Question on $\mathcal{P}$

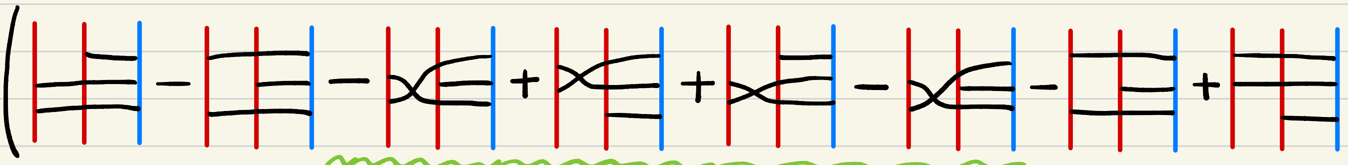
$$\begin{tikzcd}
FL\langle ps\rangle \arrow[r, "\text{symmetrization/average}"] \arrow[d, hook, "\text{natural inclusion}"] & \mathcal{P} \arrow[d, hook, "\text{natural inclusion}"] \\
FA\langle ps\rangle \arrow[r, "\text{horizontal inclusion}"] & \mathcal{A}
\end{tikzcd}$$

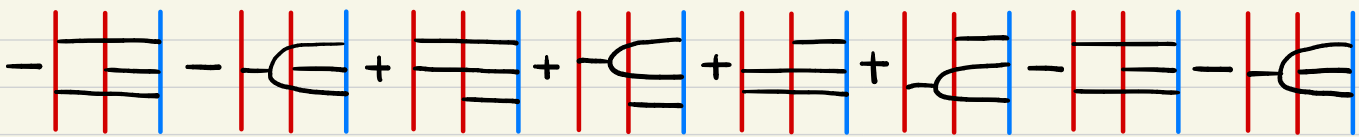
$$u \mapsto u^{hor} \qquad u \mapsto u^{sym}$$

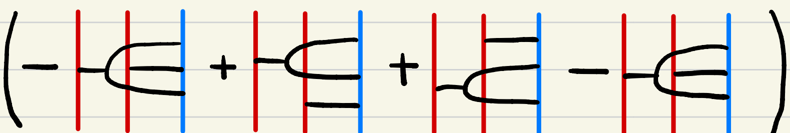
This diagram does not commute. What is $u^{sym} - u^{hor}$?

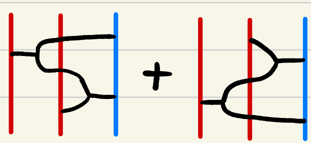
In what follows, I'll do some sample computations, and give a formula to compute it recursively (see the last two pages).

① $u = [x,[x,y]]$

$= \frac{1}{2}\Big($  $\Big)$



$= u^{hor} + \frac{1}{2}\Big($  $\Big)$



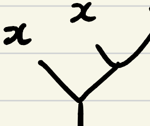
$= yx + xy \in FA\langle ps \rangle \subset \mathcal{P}$

Conclusion:

$u = [x, [x, y]]$

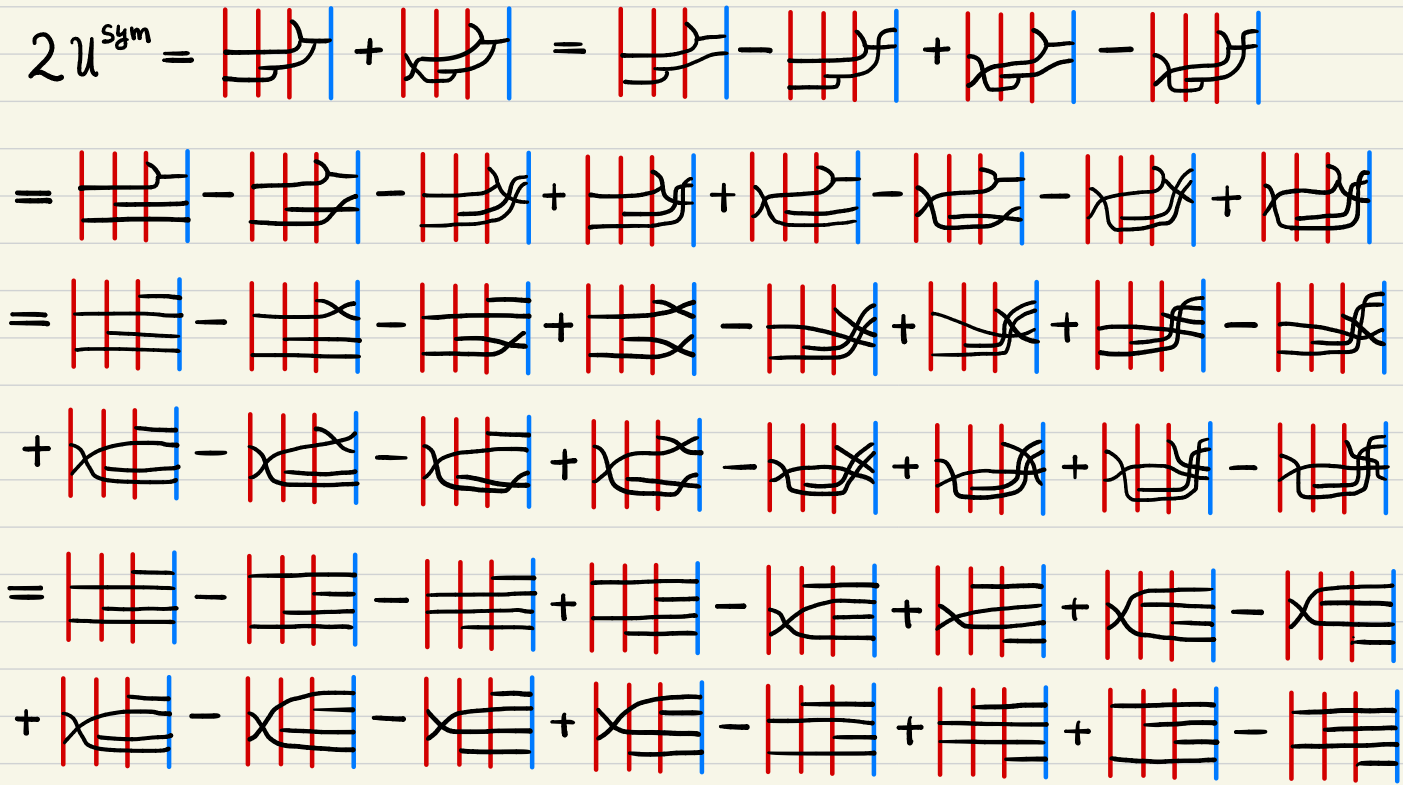
$u^{sym} - u^{hor}$

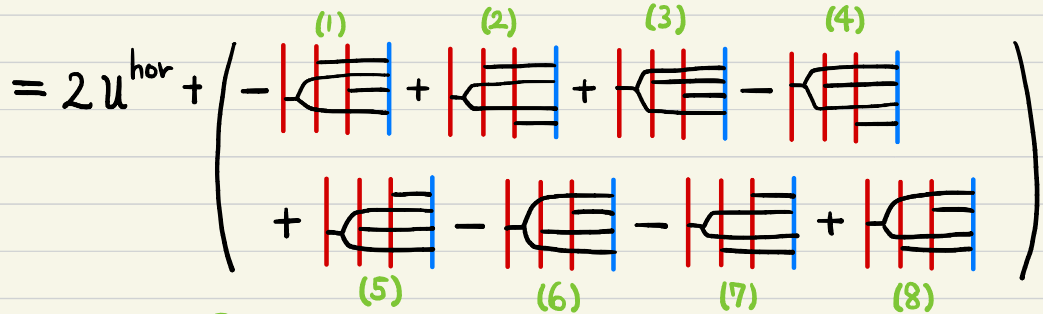
$= \frac{1}{2}(yx + xy)$

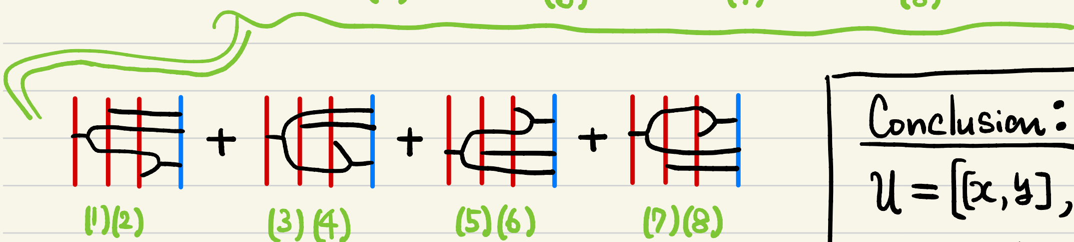


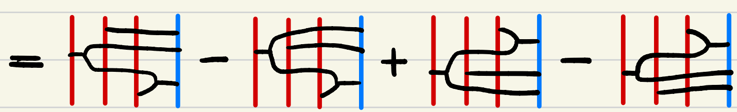
② $\mathcal{U} = \left[[x,y], [x,z] \right]$

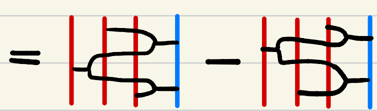
$2\mathcal{U}^{sym} =$









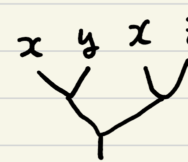


$= zxy - yxz$

Conclusion:

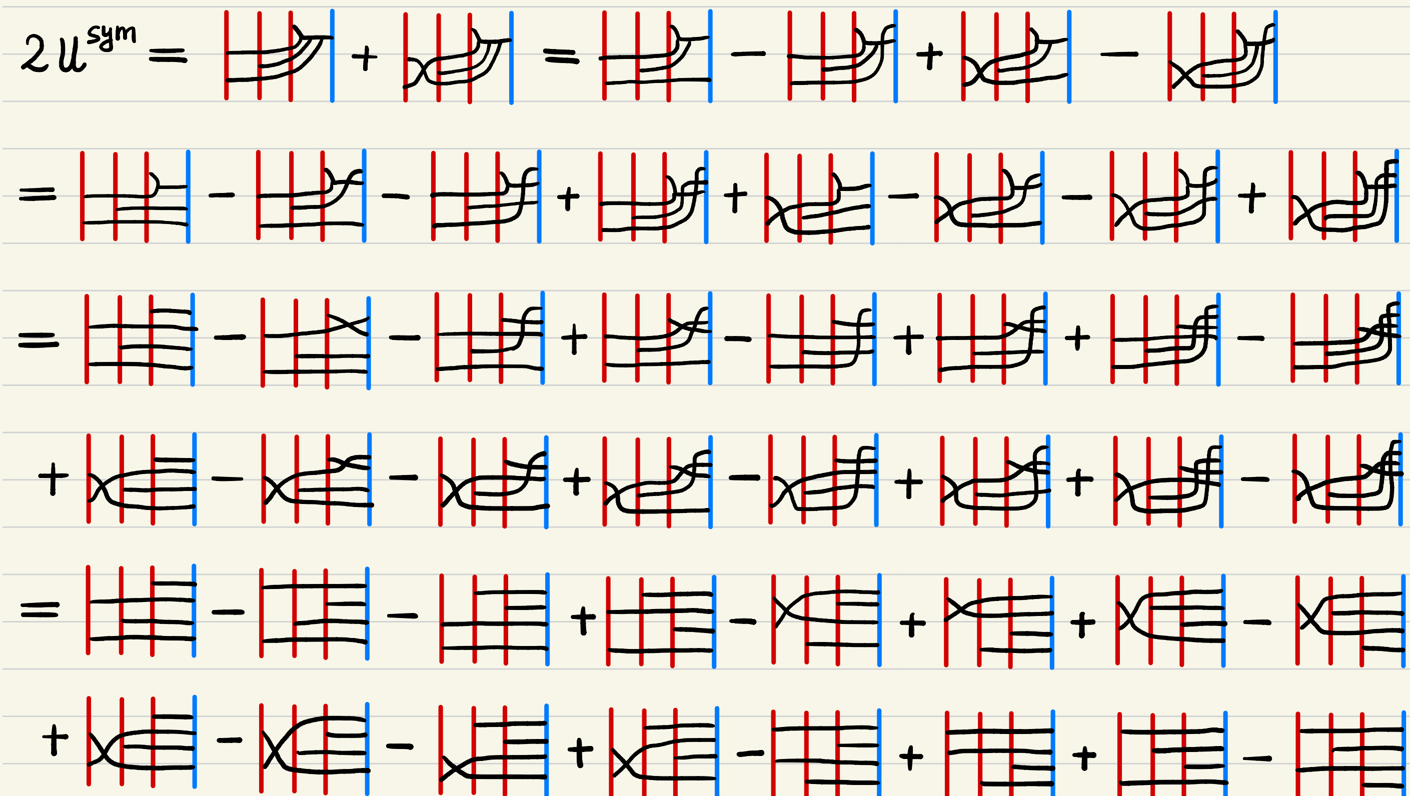
$u = [[x,y],[x,z]]$

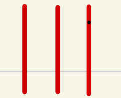
$u^{sym} - u^{hor} = \frac{1}{2}(zxy - yxz)$



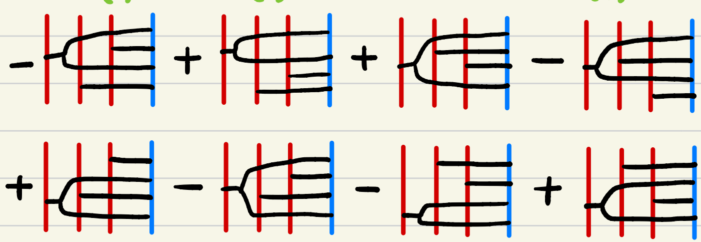
③ $u = [x,[y,[x,z]]]$

$2u^{sym} =$

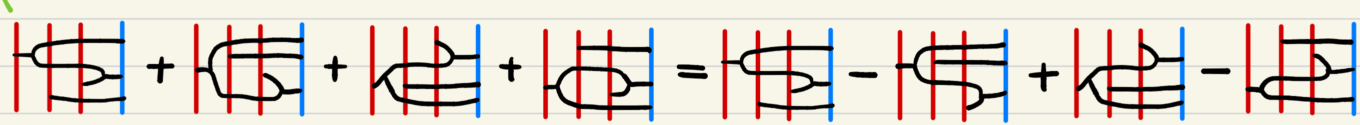




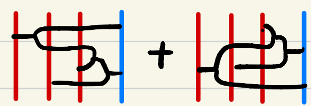
(1) (2) (3) (4)

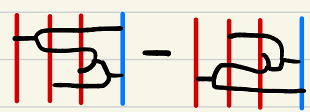
$= 2u^{hor} + \Big($  $\Big)$

(5) (6) (7) (8)



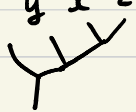
(1)(2) (3)(4) (5)(6) (7)(8)

$=$ 

$=$  $= yzx - xzy$

Conclusion:

$$u^{sym} - u^{hor} = \frac{1}{2}\left(yzx - xzy\right)$$



(changed sign!)

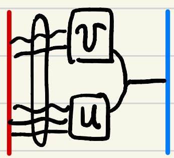
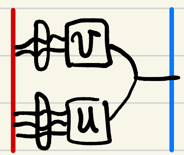
Let $R(u) := u^{hor} - u^{sym}$. We claim that $R(u) \in FA\langle ps\rangle[1] \subset \mathcal{P}$ and

Prop

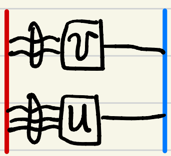
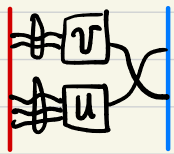
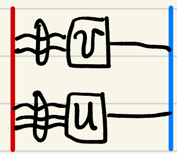
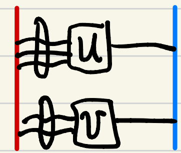
$$R([u,v]) = [u, R(v)] + [R(u), v] + \frac{1}{2} \sum_i \left((\partial_i v) x_i (\partial_i u)^* - (\partial_i u) x_i (\partial_i v)^* \right)$$

$R(x) = R(y) = 0$

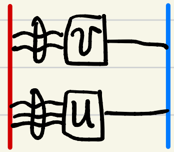
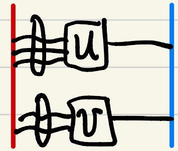
proof

$[u,v]^{sym} =$  $=$  $+ \frac{1}{2} \sum_i \left((\partial_i u) x_i (\partial_i v)^* + (\partial_i v) x_i (\partial_i u)^* \right)$

see 240117

 $-$  $=$  $-$  $- \sum_i (\partial_i v) x_i (\partial_i u)^*$

$\rightsquigarrow$

$[u,v]^{sym} =$  $-$  $+ \frac{1}{2}\sum_i \Big((\partial_i u)\, x_i\, (\partial_i v)^* - (\partial_i v)\, x_i\, (\partial_i u)^*\Big)$

$$= [u^{sym}, v^{sym}] + \frac{1}{2}\sum_i \Big((\partial_i u)\, x_i\, (\partial_i v)^* - (\partial_i v)\, x_i\, (\partial_i u)^*\Big)$$

$$[u^{hor} - R(u),\, v^{hor} - R(v)] = [u^{hor}, v^{hor}] - [u, R(v)] - [R(u), v]$$

$$= [u,v]^{hor} - [u, R(v)] - [R(u), v]$$

$\rightsquigarrow$ get the formula for $R([u,v]) = [u,v]^{hor} - [u,v]^{sym}$ //

I still don't know a closed formula....