In 240510_toward_KV1.pdf:

The emergent linearized ⬠:

1. $\varphi(x+y,0)-\varphi(y,0)=0$.
2. With $R=R(\varphi)$, $R(x+y,0)-R(y,0)=0$.
3. $(\partial_y\varphi)(x,y)+(\partial_y\varphi)(y,0)-((\partial_y\varphi)(x+y,0)=2R(x,y)$.

$R\colon FL(x,y)\to FA(x,y)$ satisfies $R(x)=R(y)=0$ and

$$R([u,v])=[u,R(v)]+[R(u),v]+\frac{1}{2}\begin{pmatrix}(\partial_x v)x(\partial_x u)^*-\partial_x u)x(\partial_x v)^*\\+(\partial_y v)x(\partial_y u)^*-\partial_y u)x(\partial_y v)^*\end{pmatrix}.$$

Lemma 1. $\forall m\geqslant 0$, $\partial_x(\mathrm{ad}_x^m(y))=-\sum_{i+j=m-1}x^i\,\mathrm{ad}_x^j(y)$.

Lemma 2. $\partial_x(\mathrm{ad}_x^m(y))=-\sum_{b+c=m-1}(-1)^c\binom{m}{b}x^byx^c$.

Lemma 3. $R(\mathrm{ad}_x^m(y))=\sum_{i+j=m-1}\frac{(-1)^j}{2}\left(\binom{m-1}{j-1}-\binom{m-1}{i-1}\right)x^iyx^j$.

Lemma 4. If $\varphi\in$ SolEMPent is degree m where $m\geqslant 4$ is even then $\varphi_1=0$, where φ_j is the y-degree j part of φ.

Lemma 5. If $\varphi\in$ SolEMPent is of degree $\geqslant 3$, then $(\partial_y\varphi)^*=\partial_y\varphi$.

Schneps' Theorem (in *Double Shuffle and Kashiwara-Vergne Lie Algebras*). Let $b\in FL(x,y)$. Then $\exists a\in FL(x,y)$ s.t. $[x,a]+[y,b]=0$ iff $\partial_y b=\partial^y b$ $(=(\partial_y b)^*)$.

Claim 1. If $\varphi\in$ SolEMPent is of degree $\geqslant 3$ and if we learn that $(\partial_x\varphi)^*=(\partial_x\varphi)(y,x)$, then $\nu(\varphi):=(\varphi(y,x),\varphi(x,y)$ satisfies KV1.