Pensieve Header: The Algebra of Emergent Chord Diagrams.

Goal: Implement $\mathcal{A}_{ps:\{\},ss:\{\}}\left[A0 : < FA[ps]^{\otimes ss} >, A1 : \Sigma t_{i,j}\, FA[ps]^{\otimes\left(ssu\left\{\overline{i},\overline{j}\right\}\right)}\right]$, including CF (Canonical Form) and HCF (HOMFLYPT Canonical Form).

In[•]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\People\\Kuno"];
<< FAA.m
```

In[•]:=
```
CF[𝒜ps_,ss_[A0_, A1_]] := Module[{l, r, u},
  𝒜ps,ss[A0, A1 /. t_i_,j_ ℰ_ :> Expand[
      t_i,j (ℰ // Δ_j→u,l // S_u→u // m_u,i→i // Δ_l→l,r // m_i,l→i // m_j,r→j)
    ]]
 ]
```

In[•]:=
```
𝒜{x,y,z},{1,2}[
  AW1[x, y, x] AW2[x, x, y],
  t1,2 AW1[x, y] AW2[y, x] AW1[z] AW2[x, y]
 ] // CF
```

Out[•]=
```
𝒜{x,y,z},{1,2}[AW1[x, y, x] AW2[x, x, y], t1,2 AW1[x, y, x, y] AW2[y, x] AW1[z] +
  t1,2 AW1[x, y, y] AW2[y, x, x] AW1[z] + t1,2 AW1[x, y, x] AW2[y, x, y] AW1[z] +
  t1,2 AW1[x, y] AW2[y, x, x, y] AW1[z] - t1,2 AW1[x, y, y] AW2[y, x] AW1[x, z] -
  t1,2 AW1[x, y] AW2[y, x, y] AW1[x, z] - t1,2 AW1[x, y, x] AW2[y, x] AW1[y, z] -
  t1,2 AW1[x, y] AW2[y, x, x] AW1[y, z] + t1,2 AW1[x, y] AW2[y, x] AW1[y, x, z]]
```

In[•]:=
```
HCF[𝒜ps_,ss_[A0_, A1_]] := 𝒜ps,ss[A0, A1 /. t_i_,j_ ℰ_ :> Expand[
      t_i,j (ℰ // m_i,j→i // m_j,i→j)
    ]]
```

In[•]:=
```
𝒜{x,y,z},{1,2}[
  AW1[x, y, x] AW2[x, x, y],
  t1,2 AW1[x, y] AW2[y, x] AW1[z] AW2[x, y]
 ] // HCF
```

Out[•]=
```
𝒜{x,y,z},{1,2}[AW1[x, y, x] AW2[x, x, y], t1,2 AW1[x, y, x, y] AW2[y, x, z]]
```

```
Unprotect[NonCommutativeMultiply];
𝒜ps_,ss_[A0_, A1_] ** 𝒜ps_,ss_[B0_, B1_] := Module[{∨, T}, 𝒜ps,ss[
    T = B0; Do[T = T // σs→∨@s, {s, ss}];
    T = Expand[A0 T]; Do[T = T // ms,∨@s→s, {s, ss}];
    T,
    Plus[
     T = B1; Do[T = T // σs→∨@s, {s, ss}];
     T = Expand[A0 T]; Do[T = T // ms,∨@s→s, {s, ss}];
     T,
     A1B0,
     A0B0
    ]
   ]]
```

In[•]:=
```
𝒜{x,y,z},{1,2}[AW1[x, y, x] AW2[x, x, y], 777] ** 𝒜{x,y,z},{1,2}[AW1[z, z, x] AW2[x, z, z], 888]
```

Out[•]=
```
𝒜{x,y,z},{1,2}[AW1[x, y, x, z, z, x] AW2[x, x, y, x, z, z], 666]
```