y] + \\
 & C_{8,247} AW_1 [y, y, y, y, x, y, y, x] + C_{8,248} AW_1 [y, y, y, y, x, y, y, y] + \\
 & C_{8,249} AW_1 [y, y, y, y, y, x, x, x] + C_{8,250} AW_1 [y, y, y, y, y, x, x, y] + \\
 & C_{8,251} AW_1 [y, y, y, y, y, x, y, x] + C_{8,252} AW_1 [y, y, y, y, y, x, y, y] + \\
 & C_{8,253} AW_1 [y, y, y, y, y, y, x, x] + C_{8,254} AW_1 [y, y, y, y, y, y, x, y] + \\
 & C_{8,255} AW_1 [y, y, y, y, y, y, y, x] + C_{8,256} AW_1 [y, y, y, y, y, y, y, y] \Big]
 \end{aligned}$$

```
In[*]:= Short[
  reIs = Union@@(List@@Pentagond[ $\Phi$ ][d]][[1]] /. {
     $\mathcal{A}_0[A_] \Rightarrow$  Table[Coefficient[A, B], {B, Basisd, {x,y} [AW1 AW2] }},
     $\mathcal{A}_{C[1,2]}[A_] \Rightarrow$  Table[Coefficient[A, B], {B, AW2[ ] Basisd-1, {x,y} [AW1 AW2 AW1] }},
  }),
  10]
```

```
Out[*]//Short=
{0, -3108 C8,1, -2808 C8,1, -1736 C8,1, -1596 C8,1, -1456 C8,1, -1023 C8,1, -960 C8,1,
-770 C8,1, -568 C8,1, -490 C8,1, -351 C8,1, -308 C8,1, -247 C8,1, -224 C8,1, -70 C8,1,
-56 C8,1, -28 C8,1, -15 C8,1, -8 C8,1, -C8,1, C8,1, 28 C8,1, 70 C8,1, 97 C8,1, 104 C8,1,
280 C8,1, <<5259>>, -2688 C8,256, -1568 C8,256, -1400 C8,256, -1002 C8,256, -952 C8,256,
-700 C8,256, -448 C8,256, -316 C8,256, -280 C8,256, -246 C8,256, -168 C8,256, -70 C8,256,
-56 C8,256, -28 C8,256, -8 C8,256, 8 C8,256, 56 C8,256, 76 C8,256, 168 C8,256, 280 C8,256,
734 C8,256, 762 C8,256, 980 C8,256, 1092 C8,256, 1512 C8,256, 2520 C8,256, 3472 C8,256}
```

```
In[*]:= eqns = # == 0 & /@ reIs;
```

```
In[*]:= vars = Union[Cases[eqns, Cd, _],  $\infty$ ]
```

```
Out[*]=
{C8,1, C8,2, C8,3, C8,4, C8,5, C8,6, C8,7, C8,8, C8,9, C8,10, C8,11, C8,12, C8,13, C8,14, C8,15, C8,16,
C8,17, C8,18, C8,19, C8,20, C8,21, C8,22, C8,23, C8,24, C8,25, C8,26, C8,27, C8,28, C8,29, C8,30,
C8,31, C8,32, C8,33, C8,34, C8,35, C8,36, C8,37, C8,38, C8,39, C8,40, C8,41, C8,42, C8,43, C8,44,
C8,45, C8,46, C8,47, C8,48, C8,49, C8,50, C8,51, C8,52, C8,53, C8,54, C8,55, C8,56, C8,57, C8,58,
C8,59, C8,60, C8,61, C8,62, C8,63, C8,64, C8,65, C8,66, C8,67, C8,68, C8,69, C8,70, C8,71, C8,72,
C8,73, C8,74, C8,75, C8,76, C8,77, C8,78, C8,79, C8,80, C8,81, C8,82, C8,83, C8,84, C8,85, C8,86,
C8,87, C8,88, C8,89, C8,90, C8,91, C8,92, C8,93, C8,94, C8,95, C8,96, C8,97, C8,98, C8,99, C8,100,
C8,101, C8,102, C8,103, C8,104, C8,105, C8,106, C8,107, C8,108, C8,109, C8,110, C8,111, C8,112, C8,113,
C8,114, C8,115, C8,116, C8,117, C8,118, C8,119, C8,120, C8,121, C8,122, C8,123, C8,124, C8,125, C8,126,
C8,127, C8,128, C8,129, C8,130, C8,131, C8,132, C8,133, C8,134, C8,135, C8,136, C8,137, C8,138, C8,139,
C8,140, C8,141, C8,142, C8,143, C8,144, C8,145, C8,146, C8,147, C8,148, C8,149, C8,150, C8,151, C8,152,
C8,153, C8,154, C8,155, C8,156, C8,157, C8,158, C8,159, C8,160, C8,161, C8,162, C8,163, C8,164, C8,165,
C8,166, C8,167, C8,168, C8,169, C8,170, C8,171, C8,172, C8,173, C8,174, C8,175, C8,176, C8,177, C8,178,
C8,179, C8,180, C8,181, C8,182, C8,183, C8,184, C8,185, C8,186, C8,187, C8,188, C8,189, C8,190, C8,191,
C8,192, C8,193, C8,194, C8,195, C8,196, C8,197, C8,198, C8,199, C8,200, C8,201, C8,202, C8,203, C8,204,
C8,205, C8,206, C8,207, C8,208, C8,209, C8,210, C8,211, C8,212, C8,213, C8,214, C8,215, C8,216, C8,217,
C8,218, C8,219, C8,220, C8,221, C8,222, C8,223, C8,224, C8,225, C8,226, C8,227, C8,228, C8,229, C8,230,
C8,231, C8,232, C8,233, C8,234, C8,235, C8,236, C8,237, C8,238, C8,239, C8,240, C8,241, C8,242, C8,243,
C8,244, C8,245, C8,246, C8,247, C8,248, C8,249, C8,250, C8,251, C8,252, C8,253, C8,254, C8,255, C8,256}
```

```
In[*]:= sol = Solve[eqns, vars][[1]]
```

 Solve: Equations may not give solutions for all "solve" variables.

```
Out[*]=
{C8,1  $\rightarrow$  0, C8,2  $\rightarrow$  - $\frac{1}{2419200}$ , C8,3  $\rightarrow$   $\frac{1}{345600}$ , C8,4  $\rightarrow$   $\frac{19}{9676800}$ , C8,5  $\rightarrow$  - $\frac{1}{115200}$ ,
C8,7  $\rightarrow$  - $\frac{19}{1612800}$  - C8,6, C8,8  $\rightarrow$  - $\frac{271}{58060800}$ , C8,9  $\rightarrow$   $\frac{1}{69120}$ , C8,10  $\rightarrow$  - $\frac{457}{58060800}$  -  $\frac{5 C_{8,6}}{2}$ ,
```

$$\begin{aligned}
 C_{8,11} &\rightarrow \frac{457}{29\,030\,400}, C_{8,12} \rightarrow \frac{587}{69\,672\,960} - \frac{3 C_{8,6}}{4}, C_{8,13} \rightarrow \frac{179}{8\,294\,400} + \frac{5 C_{8,6}}{2}, C_{8,14} \rightarrow \frac{223}{174\,182\,400}, \\
 C_{8,15} &\rightarrow \frac{1583}{116\,121\,600} + \frac{3 C_{8,6}}{4}, C_{8,16} \rightarrow \frac{2893}{464\,486\,400}, C_{8,17} \rightarrow -\frac{1}{69\,120}, C_{8,18} \rightarrow -\frac{73}{9\,676\,800}, \\
 C_{8,19} &\rightarrow \frac{1571}{29\,030\,400} + 10 C_{8,6}, C_{8,20} \rightarrow \frac{1619}{116\,121\,600} + \frac{15 C_{8,6}}{4}, C_{8,21} \rightarrow -\frac{71}{829\,440} - 10 C_{8,6}, \\
 C_{8,22} &\rightarrow -\frac{79}{1\,935\,360} - \frac{9 C_{8,6}}{2}, C_{8,23} \rightarrow -\frac{3617}{174\,182\,400}, C_{8,24} \rightarrow -\frac{83}{7\,962\,624} + \frac{3 C_{8,6}}{4}, C_{8,25} \rightarrow -\frac{1}{4\,147\,200}, \\
 C_{8,26} &\rightarrow \frac{109}{58\,060\,800}, C_{8,27} \rightarrow \frac{1391}{43\,545\,600} + \frac{9 C_{8,6}}{2}, C_{8,28} \rightarrow -\frac{35\,237}{1\,393\,459\,200} - \frac{15 C_{8,6}}{4}, \\
 C_{8,29} &\rightarrow -\frac{109}{3\,317\,760} - \frac{15 C_{8,6}}{4}, C_{8,30} \rightarrow \frac{32\,899}{1\,393\,459\,200} + \frac{15 C_{8,6}}{4}, C_{8,31} \rightarrow -\frac{5951}{464\,486\,400} - \frac{3 C_{8,6}}{4}, \\
 C_{8,32} &\rightarrow -\frac{2399}{464\,486\,400}, C_{8,33} \rightarrow \frac{1}{115\,200}, C_{8,34} \rightarrow \frac{1093}{58\,060\,800} + \frac{5 C_{8,6}}{2}, C_{8,35} \rightarrow -\frac{1529}{29\,030\,400} - 10 C_{8,6}, \\
 C_{8,36} &\rightarrow -\frac{4813}{348\,364\,800} - \frac{15 C_{8,6}}{4}, C_{8,37} \rightarrow -\frac{1}{460\,800}, C_{8,38} \rightarrow \frac{1517}{38\,707\,200} + \frac{27 C_{8,6}}{4}, \\
 C_{8,39} &\rightarrow -\frac{919}{23\,224\,320} - \frac{27 C_{8,6}}{4}, C_{8,40} \rightarrow \frac{2311}{1\,393\,459\,200}, C_{8,41} \rightarrow \frac{361}{4\,147\,200} + 10 C_{8,6}, \\
 C_{8,42} &\rightarrow \frac{113}{11\,612\,160}, C_{8,43} \rightarrow \frac{1429}{58\,060\,800}, C_{8,44} \rightarrow \frac{25\,189}{464\,486\,400} + \frac{9 C_{8,6}}{2}, C_{8,45} \rightarrow \frac{323}{5\,529\,600} + \frac{27 C_{8,6}}{4}, \\
 C_{8,46} &\rightarrow -\frac{22\,231}{464\,486\,400} - \frac{27 C_{8,6}}{4}, C_{8,47} \rightarrow \frac{4297}{199\,065\,600}, C_{8,48} \rightarrow \frac{10\,963}{1\,393\,459\,200} - C_{8,6}, \\
 C_{8,49} &\rightarrow -\frac{179}{8\,294\,400} - \frac{5 C_{8,6}}{2}, C_{8,50} \rightarrow -\frac{467}{87\,091\,200}, C_{8,51} \rightarrow \frac{1}{1\,382\,400}, C_{8,52} \rightarrow -\frac{419}{464\,486\,400}, \\
 C_{8,53} &\rightarrow -\frac{337}{5\,529\,600} - \frac{27 C_{8,6}}{4}, C_{8,54} \rightarrow -\frac{31}{2\,457\,600}, C_{8,55} \rightarrow \frac{3349}{92\,897\,280} + \frac{27 C_{8,6}}{4}, \\
 C_{8,56} &\rightarrow \frac{83}{17\,203\,200} + \frac{5 C_{8,6}}{2}, C_{8,57} \rightarrow \frac{559}{16\,588\,800} + \frac{15 C_{8,6}}{4}, C_{8,58} \rightarrow \frac{1027}{278\,691\,840}, \\
 C_{8,59} &\rightarrow -\frac{24\,697}{1\,393\,459\,200} - \frac{9 C_{8,6}}{2}, C_{8,60} \rightarrow \frac{19\,619}{1\,393\,459\,200}, C_{8,61} \rightarrow -\frac{49}{66\,355\,200}, \\
 C_{8,62} &\rightarrow -\frac{3697}{154\,828\,800} - \frac{5 C_{8,6}}{2}, C_{8,63} \rightarrow \frac{1951}{154\,828\,800} + C_{8,6}, C_{8,64} \rightarrow \frac{127}{51\,609\,600}, C_{8,65} \rightarrow -\frac{1}{345\,600}, \\
 C_{8,66} &\rightarrow -\frac{31}{4\,838\,400} - C_{8,6}, C_{8,67} \rightarrow -\frac{163}{29\,030\,400}, C_{8,68} \rightarrow \frac{29}{16\,588\,800} + \frac{3 C_{8,6}}{4}, C_{8,69} \rightarrow \frac{53}{829\,440} + 10 C_{8,6}, \\
 C_{8,70} &\rightarrow -\frac{391}{174\,182\,400}, C_{8,71} \rightarrow \frac{1993}{87\,091\,200} + \frac{9 C_{8,6}}{2}, C_{8,72} \rightarrow -\frac{13\,781}{1\,393\,459\,200} - \frac{3 C_{8,6}}{4}, \\
 C_{8,73} &\rightarrow -\frac{259}{4\,147\,200} - 10 C_{8,6}, C_{8,74} \rightarrow -\frac{4559}{116\,121\,600} - \frac{27 C_{8,6}}{4}, C_{8,75} \rightarrow \frac{19}{2\,764\,800}, \\
 C_{8,76} &\rightarrow \frac{4679}{1\,393\,459\,200}, C_{8,77} \rightarrow \frac{1}{552\,960}, C_{8,78} \rightarrow \frac{133}{2\,457\,600} + \frac{27 C_{8,6}}{4}, C_{8,79} \rightarrow -\frac{43\,369}{1\,393\,459\,200} - \frac{9 C_{8,6}}{2}, \\
 C_{8,80} &\rightarrow \frac{12\,323}{1\,393\,459\,200} + C_{8,6}, C_{8,81} \rightarrow -\frac{17}{1\,382\,400}, C_{8,82} \rightarrow \frac{661}{24\,883\,200} + \frac{9 C_{8,6}}{2}, C_{8,83} \rightarrow -\frac{599}{29\,030\,400}, \\
 C_{8,84} &\rightarrow -\frac{12\,637}{278\,691\,840} - \frac{9 C_{8,6}}{2}, C_{8,85} \rightarrow -\frac{1}{92\,160}, C_{8,86} \rightarrow -\frac{2339}{464\,486\,400}, C_{8,87} \rightarrow -\frac{1807}{92\,897\,280},
 \end{aligned}$$

$$\begin{aligned}
 C_{8,88} &\rightarrow -\frac{5687}{464\,486\,400}, C_{8,89} \rightarrow -\frac{101}{2\,764\,800} - \frac{9 C_{8,6}}{2}, C_{8,90} \rightarrow \frac{677}{464\,486\,400}, C_{8,91} \rightarrow -\frac{1609}{154\,828\,800}, \\
 C_{8,92} &\rightarrow -\frac{5257}{66\,355\,200} - 10 C_{8,6}, C_{8,93} \rightarrow \frac{541}{22\,118\,400} + \frac{9 C_{8,6}}{2}, C_{8,94} \rightarrow \frac{33\,893}{464\,486\,400} + 10 C_{8,6}, \\
 C_{8,95} &\rightarrow -\frac{6931}{464\,486\,400}, C_{8,96} \rightarrow -\frac{3881}{464\,486\,400}, C_{8,97} \rightarrow \frac{23}{2\,073\,600} + C_{8,6}, C_{8,98} \rightarrow \frac{53}{29\,030\,400}, \\
 C_{8,99} &\rightarrow -\frac{4031}{174\,182\,400} - \frac{9 C_{8,6}}{2}, C_{8,100} \rightarrow \frac{8863}{278\,691\,840} + \frac{15 C_{8,6}}{4}, C_{8,101} \rightarrow \frac{49}{1\,105\,920} + \frac{27 C_{8,6}}{4}, \\
 C_{8,102} &\rightarrow -\frac{69\,901}{1\,393\,459\,200} - \frac{27 C_{8,6}}{4}, C_{8,103} \rightarrow \frac{59}{30\,965\,760}, C_{8,104} \rightarrow -\frac{1097}{66\,355\,200} - \frac{5 C_{8,6}}{2}, \\
 C_{8,105} &\rightarrow \frac{61}{4\,147\,200}, C_{8,106} \rightarrow \frac{77\,299}{1\,393\,459\,200} + \frac{27 C_{8,6}}{4}, C_{8,107} \rightarrow \frac{365}{18\,579\,456}, \\
 C_{8,108} &\rightarrow \frac{869}{10\,321\,920} + 10 C_{8,6}, C_{8,109} \rightarrow -\frac{101}{2\,654\,208} - \frac{27 C_{8,6}}{4}, C_{8,110} \rightarrow \frac{2213}{464\,486\,400}, \\
 C_{8,111} &\rightarrow -\frac{4951}{92\,897\,280} - 10 C_{8,6}, C_{8,112} \rightarrow \frac{4927}{464\,486\,400}, C_{8,113} \rightarrow -\frac{7}{614\,400} - \frac{3 C_{8,6}}{4}, \\
 C_{8,114} &\rightarrow -\frac{13\,549}{464\,486\,400} - \frac{15 C_{8,6}}{4}, C_{8,115} \rightarrow \frac{32\,341}{1\,393\,459\,200} + \frac{9 C_{8,6}}{2}, C_{8,116} \rightarrow -\frac{8417}{1\,393\,459\,200}, \\
 C_{8,117} &\rightarrow -\frac{671}{66\,355\,200}, C_{8,118} \rightarrow -\frac{35\,459}{464\,486\,400} - 10 C_{8,6}, C_{8,119} \rightarrow \frac{5137}{92\,897\,280} + 10 C_{8,6}, \\
 C_{8,120} &\rightarrow -\frac{683}{92\,897\,280}, C_{8,121} \rightarrow \frac{557}{66\,355\,200} + \frac{3 C_{8,6}}{4}, C_{8,122} \rightarrow \frac{359}{17\,203\,200} + \frac{5 C_{8,6}}{2}, \\
 C_{8,123} &\rightarrow \frac{307}{66\,355\,200}, C_{8,124} \rightarrow -\frac{319}{464\,486\,400}, C_{8,125} \rightarrow -\frac{187}{22\,118\,400} - C_{8,6}, C_{8,126} \rightarrow \frac{13}{4\,423\,680}, \\
 C_{8,127} &\rightarrow -\frac{107}{51\,609\,600}, C_{8,128} \rightarrow -\frac{127}{154\,828\,800}, C_{8,129} \rightarrow \frac{1}{2\,419\,200}, C_{8,130} \rightarrow -\frac{1}{1\,075\,200}, \\
 C_{8,131} &\rightarrow \frac{29}{2\,419\,200} + C_{8,6}, C_{8,132} \rightarrow \frac{43}{11\,612\,160}, C_{8,133} \rightarrow -\frac{1577}{58\,060\,800} - \frac{5 C_{8,6}}{2}, \\
 C_{8,134} &\rightarrow -\frac{5413}{348\,364\,800} - \frac{3 C_{8,6}}{4}, C_{8,135} \rightarrow -\frac{823}{174\,182\,400}, C_{8,136} \rightarrow -\frac{323}{51\,609\,600}, C_{8,137} \rightarrow \frac{433}{29\,030\,400}, \\
 C_{8,138} &\rightarrow \frac{899}{23\,224\,320} + \frac{15 C_{8,6}}{4}, C_{8,139} \rightarrow -\frac{1}{76\,800} - \frac{9 C_{8,6}}{2}, C_{8,140} \rightarrow \frac{4829}{199\,065\,600} + \frac{3 C_{8,6}}{4}, \\
 C_{8,141} &\rightarrow \frac{787}{174\,182\,400}, C_{8,142} \rightarrow -\frac{36\,941}{1\,393\,459\,200} - \frac{15 C_{8,6}}{4}, C_{8,143} \rightarrow \frac{51\,803}{1\,393\,459\,200} + \frac{15 C_{8,6}}{4}, \\
 C_{8,144} &\rightarrow \frac{1411}{154\,828\,800}, C_{8,145} \rightarrow \frac{257}{58\,060\,800} + \frac{5 C_{8,6}}{2}, C_{8,146} \rightarrow -\frac{2587}{69\,672\,960} - \frac{15 C_{8,6}}{4}, \\
 C_{8,147} &\rightarrow \frac{4009}{116\,121\,600} + \frac{27 C_{8,6}}{4}, C_{8,148} \rightarrow -\frac{16\,801}{1\,393\,459\,200}, C_{8,149} \rightarrow -\frac{107}{5\,806\,080}, \\
 C_{8,150} &\rightarrow \frac{1163}{66\,355\,200} + \frac{9 C_{8,6}}{2}, C_{8,151} \rightarrow -\frac{32\,303}{464\,486\,400} - \frac{27 C_{8,6}}{4}, C_{8,152} \rightarrow -\frac{40\,237}{1\,393\,459\,200} - C_{8,6}, \\
 C_{8,153} &\rightarrow \frac{89}{87\,091\,200}, C_{8,154} \rightarrow \frac{59}{30\,965\,760}, C_{8,155} \rightarrow \frac{631}{154\,828\,800}, C_{8,156} \rightarrow \frac{10\,457}{464\,486\,400} + \frac{5 C_{8,6}}{2}, \\
 C_{8,157} &\rightarrow -\frac{2081}{278\,691\,840}, C_{8,158} \rightarrow -\frac{313}{55\,738\,368}, C_{8,159} \rightarrow -\frac{11\,243}{464\,486\,400} - \frac{5 C_{8,6}}{2},
 \end{aligned}$$

$$\begin{aligned}
 C_{8,160} &\rightarrow -\frac{2977}{464\,486\,400}, C_{8,161} \rightarrow \frac{1}{1\,451\,520} - C_{8,6}, C_{8,162} \rightarrow \frac{181}{23\,224\,320} + \frac{3 C_{8,6}}{4}, C_{8,163} \rightarrow \frac{1439}{87\,091\,200}, \\
 C_{8,164} &\rightarrow \frac{11\,219}{1\,393\,459\,200} - \frac{3 C_{8,6}}{4}, C_{8,165} \rightarrow -\frac{5689}{116\,121\,600} - \frac{27 C_{8,6}}{4}, C_{8,166} \rightarrow -\frac{13\,361}{1\,393\,459\,200}, \\
 C_{8,167} &\rightarrow \frac{2833}{51\,609\,600} + \frac{27 C_{8,6}}{4}, C_{8,168} \rightarrow \frac{13\,843}{1\,393\,459\,200} + C_{8,6}, C_{8,169} \rightarrow \frac{8459}{174\,182\,400} + \frac{9 C_{8,6}}{2}, \\
 C_{8,170} &\rightarrow -\frac{25\,273}{1\,393\,459\,200} - \frac{9 C_{8,6}}{2}, C_{8,171} \rightarrow \frac{11\,813}{464\,486\,400}, C_{8,172} \rightarrow \frac{12\,449}{464\,486\,400}, \\
 C_{8,173} &\rightarrow \frac{5417}{464\,486\,400}, C_{8,174} \rightarrow -\frac{32\,507}{464\,486\,400} - 10 C_{8,6}, C_{8,175} \rightarrow \frac{14\,447}{154\,828\,800} + 10 C_{8,6}, \\
 C_{8,176} &\rightarrow \frac{9551}{464\,486\,400}, C_{8,177} \rightarrow -\frac{613}{174\,182\,400}, C_{8,178} \rightarrow \frac{25\,913}{1\,393\,459\,200} + \frac{15 C_{8,6}}{4}, \\
 C_{8,179} &\rightarrow -\frac{59\,807}{1\,393\,459\,200} - \frac{27 C_{8,6}}{4}, C_{8,180} \rightarrow -\frac{1093}{66\,355\,200} - \frac{5 C_{8,6}}{2}, C_{8,181} \rightarrow \frac{43\,649}{1\,393\,459\,200} + \frac{27 C_{8,6}}{4}, \\
 C_{8,182} &\rightarrow \frac{23\,741}{464\,486\,400} + 10 C_{8,6}, C_{8,183} \rightarrow -\frac{5003}{464\,486\,400}, C_{8,184} \rightarrow -\frac{9463}{464\,486\,400}, \\
 C_{8,185} &\rightarrow -\frac{24\,391}{1\,393\,459\,200} - \frac{15 C_{8,6}}{4}, C_{8,186} \rightarrow \frac{197}{39\,813\,120}, C_{8,187} \rightarrow -\frac{389}{7\,372\,800} - 10 C_{8,6}, \\
 C_{8,188} &\rightarrow \frac{11\,521}{464\,486\,400}, C_{8,189} \rightarrow \frac{7793}{464\,486\,400} + \frac{5 C_{8,6}}{2}, C_{8,190} \rightarrow -\frac{6187}{464\,486\,400}, C_{8,191} \rightarrow \frac{1471}{154\,828\,800}, \\
 C_{8,192} &\rightarrow \frac{127}{22\,118\,400}, C_{8,193} \rightarrow -\frac{19}{9\,676\,800}, C_{8,194} \rightarrow -\frac{1}{1\,612\,800}, C_{8,195} \rightarrow -\frac{757}{116\,121\,600} - \frac{3 C_{8,6}}{4}, \\
 C_{8,196} &\rightarrow -\frac{49}{66\,355\,200}, C_{8,197} \rightarrow \frac{1139}{69\,672\,960} + \frac{15 C_{8,6}}{4}, C_{8,198} \rightarrow -\frac{2393}{1\,393\,459\,200} + \frac{3 C_{8,6}}{4}, \\
 C_{8,199} &\rightarrow -\frac{1961}{55\,738\,368} - \frac{15 C_{8,6}}{4}, C_{8,200} \rightarrow -\frac{581}{66\,355\,200}, C_{8,201} \rightarrow -\frac{1717}{116\,121\,600} - \frac{15 C_{8,6}}{4}, \\
 C_{8,202} &\rightarrow \frac{1619}{154\,828\,800}, C_{8,203} \rightarrow \frac{973}{22\,118\,400} + \frac{9 C_{8,6}}{2}, C_{8,204} \rightarrow \frac{491}{66\,355\,200} - C_{8,6}, \\
 C_{8,205} &\rightarrow \frac{221}{92\,897\,280}, C_{8,206} \rightarrow \frac{1387}{66\,355\,200} + \frac{5 C_{8,6}}{2}, C_{8,207} \rightarrow \frac{6359}{1\,393\,459\,200}, C_{8,208} \rightarrow \frac{127}{22\,118\,400}, \\
 C_{8,209} &\rightarrow -\frac{587}{69\,672\,960} + \frac{3 C_{8,6}}{4}, C_{8,210} \rightarrow -\frac{509}{39\,813\,120} - \frac{3 C_{8,6}}{4}, C_{8,211} \rightarrow -\frac{9241}{464\,486\,400}, \\
 C_{8,212} &\rightarrow -\frac{157}{13\,271\,040} + C_{8,6}, C_{8,213} \rightarrow -\frac{21\,661}{464\,486\,400} - \frac{9 C_{8,6}}{2}, C_{8,214} \rightarrow -\frac{3683}{464\,486\,400}, \\
 C_{8,215} &\rightarrow -\frac{2917}{30\,965\,760} - 10 C_{8,6}, C_{8,216} \rightarrow -\frac{17}{819\,200}, C_{8,217} \rightarrow \frac{31\,709}{1\,393\,459\,200} + \frac{15 C_{8,6}}{4}, \\
 C_{8,218} &\rightarrow -\frac{7667}{464\,486\,400} - \frac{5 C_{8,6}}{2}, C_{8,219} \rightarrow \frac{29\,893}{464\,486\,400} + 10 C_{8,6}, C_{8,220} \rightarrow -\frac{49}{7\,372\,800}, \\
 C_{8,221} &\rightarrow -\frac{2689}{278\,691\,840}, C_{8,222} \rightarrow \frac{853}{464\,486\,400}, C_{8,223} \rightarrow -\frac{7939}{464\,486\,400}, C_{8,224} \rightarrow -\frac{127}{7\,372\,800}, \\
 C_{8,225} &\rightarrow \frac{271}{58\,060\,800}, C_{8,226} \rightarrow \frac{19}{10\,321\,920}, C_{8,227} \rightarrow \frac{22\,289}{1\,393\,459\,200} + \frac{3 C_{8,6}}{4}, C_{8,228} \rightarrow \frac{679}{66\,355\,200}, \\
 C_{8,229} &\rightarrow -\frac{5839}{1\,393\,459\,200}, C_{8,230} \rightarrow -\frac{22\,609}{1\,393\,459\,200} - C_{8,6}, C_{8,231} \rightarrow \frac{9229}{464\,486\,400} + \frac{5 C_{8,6}}{2},
 \end{aligned}$$

$$\begin{aligned}
 C_{8,232} &\rightarrow -\frac{49}{66\,355\,200}, C_{8,233} \rightarrow \frac{2243}{199\,065\,600} - \frac{3 C_{8,6}}{4}, C_{8,234} \rightarrow \frac{30\,007}{1\,393\,459\,200} + C_{8,6}, C_{8,235} \rightarrow \frac{10\,469}{464\,486\,400}, \\
 C_{8,236} &\rightarrow \frac{557}{22\,118\,400}, C_{8,237} \rightarrow -\frac{2927}{464\,486\,400} - \frac{5 C_{8,6}}{2}, C_{8,238} \rightarrow \frac{241}{92\,897\,280}, C_{8,239} \rightarrow \frac{10\,301}{464\,486\,400}, \\
 C_{8,240} &\rightarrow \frac{127}{4\,423\,680}, C_{8,241} \rightarrow -\frac{2893}{464\,486\,400}, C_{8,242} \rightarrow -\frac{2993}{464\,486\,400}, C_{8,243} \rightarrow -\frac{16\,043}{1\,393\,459\,200} - C_{8,6}, \\
 C_{8,244} &\rightarrow -\frac{127}{22\,118\,400}, C_{8,245} \rightarrow -\frac{10\,963}{1\,393\,459\,200} + C_{8,6}, C_{8,246} \rightarrow -\frac{6451}{464\,486\,400}, C_{8,247} \rightarrow -\frac{8027}{464\,486\,400}, \\
 C_{8,248} &\rightarrow -\frac{127}{4\,423\,680}, C_{8,249} \rightarrow \frac{2399}{464\,486\,400}, C_{8,250} \rightarrow \frac{2357}{464\,486\,400}, C_{8,251} \rightarrow \frac{643}{66\,355\,200}, \\
 C_{8,252} &\rightarrow \frac{127}{7\,372\,800}, C_{8,253} \rightarrow -\frac{127}{51\,609\,600}, C_{8,254} \rightarrow -\frac{127}{22\,118\,400}, C_{8,255} \rightarrow \frac{127}{154\,828\,800}, C_{8,256} \rightarrow 0 \}
 \end{aligned}$$

In[*]:= sol /. Rule -> Set

Out[*]=

$$\left\{ 0, -\frac{1}{2\,419\,200}, \frac{1}{345\,600}, \frac{19}{9\,676\,800}, -\frac{1}{115\,200}, -\frac{19}{1\,612\,800} - C_{8,6}, -\frac{271}{58\,060\,800}, \frac{1}{69\,120}, \right. \\
 -\frac{457}{58\,060\,800} + \frac{5 C_{8,6}}{2}, \frac{1}{29\,030\,400}, \frac{19}{69\,672\,960} - \frac{3 C_{8,6}}{4}, \frac{179}{8\,294\,400} + \frac{5 C_{8,6}}{2}, \frac{223}{174\,182\,400}, \\
 \frac{1583}{116\,121\,600} + \frac{3 C_{8,6}}{4}, \frac{2893}{464\,486\,400}, -\frac{1}{69\,120}, -\frac{73}{9\,676\,800}, \frac{1571}{29\,030\,400} + 10 C_{8,6}, \frac{1619}{116\,121\,600} + \frac{15 C_{8,6}}{4}, \\
 -\frac{71}{829\,440} - 10 C_{8,6}, -\frac{79}{1\,935\,360} - \frac{9 C_{8,6}}{2}, -\frac{3617}{174\,182\,400}, -\frac{83}{7\,962\,624} + \frac{3 C_{8,6}}{4}, -\frac{1}{4\,147\,200}, \frac{109}{58\,060\,800}, \\
 \frac{1391}{43\,545\,600} + \frac{9 C_{8,6}}{2}, -\frac{35\,237}{1\,393\,459\,200} - \frac{15 C_{8,6}}{4}, -\frac{109}{3\,317\,760} - \frac{15 C_{8,6}}{4}, \frac{32\,899}{1\,393\,459\,200} + \frac{15 C_{8,6}}{4}, \\
 -\frac{5951}{464\,486\,400} - \frac{3 C_{8,6}}{4}, -\frac{2399}{464\,486\,400}, \frac{1}{115\,200}, \frac{1093}{58\,060\,800} + \frac{5 C_{8,6}}{2}, -\frac{1529}{29\,030\,400} - 10 C_{8,6}, \\
 -\frac{4813}{348\,364\,800} - \frac{15 C_{8,6}}{4}, -\frac{1}{460\,800}, \frac{1517}{38\,707\,200} + \frac{27 C_{8,6}}{4}, \frac{919}{23\,224\,320} - \frac{27 C_{8,6}}{4}, \frac{2311}{1\,393\,459\,200}, \\
 \frac{361}{4\,147\,200} + 10 C_{8,6}, \frac{113}{11\,612\,160}, \frac{1429}{58\,060\,800}, \frac{25\,189}{464\,486\,400} + \frac{9 C_{8,6}}{2}, \frac{323}{5\,529\,600} + \frac{27 C_{8,6}}{4}, \\
 -\frac{22\,231}{464\,486\,400} - \frac{27 C_{8,6}}{4}, \frac{4297}{199\,065\,600}, \frac{10\,963}{1\,393\,459\,200} - C_{8,6}, -\frac{179}{8\,294\,400} - \frac{5 C_{8,6}}{2}, -\frac{467}{87\,091\,200}, \\
 \frac{1}{1\,382\,400}, -\frac{419}{464\,486\,400}, -\frac{337}{5\,529\,600} - \frac{27 C_{8,6}}{4}, -\frac{31}{2\,457\,600}, \frac{3349}{92\,897\,280} + \frac{27 C_{8,6}}{4}, \\
 \frac{83}{17\,203\,200} + \frac{5 C_{8,6}}{2}, \frac{559}{16\,588\,800} + \frac{15 C_{8,6}}{4}, \frac{1027}{278\,691\,840}, -\frac{24\,697}{1\,393\,459\,200} - \frac{9 C_{8,6}}{2}, \frac{19\,619}{1\,393\,459\,200}, \\
 -\frac{49}{66\,355\,200}, -\frac{3697}{154\,828\,800} - \frac{5 C_{8,6}}{2}, \frac{1951}{154\,828\,800} + C_{8,6}, \frac{127}{51\,609\,600}, -\frac{1}{345\,600}, -\frac{31}{4\,838\,400} - C_{8,6}, \\
 -\frac{163}{29\,030\,400}, \frac{29}{16\,588\,800} + \frac{3 C_{8,6}}{4}, \frac{53}{829\,440} + 10 C_{8,6}, -\frac{391}{174\,182\,400}, \frac{1993}{87\,091\,200} + \frac{9 C_{8,6}}{2}, \\
 \frac{13\,781}{1\,393\,459\,200} - \frac{3 C_{8,6}}{4}, -\frac{259}{4\,147\,200} - 10 C_{8,6}, -\frac{4559}{116\,121\,600} - \frac{27 C_{8,6}}{4}, \frac{19}{2\,764\,800}, \frac{4679}{1\,393\,459\,200}, \\
 \frac{1}{552\,960}, \frac{133}{2\,457\,600} + \frac{27 C_{8,6}}{4}, -\frac{43\,369}{1\,393\,459\,200} - \frac{9 C_{8,6}}{2}, \frac{12\,323}{1\,393\,459\,200} + C_{8,6}, -\frac{17}{1\,382\,400},
 \left. \right\}$$

$$\begin{array}{r}
 \frac{661}{24883200} + \frac{9 c_{8,6}}{2}, - \frac{599}{29030400}, - \frac{12637}{278691840} - \frac{9 c_{8,6}}{2}, - \frac{1}{92160}, - \frac{2339}{464486400}, - \frac{1807}{92897280}, \\
 \frac{5687}{464486400}, - \frac{101}{2764800} - \frac{9 c_{8,6}}{2}, \frac{677}{464486400}, - \frac{1609}{154828800}, - \frac{5257}{66355200} - 10 c_{8,6}, \\
 \frac{541}{22118400} + \frac{9 c_{8,6}}{2}, \frac{33893}{464486400} + 10 c_{8,6}, - \frac{6931}{464486400}, - \frac{3881}{464486400}, \frac{23}{2073600} + c_{8,6}, \\
 \frac{53}{29030400}, - \frac{4031}{174182400} - \frac{9 c_{8,6}}{2}, \frac{8863}{278691840} + \frac{15 c_{8,6}}{4}, \frac{49}{1105920} + \frac{27 c_{8,6}}{4}, \\
 \frac{69901}{1393459200} - \frac{27 c_{8,6}}{4}, \frac{59}{30965760}, - \frac{1097}{66355200} - \frac{5 c_{8,6}}{2}, \frac{61}{4147200}, \frac{77299}{1393459200} + \frac{27 c_{8,6}}{4}, \\
 \frac{365}{18579456}, - \frac{869}{10321920} + 10 c_{8,6}, - \frac{101}{2654208} - \frac{27 c_{8,6}}{4}, \frac{2213}{464486400}, - \frac{4951}{92897280} - 10 c_{8,6}, \\
 \frac{4927}{464486400}, - \frac{7}{614400} - \frac{3 c_{8,6}}{4}, \frac{13549}{464486400} - \frac{15 c_{8,6}}{4}, \frac{32341}{1393459200} + \frac{9 c_{8,6}}{2}, - \frac{8417}{1393459200}, \\
 \frac{671}{66355200}, - \frac{35459}{464486400} - 10 c_{8,6}, \frac{5137}{92897280} + 10 c_{8,6}, - \frac{683}{92897280}, \frac{557}{66355200} + \frac{3 c_{8,6}}{4}, \\
 \frac{359}{17203200} + \frac{5 c_{8,6}}{2}, \frac{307}{66355200}, - \frac{319}{464486400}, - \frac{187}{22118400} - c_{8,6}, \frac{13}{4423680}, - \frac{107}{51609600}, \\
 \frac{127}{154828800}, - \frac{1}{2419200} - \frac{1}{1075200}, \frac{29}{2419200} + c_{8,6}, \frac{43}{11612160}, - \frac{1577}{58060800} - \frac{5 c_{8,6}}{2}, \\
 \frac{5413}{348364800} - \frac{3 c_{8,6}}{4}, - \frac{823}{174182400}, - \frac{323}{51609600}, \frac{433}{29030400}, \frac{899}{23224320} + \frac{15 c_{8,6}}{4}, \\
 \frac{76800}{1393459200} - \frac{2}{154828800} + \frac{4}{58060800}, \frac{174182400}{1393459200} - \frac{4}{69672960} - \frac{15 c_{8,6}}{4}, \\
 \frac{51803}{1393459200} + \frac{15 c_{8,6}}{4}, \frac{1411}{154828800}, \frac{257}{58060800} + \frac{5 c_{8,6}}{2}, - \frac{2587}{69672960} - \frac{15 c_{8,6}}{4}, \\
 \frac{4009}{116121600} + \frac{27 c_{8,6}}{4}, - \frac{16801}{1393459200}, - \frac{107}{58060800}, \frac{1163}{66355200} + \frac{9 c_{8,6}}{2}, - \frac{32303}{464486400} - \frac{27 c_{8,6}}{4}, \\
 \frac{40237}{1393459200} - c_{8,6}, \frac{89}{87091200}, \frac{59}{30965760}, \frac{631}{154828800}, \frac{10457}{464486400} + \frac{5 c_{8,6}}{2}, - \frac{2081}{278691840}, \\
 \frac{313}{55738368}, - \frac{11243}{464486400} - \frac{5 c_{8,6}}{2}, \frac{2977}{464486400}, \frac{1}{1451520} - c_{8,6}, \frac{181}{23224320} + \frac{3 c_{8,6}}{4}, \\
 \frac{1439}{87091200}, \frac{11219}{1393459200} - \frac{3 c_{8,6}}{4}, \frac{5689}{116121600} - \frac{27 c_{8,6}}{4}, \frac{13361}{1393459200}, \frac{2833}{51609600} + \frac{27 c_{8,6}}{4}, \\
 \frac{13843}{1393459200} + c_{8,6}, \frac{8459}{174182400} + \frac{9 c_{8,6}}{2}, - \frac{25273}{1393459200} - \frac{9 c_{8,6}}{2}, \frac{11813}{464486400}, \frac{12449}{464486400}, \\
 \frac{5417}{464486400}, - \frac{32507}{464486400} - 10 c_{8,6}, \frac{14447}{154828800} + 10 c_{8,6}, \frac{9551}{464486400}, - \frac{613}{174182400}, \\
 \frac{25913}{464486400} + \frac{15 c_{8,6}}{4}, - \frac{59807}{154828800} - \frac{27 c_{8,6}}{4}, \frac{1093}{464486400} - \frac{5 c_{8,6}}{2}, \frac{43649}{174182400} + \frac{27 c_{8,6}}{4}, \\
 \frac{23741}{1393459200} + \frac{4}{1393459200}, - \frac{5003}{1393459200} - \frac{4}{66355200} - \frac{2}{1393459200} + \frac{15 c_{8,6}}{4}, \frac{197}{464486400} + 10 c_{8,6}, - \frac{5003}{464486400}, - \frac{9463}{464486400}, - \frac{24391}{1393459200} - \frac{15 c_{8,6}}{4}, \frac{197}{39813120}, \\
 \frac{389}{7372800} - 10 c_{8,6}, \frac{11521}{464486400}, \frac{7793}{464486400} + \frac{5 c_{8,6}}{2}, - \frac{6187}{464486400}, \frac{1471}{154828800},
 \end{array}$$

$$\begin{array}{r}
 \frac{127}{22\,118\,400} - \frac{19}{9\,676\,800} - \frac{1}{1\,612\,800} - \frac{757}{116\,121\,600} - \frac{3\,c_{8,6}}{4} - \frac{49}{66\,355\,200} - \frac{1139}{69\,672\,960} + \frac{15\,c_{8,6}}{4}, \\
 - \frac{2393}{1\,393\,459\,200} + \frac{3\,c_{8,6}}{4} - \frac{1961}{55\,738\,368} - \frac{15\,c_{8,6}}{4} - \frac{581}{66\,355\,200} - \frac{1717}{116\,121\,600} - \frac{15\,c_{8,6}}{4}, \\
 \frac{1619}{154\,828\,800} - \frac{973}{22\,118\,400} + \frac{9\,c_{8,6}}{2} - \frac{491}{66\,355\,200} - \frac{221}{92\,897\,280} - \frac{1387}{66\,355\,200} + \frac{5\,c_{8,6}}{2}, \\
 \frac{6359}{1\,393\,459\,200} - \frac{127}{22\,118\,400} - \frac{587}{69\,672\,960} + \frac{3\,c_{8,6}}{4} - \frac{509}{39\,813\,120} - \frac{3\,c_{8,6}}{4} - \frac{9241}{464\,486\,400}, \\
 - \frac{157}{13\,271\,040} + \frac{21\,661}{464\,486\,400} - \frac{9\,c_{8,6}}{2} - \frac{3683}{464\,486\,400} - \frac{2917}{30\,965\,760} - \frac{17}{819\,200}, \\
 \frac{31\,709}{1\,393\,459\,200} + \frac{15\,c_{8,6}}{4} - \frac{7667}{464\,486\,400} - \frac{5\,c_{8,6}}{2} - \frac{29\,893}{464\,486\,400} + \frac{10\,c_{8,6}}{7\,372\,800} - \frac{49}{278\,691\,840}, \\
 \frac{853}{464\,486\,400} - \frac{7939}{464\,486\,400} - \frac{127}{7\,372\,800} - \frac{271}{58\,060\,800} - \frac{19}{10\,321\,920} - \frac{22\,289}{1\,393\,459\,200} + \frac{3\,c_{8,6}}{4}, \\
 \frac{679}{66\,355\,200} - \frac{5839}{1\,393\,459\,200} - \frac{22\,609}{1\,393\,459\,200} - \frac{9229}{464\,486\,400} + \frac{5\,c_{8,6}}{2} - \frac{49}{66\,355\,200}, \\
 \frac{2243}{199\,065\,600} - \frac{3\,c_{8,6}}{4} - \frac{30\,007}{1\,393\,459\,200} + \frac{10\,469}{464\,486\,400} - \frac{557}{22\,118\,400} - \frac{2927}{464\,486\,400} - \frac{5\,c_{8,6}}{2}, \\
 \frac{241}{92\,897\,280} - \frac{10\,301}{464\,486\,400} - \frac{127}{4\,423\,680} - \frac{2893}{464\,486\,400} - \frac{2993}{464\,486\,400} - \frac{16\,043}{1\,393\,459\,200} - \frac{c_{8,6}}{2}, \\
 - \frac{127}{22\,118\,400} - \frac{10\,963}{1\,393\,459\,200} + \frac{6451}{464\,486\,400} - \frac{8027}{464\,486\,400} - \frac{127}{4\,423\,680} - \frac{2399}{464\,486\,400}, \\
 \frac{2357}{464\,486\,400} - \frac{643}{66\,355\,200} - \frac{127}{7\,372\,800} - \frac{127}{51\,609\,600} - \frac{127}{22\,118\,400} - \frac{127}{154\,828\,800}, \left. \vphantom{\frac{2357}{464\,486\,400}} \right\} 0
 \end{array}$$

In[*]:= $c_{8,6} = -\frac{13}{3\,628\,800}$

Out[*]= $-\frac{13}{3\,628\,800}$

In[*]:= \mathfrak{d}

Out[*]=
$$\begin{aligned}
 & \mathcal{O}_{AR, \{x, y\}, \{1\}} \left[\mathcal{F}_0 \left[\right. \right. \\
 & \quad \frac{1}{24} AW_1[x, y] - \frac{1}{24} AW_1[y, x] - \frac{AW_1[x, x, x, y]}{1440} + \frac{1}{480} AW_1[x, x, y, x] + \frac{7 AW_1[x, x, y, y]}{5760} - \\
 & \quad \frac{1}{480} AW_1[x, y, x, x] - \frac{1}{640} AW_1[x, y, x, y] - \frac{AW_1[x, y, y, x]}{1152} - \frac{7 AW_1[x, y, y, y]}{5760} + \\
 & \quad \frac{AW_1[y, x, x, x]}{1440} - \frac{AW_1[y, x, x, y]}{1152} + \frac{19 AW_1[y, x, y, x]}{5760} + \frac{7 AW_1[y, x, y, y]}{1920} - \frac{7 AW_1[y, y, x, x]}{5760} - \\
 & \quad \frac{7 AW_1[y, y, x, y]}{1920} + \frac{7 AW_1[y, y, y, x]}{5760} + \frac{AW_1[x, x, x, x, x, y]}{60480} - \frac{AW_1[x, x, x, x, y, x]}{12096} - \\
 & \quad \frac{13 AW_1[x, x, x, x, y, y]}{241920} + \frac{AW_1[x, x, x, y, x, x]}{6048} + \frac{19 AW_1[x, x, x, y, x, y]}{145152} + \left. \left. \right. \right]
 \end{aligned}$$

$$\begin{aligned}
 & \frac{61 \text{ AW}_1[x, x, x, y, y, x]}{725\,760} + \frac{83 \text{ AW}_1[x, x, x, y, y, y]}{967\,680} - \frac{\text{AW}_1[x, x, y, x, x, x]}{6048} - \\
 & \frac{17 \text{ AW}_1[x, x, y, x, x, y]}{241\,920} - \frac{61 \text{ AW}_1[x, x, y, x, y, x]}{241\,920} - \frac{89 \text{ AW}_1[x, x, y, x, y, y]}{414\,720} + \\
 & \frac{71 \text{ AW}_1[x, x, y, y, x, y]}{967\,680} - \frac{337 \text{ AW}_1[x, x, y, y, y, x]}{2\,903\,040} - \frac{31 \text{ AW}_1[x, x, y, y, y, y]}{483\,840} + \\
 & \frac{\text{AW}_1[x, y, x, x, x, x]}{12\,096} + \frac{13 \text{ AW}_1[x, y, x, x, x, y]}{725\,760} + \frac{\text{AW}_1[x, y, x, x, y, x]}{11\,520} + \\
 & \frac{37 \text{ AW}_1[x, y, x, x, y, y]}{580\,608} + \frac{\text{AW}_1[x, y, x, y, x, x]}{6048} + \frac{79 \text{ AW}_1[x, y, x, y, x, y]}{967\,680} + \\
 & \frac{71 \text{ AW}_1[x, y, x, y, y, x]}{322\,560} + \frac{73 \text{ AW}_1[x, y, x, y, y, y]}{483\,840} - \frac{\text{AW}_1[x, y, y, x, x, x]}{18\,144} - \\
 & \frac{53 \text{ AW}_1[x, y, y, x, x, y]}{967\,680} - \frac{23 \text{ AW}_1[x, y, y, x, y, x]}{193\,536} - \frac{11 \text{ AW}_1[x, y, y, x, y, y]}{161\,280} + \\
 & \frac{19 \text{ AW}_1[x, y, y, y, x, x]}{290\,304} - \frac{\text{AW}_1[x, y, y, y, x, y]}{193\,536} + \frac{7 \text{ AW}_1[x, y, y, y, y, x]}{138\,240} + \\
 & \frac{31 \text{ AW}_1[x, y, y, y, y, y]}{967\,680} - \frac{\text{AW}_1[y, x, x, x, x, x]}{60\,480} + \frac{\text{AW}_1[y, x, x, x, x, y]}{34\,560} - \\
 & \frac{97 \text{ AW}_1[y, x, x, x, y, x]}{725\,760} - \frac{103 \text{ AW}_1[y, x, x, x, y, y]}{967\,680} + \frac{19 \text{ AW}_1[y, x, x, y, x, x]}{120\,960} + \\
 & \frac{583 \text{ AW}_1[y, x, x, y, x, y]}{2\,903\,040} + \frac{53 \text{ AW}_1[y, x, x, y, y, x]}{967\,680} + \frac{17 \text{ AW}_1[y, x, x, y, y, y]}{161\,280} - \\
 & \frac{29 \text{ AW}_1[y, x, y, x, x, x]}{181\,440} - \frac{289 \text{ AW}_1[y, x, y, x, x, y]}{2\,903\,040} - \frac{55 \text{ AW}_1[y, x, y, x, y, x]}{193\,536} - \\
 & \frac{17 \text{ AW}_1[y, x, y, x, y, y]}{53\,760} - \frac{11 \text{ AW}_1[y, x, y, y, x, x]}{483\,840} + \frac{7 \text{ AW}_1[y, x, y, y, x, y]}{46\,080} - \\
 & \frac{191 \text{ AW}_1[y, x, y, y, y, x]}{967\,680} - \frac{31 \text{ AW}_1[y, x, y, y, y, y]}{193\,536} + \frac{13 \text{ AW}_1[y, y, x, x, x, x]}{241\,920} + \\
 & \frac{\text{AW}_1[y, y, x, x, x, y]}{17\,920} - \frac{19 \text{ AW}_1[y, y, x, x, y, x]}{1\,451\,520} + \frac{89 \text{ AW}_1[y, y, x, y, x, x]}{414\,720} + \\
 & \frac{53 \text{ AW}_1[y, y, x, y, x, y]}{322\,560} + \frac{71 \text{ AW}_1[y, y, x, y, y, x]}{322\,560} + \frac{31 \text{ AW}_1[y, y, x, y, y, y]}{96\,768} - \\
 & \frac{83 \text{ AW}_1[y, y, y, x, x, x]}{967\,680} - \frac{53 \text{ AW}_1[y, y, y, x, x, y]}{967\,680} - \frac{13 \text{ AW}_1[y, y, y, x, y, x]}{64\,512} - \\
 & \frac{31 \text{ AW}_1[y, y, y, x, y, y]}{96\,768} + \frac{31 \text{ AW}_1[y, y, y, y, x, x]}{483\,840} + \frac{31 \text{ AW}_1[y, y, y, y, x, y]}{193\,536} - \\
 & \frac{31 \text{ AW}_1[y, y, y, y, y, x]}{967\,680} - \frac{\text{AW}_1[x, x, x, x, x, x, y]}{2\,419\,200} + \frac{\text{AW}_1[x, x, x, x, x, x, y, x]}{345\,600} + \\
 & \frac{19 \text{ AW}_1[x, x, x, x, x, x, y, y]}{9\,676\,800} - \frac{\text{AW}_1[x, x, x, x, x, y, x, x]}{115\,200} - \frac{13 \text{ AW}_1[x, x, x, x, x, y, x, y]}{3\,628\,800} - \\
 & \frac{17 \text{ AW}_1[x, x, x, x, x, y, y, x]}{2\,073\,600} - \frac{271 \text{ AW}_1[x, x, x, x, x, y, y, y]}{58\,060\,800} + \\
 & \frac{\text{AW}_1[x, x, x, x, y, x, x, x]}{69\,120} + \frac{\text{AW}_1[x, x, x, x, y, x, x, y]}{921\,600} + \frac{457 \text{ AW}_1[x, x, x, x, y, x, y, x]}{29\,030\,400} +
 \end{aligned}$$

$$\begin{aligned}
 & \frac{553 \text{ AW}_1[x, x, x, x, y, x, y, y]}{49766400} + \frac{733 \text{ AW}_1[x, x, x, x, y, y, x, x]}{58060800} + \\
 & \frac{223 \text{ AW}_1[x, x, x, x, y, y, x, y]}{174182400} + \frac{1271 \text{ AW}_1[x, x, x, x, y, y, y, x]}{116121600} + \\
 & \frac{2893 \text{ AW}_1[x, x, x, x, y, y, y, y]}{464486400} - \frac{\text{AW}_1[x, x, x, y, x, x, x, x]}{69120} - \frac{73 \text{ AW}_1[x, x, x, y, x, x, x, y]}{9676800} + \\
 & \frac{59 \text{ AW}_1[x, x, x, y, x, x, y, x]}{3225600} + \frac{59 \text{ AW}_1[x, x, x, y, x, x, y, y]}{116121600} - \\
 & \frac{289 \text{ AW}_1[x, x, x, y, x, y, x, x]}{5806080} - \frac{239 \text{ AW}_1[x, x, x, y, x, y, x, y]}{9676800} - \\
 & \frac{3617 \text{ AW}_1[x, x, x, y, x, y, y, x]}{174182400} - \frac{18269 \text{ AW}_1[x, x, x, y, x, y, y, y]}{1393459200} - \\
 & \frac{\text{AW}_1[x, x, x, y, y, x, x, x]}{4147200} + \frac{109 \text{ AW}_1[x, x, x, y, y, x, x, y]}{58060800} + \frac{689 \text{ AW}_1[x, x, x, y, y, x, y, x]}{43545600} - \\
 & \frac{16517 \text{ AW}_1[x, x, x, y, y, x, y, y]}{1393459200} - \frac{451 \text{ AW}_1[x, x, x, y, y, y, x, x]}{23224320} + \\
 & \frac{14179 \text{ AW}_1[x, x, x, y, y, y, x, y]}{1393459200} - \frac{4703 \text{ AW}_1[x, x, x, y, y, y, y, x]}{464486400} - \\
 & \frac{2399 \text{ AW}_1[x, x, x, y, y, y, y, y]}{464486400} + \frac{\text{AW}_1[x, x, y, x, x, x, x, x]}{115200} + \\
 & \frac{191 \text{ AW}_1[x, x, y, x, x, x, x, y]}{19353600} - \frac{163 \text{ AW}_1[x, x, y, x, x, x, y, x]}{9676800} - \\
 & \frac{19 \text{ AW}_1[x, x, y, x, x, x, y, y]}{49766400} - \frac{\text{AW}_1[x, x, y, x, x, y, x, x]}{460800} + \frac{83 \text{ AW}_1[x, x, y, x, x, y, x, y]}{5529600} - \\
 & \frac{1787 \text{ AW}_1[x, x, y, x, x, y, y, x]}{116121600} + \frac{2311 \text{ AW}_1[x, x, y, x, x, y, y, y]}{1393459200} + \\
 & \frac{1487 \text{ AW}_1[x, x, y, x, y, x, x, x]}{29030400} + \frac{113 \text{ AW}_1[x, x, y, x, y, x, x, y]}{11612160} + \\
 & \frac{1429 \text{ AW}_1[x, x, y, x, y, x, y, x]}{58060800} + \frac{17701 \text{ AW}_1[x, x, y, x, y, x, y, y]}{464486400} + \\
 & \frac{53 \text{ AW}_1[x, x, y, x, y, y, x, x]}{1548288} - \frac{10999 \text{ AW}_1[x, x, y, x, y, y, x, y]}{464486400} + \\
 & \frac{4297 \text{ AW}_1[x, x, y, x, y, y, y, x]}{199065600} + \frac{3191 \text{ AW}_1[x, x, y, x, y, y, y, y]}{278691840} - \\
 & \frac{733 \text{ AW}_1[x, x, y, y, x, x, x, x]}{58060800} - \frac{467 \text{ AW}_1[x, x, y, y, x, x, x, y]}{87091200} + \frac{\text{AW}_1[x, x, y, y, x, x, y, x]}{1382400} - \\
 & \frac{419 \text{ AW}_1[x, x, y, y, x, x, y, y]}{464486400} - \frac{1423 \text{ AW}_1[x, x, y, y, x, y, x, x]}{38707200} - \\
 & \frac{31 \text{ AW}_1[x, x, y, y, x, y, x, y]}{2457600} + \frac{5513 \text{ AW}_1[x, x, y, y, x, y, y, x]}{464486400} - \\
 & \frac{1919 \text{ AW}_1[x, x, y, y, x, y, y, y]}{464486400} + \frac{2353 \text{ AW}_1[x, x, y, y, y, x, x, x]}{116121600} + \\
 & \frac{1027 \text{ AW}_1[x, x, y, y, y, x, x, y]}{278691840} - \frac{319 \text{ AW}_1[x, x, y, y, y, x, y, x]}{199065600} +
 \end{aligned}$$

$$\begin{aligned}
 & \frac{19\,619\,AW_1[x, x, y, y, y, x, y, y]}{1\,393\,459\,200} - \frac{49\,AW_1[x, x, y, y, y, y, x, x]}{66\,355\,200} - \\
 & \frac{6931\,AW_1[x, x, y, y, y, y, x, y]}{464\,486\,400} + \frac{4189\,AW_1[x, x, y, y, y, y, y, x]}{464\,486\,400} + \\
 & \frac{127\,AW_1[x, x, y, y, y, y, y, y]}{51\,609\,600} - \frac{AW_1[x, y, x, x, x, x, x, x]}{345\,600} - \frac{41\,AW_1[x, y, x, x, x, x, x, y]}{14\,515\,200} - \\
 & \frac{163\,AW_1[x, y, x, x, x, x, y, x]}{29\,030\,400} - \frac{109\,AW_1[x, y, x, x, x, x, y, y]}{116\,121\,600} + \\
 & \frac{163\,AW_1[x, y, x, x, x, y, x, x]}{5\,806\,080} - \frac{391\,AW_1[x, y, x, x, x, y, x, y]}{174\,182\,400} + \\
 & \frac{589\,AW_1[x, y, x, x, x, y, y, x]}{87\,091\,200} - \frac{10\,037\,AW_1[x, y, x, x, x, y, y, y]}{1\,393\,459\,200} - \\
 & \frac{773\,AW_1[x, y, x, x, y, x, x, x]}{29\,030\,400} - \frac{1751\,AW_1[x, y, x, x, y, x, x, y]}{116\,121\,600} + \\
 & \frac{19\,AW_1[x, y, x, x, y, x, y, x]}{2\,764\,800} + \frac{4679\,AW_1[x, y, x, x, y, x, y, y]}{1\,393\,459\,200} + \frac{AW_1[x, y, x, x, y, y, x, x]}{552\,960} + \\
 & \frac{103\,AW_1[x, y, x, x, y, y, x, y]}{3\,440\,640} - \frac{4181\,AW_1[x, y, x, x, y, y, y, x]}{278\,691\,840} + \\
 & \frac{7331\,AW_1[x, y, x, x, y, y, y, y]}{17\,AW_1[x, y, x, y, x, x, x, x]} + \\
 & \frac{1819\,AW_1[x, y, x, y, x, x, x, y]}{1\,393\,459\,200} - \frac{599\,AW_1[x, y, x, y, x, x, y, x]}{1\,382\,400} - \\
 & \frac{174\,182\,400}{40\,721\,AW_1[x, y, x, y, x, x, y, y]} - \frac{29\,030\,400}{AW_1[x, y, x, y, x, y, x, x]} - \\
 & \frac{2339\,AW_1[x, y, x, y, x, y, x, y]}{1\,393\,459\,200} - \frac{1807\,AW_1[x, y, x, y, x, y, y, x]}{92\,160} - \\
 & \frac{5687\,AW_1[x, y, x, y, x, y, y, y]}{464\,486\,400} - \frac{79\,AW_1[x, y, x, y, y, x, x, x]}{92\,897\,280} + \\
 & \frac{677\,AW_1[x, y, x, y, y, x, x, y]}{464\,486\,400} - \frac{1609\,AW_1[x, y, x, y, y, x, y, x]}{3\,870\,720} - \\
 & \frac{20\,159\,AW_1[x, y, x, y, y, x, y, y]}{464\,486\,400} - \frac{1291\,AW_1[x, y, x, y, y, y, x, x]}{154\,828\,800} + \\
 & \frac{213\,AW_1[x, y, x, y, y, y, x, y]}{5\,734\,400} - \frac{6931\,AW_1[x, y, x, y, y, y, y, x]}{154\,828\,800} - \\
 & \frac{3881\,AW_1[x, y, x, y, y, y, y, y]}{5\,734\,400} + \frac{109\,AW_1[x, y, y, x, x, x, x, x]}{464\,486\,400} + \\
 & \frac{53\,AW_1[x, y, y, x, x, x, x, y]}{464\,486\,400} - \frac{1223\,AW_1[x, y, y, x, x, x, y, x]}{14\,515\,200} + \\
 & \frac{5119\,AW_1[x, y, y, x, x, x, y, y]}{29\,030\,400} - \frac{779\,AW_1[x, y, y, x, x, y, x, x]}{174\,182\,400} - \\
 & \frac{7241\,AW_1[x, y, y, x, x, y, x, y]}{278\,691\,840} + \frac{59\,AW_1[x, y, y, x, x, y, y, x]}{38\,707\,200} - \\
 & \frac{391\,AW_1[x, y, y, x, x, y, y, y]}{278\,691\,840} + \frac{61\,AW_1[x, y, y, x, y, x, x, x]}{30\,965\,760} + \\
 & \frac{51\,609\,600}{4\,147\,200}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{6229 \text{ AW}_1[x, y, y, x, y, x, x, y]}{199065600} + \frac{365 \text{ AW}_1[x, y, y, x, y, x, y, x]}{18579456} + \\
 & \frac{4493 \text{ AW}_1[x, y, y, x, y, x, y, y]}{92897280} - \frac{6443 \text{ AW}_1[x, y, y, x, y, y, x, x]}{464486400} + \\
 & \frac{2213 \text{ AW}_1[x, y, y, x, y, y, x, y]}{464486400} - \frac{541 \text{ AW}_1[x, y, y, x, y, y, y, x]}{30965760} + \\
 & \frac{4927 \text{ AW}_1[x, y, y, x, y, y, y, y]}{464486400} - \frac{337 \text{ AW}_1[x, y, y, y, x, x, x, x]}{38707200} - \\
 & \frac{7309 \text{ AW}_1[x, y, y, y, x, x, x, y]}{464486400} + \frac{1411 \text{ AW}_1[x, y, y, y, x, x, y, x]}{199065600} - \\
 & \frac{8417 \text{ AW}_1[x, y, y, y, x, x, y, y]}{1393459200} - \frac{671 \text{ AW}_1[x, y, y, y, x, y, x, x]}{66355200} - \\
 & \frac{697 \text{ AW}_1[x, y, y, y, x, y, x, y]}{17203200} + \frac{67 \text{ AW}_1[x, y, y, y, x, y, y, x]}{3440640} - \\
 & \frac{683 \text{ AW}_1[x, y, y, y, x, y, y, y]}{92897280} + \frac{2651 \text{ AW}_1[x, y, y, y, y, x, x, x]}{464486400} + \\
 & \frac{5533 \text{ AW}_1[x, y, y, y, y, x, x, y]}{464486400} + \frac{307 \text{ AW}_1[x, y, y, y, y, x, y, x]}{66355200} - \\
 & \frac{319 \text{ AW}_1[x, y, y, y, y, x, y, y]}{464486400} - \frac{2263 \text{ AW}_1[x, y, y, y, y, y, x, x]}{464486400} + \\
 & \frac{13 \text{ AW}_1[x, y, y, y, y, y, x, y]}{4423680} - \frac{107 \text{ AW}_1[x, y, y, y, y, y, y, x]}{51609600} - \\
 & \frac{127 \text{ AW}_1[x, y, y, y, y, y, y, y]}{154828800} + \frac{\text{AW}_1[y, x, x, x, x, x, x, x]}{2419200} - \frac{\text{AW}_1[y, x, x, x, x, x, x, y]}{1075200} + \\
 & \frac{61 \text{ AW}_1[y, x, x, x, x, x, y, x]}{7257600} + \frac{43 \text{ AW}_1[y, x, x, x, x, x, y, y]}{11612160} - \\
 & \frac{151 \text{ AW}_1[y, x, x, x, x, y, x, x]}{8294400} - \frac{4477 \text{ AW}_1[y, x, x, x, x, y, x, y]}{348364800} - \\
 & \frac{823 \text{ AW}_1[y, x, x, x, x, y, y, x]}{174182400} - \frac{323 \text{ AW}_1[y, x, x, x, x, y, y, y]}{51609600} + \\
 & \frac{433 \text{ AW}_1[y, x, x, x, y, x, x, x]}{29030400} + \frac{587 \text{ AW}_1[y, x, x, x, y, x, x, y]}{23224320} + \frac{\text{AW}_1[y, x, x, x, y, x, y, x]}{322560} + \\
 & \frac{30059 \text{ AW}_1[y, x, x, x, y, x, y, y]}{1393459200} + \frac{787 \text{ AW}_1[y, x, x, x, y, y, x, x]}{174182400} - \\
 & \frac{2603 \text{ AW}_1[y, x, x, x, y, y, x, y]}{199065600} + \frac{33083 \text{ AW}_1[y, x, x, x, y, y, y, x]}{1393459200} + \\
 & \frac{1411 \text{ AW}_1[y, x, x, x, y, y, y, y]}{154828800} - \frac{263 \text{ AW}_1[y, x, x, y, x, x, x, x]}{58060800} - \\
 & \frac{1651 \text{ AW}_1[y, x, x, y, x, x, x, y]}{69672960} + \frac{1201 \text{ AW}_1[y, x, x, y, x, x, y, x]}{116121600} - \\
 & \frac{16801 \text{ AW}_1[y, x, x, y, x, x, y, y]}{1393459200} - \frac{107 \text{ AW}_1[y, x, x, y, x, y, x, x]}{5806080} + \\
 & \frac{653 \text{ AW}_1[y, x, x, y, x, y, x, y]}{464486400} - \frac{21071 \text{ AW}_1[y, x, x, y, x, y, y, x]}{464486400} -
 \end{aligned}$$

$$\begin{array}{r}
 \frac{1007 \text{ AW}_1 [y, x, x, y, x, y, y, y]}{39\,813\,120} + \frac{89 \text{ AW}_1 [y, x, x, y, y, x, x, x]}{87\,091\,200} + \\
 \frac{59 \text{ AW}_1 [y, x, x, y, y, x, x, y]}{30\,965\,760} + \frac{631 \text{ AW}_1 [y, x, x, y, y, x, y, x]}{154\,828\,800} + \\
 \frac{2099 \text{ AW}_1 [y, x, x, y, y, x, y, y]}{154\,828\,800} - \frac{2081 \text{ AW}_1 [y, x, x, y, y, y, x, x]}{278\,691\,840} - \\
 \frac{313 \text{ AW}_1 [y, x, x, y, y, y, x, y]}{55\,738\,368} - \frac{787 \text{ AW}_1 [y, x, x, y, y, y, y, x]}{51\,609\,600} - \\
 \frac{2977 \text{ AW}_1 [y, x, x, y, y, y, y, y]}{464\,486\,400} + \frac{31 \text{ AW}_1 [y, x, y, x, x, x, x, x]}{7\,257\,600} + \\
 \frac{593 \text{ AW}_1 [y, x, y, x, x, x, x, y]}{116\,121\,600} + \frac{1439 \text{ AW}_1 [y, x, y, x, x, x, y, x]}{87\,091\,200} + \\
 \frac{14\,963 \text{ AW}_1 [y, x, y, x, x, x, y, y]}{1\,393\,459\,200} - \frac{2881 \text{ AW}_1 [y, x, y, x, x, y, x, x]}{116\,121\,600} - \\
 \frac{13\,361 \text{ AW}_1 [y, x, y, x, x, y, x, y]}{1\,393\,459\,200} + \frac{317 \text{ AW}_1 [y, x, y, x, x, y, y, x]}{10\,321\,920} + \\
 \frac{8851 \text{ AW}_1 [y, x, y, x, x, y, y, y]}{1\,393\,459\,200} + \frac{5651 \text{ AW}_1 [y, x, y, x, y, x, x, x]}{174\,182\,400} - \\
 \frac{2809 \text{ AW}_1 [y, x, y, x, y, x, x, y]}{1\,393\,459\,200} + \frac{11\,813 \text{ AW}_1 [y, x, y, x, y, x, y, x]}{464\,486\,400} + \\
 \frac{12\,449 \text{ AW}_1 [y, x, y, x, y, x, y, y]}{464\,486\,400} + \frac{5417 \text{ AW}_1 [y, x, y, x, y, y, x, x]}{464\,486\,400} - \\
 \frac{1763 \text{ AW}_1 [y, x, y, x, y, y, x, y]}{51\,609\,600} + \frac{26\,701 \text{ AW}_1 [y, x, y, x, y, y, y, x]}{464\,486\,400} + \\
 \frac{9551 \text{ AW}_1 [y, x, y, x, y, y, y, y]}{464\,486\,400} - \frac{613 \text{ AW}_1 [y, x, y, y, x, x, x, x]}{174\,182\,400} + \\
 \frac{7193 \text{ AW}_1 [y, x, y, y, x, x, x, y]}{1\,393\,459\,200} - \frac{26\,111 \text{ AW}_1 [y, x, y, y, x, x, y, x]}{1\,393\,459\,200} - \\
 \frac{3491 \text{ AW}_1 [y, x, y, y, x, x, y, y]}{464\,486\,400} + \frac{9953 \text{ AW}_1 [y, x, y, y, x, y, x, x]}{1\,393\,459\,200} + \\
 \frac{263 \text{ AW}_1 [y, x, y, y, x, y, x, y]}{17\,203\,200} - \frac{5003 \text{ AW}_1 [y, x, y, y, x, y, y, x]}{464\,486\,400} - \\
 \frac{9463 \text{ AW}_1 [y, x, y, y, x, y, y, y]}{464\,486\,400} - \frac{5671 \text{ AW}_1 [y, x, y, y, y, x, x, x]}{1\,393\,459\,200} + \\
 \frac{197 \text{ AW}_1 [y, x, y, y, y, x, x, y]}{39\,813\,120} - \frac{7867 \text{ AW}_1 [y, x, y, y, y, x, y, x]}{464\,486\,400} + \\
 \frac{11\,521 \text{ AW}_1 [y, x, y, y, y, x, y, y]}{464\,486\,400} + \frac{173 \text{ AW}_1 [y, x, y, y, y, y, x, x]}{22\,118\,400} - \\
 \frac{6187 \text{ AW}_1 [y, x, y, y, y, y, x, y]}{464\,486\,400} + \frac{1471 \text{ AW}_1 [y, x, y, y, y, y, y, x]}{154\,828\,800} + \\
 \frac{127 \text{ AW}_1 [y, x, y, y, y, y, y, y]}{22\,118\,400} - \frac{19 \text{ AW}_1 [y, y, x, x, x, x, x, x]}{9\,676\,800} - \frac{\text{AW}_1 [y, y, x, x, x, x, x, y]}{1\,612\,800} \\
 \frac{89 \text{ AW}_1 [y, y, x, x, x, x, y, x]}{23\,224\,320} - \frac{49 \text{ AW}_1 [y, y, x, x, x, x, y, y]}{66\,355\,200} + \frac{29 \text{ AW}_1 [y, y, x, x, x, y, x, x]}{9\,953\,280} -
 \end{array}$$

$$\begin{array}{r}
 \frac{6137 \text{ AW}_1 [y, y, x, x, x, y, x, y]}{1393459200} - \frac{6061 \text{ AW}_1 [y, y, x, x, x, y, y, x]}{278691840} - \\
 \frac{581 \text{ AW}_1 [y, y, x, x, x, y, y, y]}{66355200} - \frac{157 \text{ AW}_1 [y, y, x, x, y, x, x, x]}{116121600} + \\
 \frac{1619 \text{ AW}_1 [y, y, x, x, y, x, x, y]}{154828800} + \frac{863 \text{ AW}_1 [y, y, x, x, y, x, y, x]}{30965760} + \\
 \frac{5101 \text{ AW}_1 [y, y, x, x, y, x, y, y]}{464486400} + \frac{221 \text{ AW}_1 [y, y, x, x, y, y, x, x]}{92897280} + \\
 \frac{5549 \text{ AW}_1 [y, y, x, x, y, y, x, y]}{464486400} + \frac{6359 \text{ AW}_1 [y, y, x, x, y, y, y, x]}{1393459200} + \\
 \frac{127 \text{ AW}_1 [y, y, x, x, y, y, y, y]}{22118400} - \frac{553 \text{ AW}_1 [y, y, x, y, x, x, x, x]}{49766400} - \\
 \frac{14071 \text{ AW}_1 [y, y, x, y, x, x, x, y]}{1393459200} - \frac{9241 \text{ AW}_1 [y, y, x, y, x, x, y, x]}{464486400} - \\
 \frac{7159 \text{ AW}_1 [y, y, x, y, x, x, y, y]}{464486400} - \frac{14173 \text{ AW}_1 [y, y, x, y, x, y, x, x]}{464486400} - \\
 \frac{3683 \text{ AW}_1 [y, y, x, y, x, y, x, y]}{464486400} - \frac{5423 \text{ AW}_1 [y, y, x, y, x, y, y, x]}{92897280} - \\
 \frac{17 \text{ AW}_1 [y, y, x, y, x, y, y, y]}{819200} + \frac{12989 \text{ AW}_1 [y, y, x, y, y, x, x, x]}{1393459200} - \\
 \frac{167 \text{ AW}_1 [y, y, x, y, y, x, x, y]}{22118400} + \frac{13253 \text{ AW}_1 [y, y, x, y, y, x, y, x]}{464486400} - \\
 \frac{49 \text{ AW}_1 [y, y, x, y, y, x, y, y]}{7372800} - \frac{2689 \text{ AW}_1 [y, y, x, y, y, y, x, x]}{278691840} + \\
 \frac{853 \text{ AW}_1 [y, y, x, y, y, y, x, y]}{464486400} - \frac{7939 \text{ AW}_1 [y, y, x, y, y, y, y, x]}{464486400} - \\
 \frac{127 \text{ AW}_1 [y, y, x, y, y, y, y, y]}{7372800} + \frac{271 \text{ AW}_1 [y, y, y, x, x, x, x, x]}{58060800} + \\
 \frac{19 \text{ AW}_1 [y, y, y, x, x, x, x, y]}{10321920} + \frac{3709 \text{ AW}_1 [y, y, y, x, x, x, y, x]}{278691840} + \\
 \frac{679 \text{ AW}_1 [y, y, y, x, x, x, y, y]}{66355200} - \frac{5839 \text{ AW}_1 [y, y, y, x, x, y, x, x]}{1393459200} - \\
 \frac{17617 \text{ AW}_1 [y, y, y, x, x, y, x, y]}{1393459200} + \frac{5069 \text{ AW}_1 [y, y, y, x, x, y, y, x]}{464486400} - \\
 \frac{49 \text{ AW}_1 [y, y, y, x, x, y, y, y]}{66355200} + \frac{3889 \text{ AW}_1 [y, y, y, x, y, x, x, x]}{278691840} + \\
 \frac{5003 \text{ AW}_1 [y, y, y, x, y, x, x, y]}{278691840} + \frac{10469 \text{ AW}_1 [y, y, y, x, y, x, y, x]}{464486400} + \\
 \frac{557 \text{ AW}_1 [y, y, y, x, y, x, y, y]}{22118400} + \frac{137 \text{ AW}_1 [y, y, y, x, y, y, x, x]}{51609600} + \\
 \frac{241 \text{ AW}_1 [y, y, y, x, y, y, x, y]}{92897280} + \frac{10301 \text{ AW}_1 [y, y, y, x, y, y, y, x]}{464486400} + \\
 \frac{127 \text{ AW}_1 [y, y, y, x, y, y, y, y]}{4423680} - \frac{2893 \text{ AW}_1 [y, y, y, y, x, x, x, x]}{464486400} -
 \end{array}$$

$$\begin{array}{r}
 \frac{2993 \text{ AW}_1 [y, y, y, y, x, x, x, y]}{464486400} - \frac{11051 \text{ AW}_1 [y, y, y, y, x, x, y, x]}{1393459200} - \\
 \frac{127 \text{ AW}_1 [y, y, y, y, x, x, y, y]}{22118400} - \frac{3191 \text{ AW}_1 [y, y, y, y, x, y, x, x]}{278691840} - \\
 \frac{6451 \text{ AW}_1 [y, y, y, y, x, y, x, y]}{464486400} - \frac{8027 \text{ AW}_1 [y, y, y, y, x, y, y, x]}{464486400} - \\
 \frac{127 \text{ AW}_1 [y, y, y, y, x, y, y, y]}{4423680} + \frac{2399 \text{ AW}_1 [y, y, y, y, y, x, x, x]}{464486400} + \\
 \frac{2357 \text{ AW}_1 [y, y, y, y, y, x, x, y]}{464486400} + \frac{643 \text{ AW}_1 [y, y, y, y, y, x, y, x]}{66355200} + \\
 \frac{127 \text{ AW}_1 [y, y, y, y, y, x, y, y]}{7372800} - \frac{127 \text{ AW}_1 [y, y, y, y, y, y, x, x]}{51609600} - \\
 \frac{127 \text{ AW}_1 [y, y, y, y, y, y, x, y]}{22118400} + \frac{127 \text{ AW}_1 [y, y, y, y, y, y, y, x]}{154828800} \Big] \Big]
 \end{array}$$