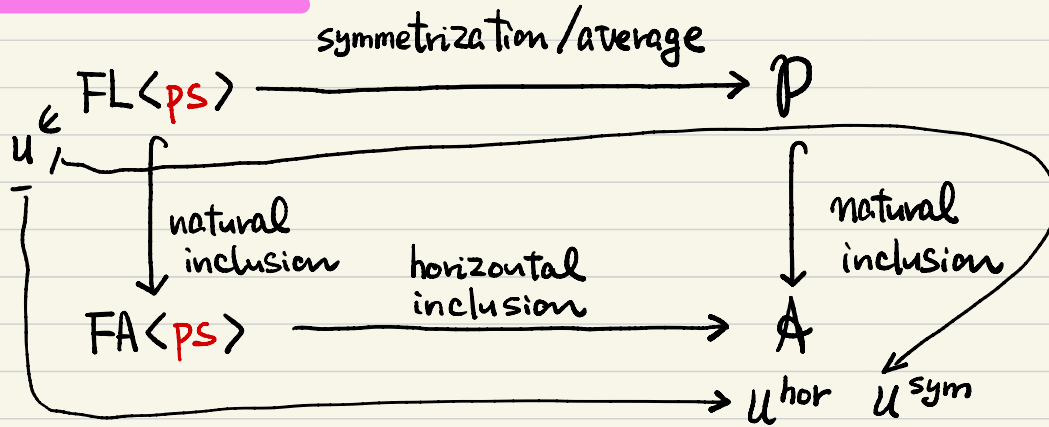


Question on \mathcal{P}



This diagram does not commute. What is $u^{\text{sym}} - u^{\text{hor}}$?

In what follows, I'll do some sample computations, and give a formula to compute it recursively (see the last two pages).

$$\textcircled{1} \quad u = [x, [x, y]]$$

$$u^{\text{sym}} = \frac{1}{2} \left(\begin{array}{c} | \quad | \quad | \\ \text{---} \diagup \quad \diagdown \\ | \quad | \quad | \end{array} + \begin{array}{c} | \quad | \quad | \\ \text{---} \diagdown \quad \diagup \\ | \quad | \quad | \end{array} \right)$$

$$= \frac{1}{2} \left(\begin{array}{c} | \quad | \quad | \\ \text{---} \diagup \quad \diagdown \\ | \quad | \quad | \end{array} - \begin{array}{c} | \quad | \quad | \\ \text{---} \diagdown \quad \diagup \\ | \quad | \quad | \end{array} + \begin{array}{c} | \quad | \quad | \\ \text{---} \diagdown \quad \diagup \\ | \quad | \quad | \end{array} - \begin{array}{c} | \quad | \quad | \\ \text{---} \diagup \quad \diagdown \\ | \quad | \quad | \end{array} \right)$$

$$= \frac{1}{2} \left(\begin{array}{c} | \quad | \quad | \\ \text{---} \quad \text{---} \quad \text{---} \\ | \quad | \quad | \end{array} - \begin{array}{c} | \quad | \quad | \\ \text{---} \diagdown \quad \diagup \\ | \quad | \quad | \end{array} - \begin{array}{c} | \quad | \quad | \\ \text{---} \diagup \quad \diagdown \\ | \quad | \quad | \end{array} + \begin{array}{c} | \quad | \quad | \\ \text{---} \diagdown \quad \diagup \\ | \quad | \quad | \end{array} \right) \\
\left(\begin{array}{c} + \begin{array}{c} | \quad | \quad | \\ \text{---} \diagdown \quad \diagup \\ | \quad | \quad | \end{array} - \begin{array}{c} | \quad | \quad | \\ \text{---} \diagup \quad \diagdown \\ | \quad | \quad | \end{array} - \begin{array}{c} | \quad | \quad | \\ \text{---} \diagdown \quad \diagup \\ | \quad | \quad | \end{array} + \begin{array}{c} | \quad | \quad | \\ \text{---} \diagup \quad \diagdown \\ | \quad | \quad | \end{array} \end{array} \right)$$

$$= \frac{1}{2} \left(\begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \\ \text{Diagram 3} \\ \text{Diagram 4} \\ \text{Diagram 5} \\ \text{Diagram 6} \\ \text{Diagram 7} \\ \text{Diagram 8} \end{array} \right)$$

$$- \begin{array}{c} \text{Diagram 9} \\ \text{Diagram 10} \\ \text{Diagram 11} \\ \text{Diagram 12} \\ \text{Diagram 13} \\ \text{Diagram 14} \\ \text{Diagram 15} \\ \text{Diagram 16} \end{array}$$

$$= u^{\text{hor}} + \frac{1}{2} \left(\begin{array}{c} \text{Diagram 17} \\ \text{Diagram 18} \\ \text{Diagram 19} \\ \text{Diagram 20} \end{array} \right)$$

$$\begin{array}{c} \text{Diagram 21} \\ \text{Diagram 22} \end{array} + \begin{array}{c} \text{Diagram 23} \\ \text{Diagram 24} \end{array} = yx + xy \in \text{FA}(\text{ps}) \subset \mathcal{P}$$

Conclusion:
 $u = [x, [x, y]]$
 $u^{\text{sym}} - u^{\text{hor}}$
 $= \frac{1}{2}(yx + xy)$

$$\textcircled{2} u = [x, y], [x, z]$$

$$2u^{\text{sym}} = \text{diagram 1} + \text{diagram 2} = \text{diagram 3} - \text{diagram 4} + \text{diagram 5} - \text{diagram 6}$$

$$= \text{diagram 7} - \text{diagram 8} - \text{diagram 9} + \text{diagram 10} + \text{diagram 11} - \text{diagram 12} - \text{diagram 13} + \text{diagram 14}$$

$$= \text{diagram 15} - \text{diagram 16} - \text{diagram 17} + \text{diagram 18} - \text{diagram 19} + \text{diagram 20} + \text{diagram 21} - \text{diagram 22}$$

$$+ \text{diagram 23} - \text{diagram 24} - \text{diagram 25} + \text{diagram 26} - \text{diagram 27} + \text{diagram 28} + \text{diagram 29} - \text{diagram 30}$$

$$= \text{diagram 31} - \text{diagram 32} - \text{diagram 33} + \text{diagram 34} - \text{diagram 35} + \text{diagram 36} + \text{diagram 37} - \text{diagram 38}$$

$$+ \text{diagram 39} - \text{diagram 40} - \text{diagram 41} + \text{diagram 42} - \text{diagram 43} + \text{diagram 44} + \text{diagram 45} - \text{diagram 46}$$

$$= 2u^{\text{hor}} + \left(\begin{array}{cccc} \text{(1)} & \text{(2)} & \text{(3)} & \text{(4)} \\ - & + & + & - \\ \text{(5)} & \text{(6)} & \text{(7)} & \text{(8)} \end{array} \right)$$

$$= \text{(1)(2)} - \text{(3)(4)} + \text{(5)(6)} - \text{(7)(8)} = zxy - yxz$$

Conclusion:
 $u = [x, y], [x, z]$
 $u^{\text{sym}} - u^{\text{hor}} = \frac{1}{2}(zxy - yxz)$

$$\textcircled{3} \quad u = [x, [y, [x, z]]$$

$$2u^{\text{sym}} = \text{diagram 1} + \text{diagram 2} = \text{diagram 3} - \text{diagram 4} + \text{diagram 5} - \text{diagram 6}$$

$$= \text{diagram 7} - \text{diagram 8} - \text{diagram 9} + \text{diagram 10} + \text{diagram 11} - \text{diagram 12} - \text{diagram 13} + \text{diagram 14}$$

$$= \text{diagram 15} - \text{diagram 16} - \text{diagram 17} + \text{diagram 18} - \text{diagram 19} + \text{diagram 20} + \text{diagram 21} - \text{diagram 22}$$

$$+ \text{diagram 23} - \text{diagram 24} - \text{diagram 25} + \text{diagram 26} - \text{diagram 27} + \text{diagram 28} + \text{diagram 29} - \text{diagram 30}$$

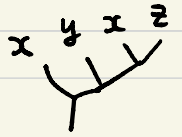
$$= \text{diagram 31} - \text{diagram 32} - \text{diagram 33} + \text{diagram 34} - \text{diagram 35} + \text{diagram 36} + \text{diagram 37} - \text{diagram 38}$$

$$+ \text{diagram 39} - \text{diagram 40} - \text{diagram 41} + \text{diagram 42} - \text{diagram 43} + \text{diagram 44} + \text{diagram 45} - \text{diagram 46}$$

$$= 2u^{hor} + \left(\begin{array}{cccc} \text{(1)} & \text{(2)} & \text{(3)} & \text{(4)} \\ - & + & + & - \\ \text{(5)} & \text{(6)} & \text{(7)} & \text{(8)} \end{array} \right)$$

Conclusion:

$$u^{sym} - u^{hor} = \frac{1}{2} (yzx - xzy)$$



$$= yzx - xzy$$

(changed sign!)

Let $R(u) := u^{\text{hor}} - u^{\text{sym}}$. We claim that $R(u) \in \text{FA}(\text{ps})[1] \subset \mathcal{P}$ and

$$\left[\begin{array}{l} \text{Prop} \\ R([u, v]) = [u, R(v)] + [R(u), v] \\ \quad + \frac{1}{2} \sum_i \left((\partial_i v) x_i (\partial_i u)^* - (\partial_i u) x_i (\partial_i v)^* \right) \end{array} \right]$$

$R(x) = R(y) = 0$

proof

$$[u, v]^{\text{sym}} = \left(\begin{array}{c} \text{diagram 1} \end{array} \right) = \left(\begin{array}{c} \text{diagram 2} \end{array} \right) + \frac{1}{2} \sum_i \left((\partial_i u) x_i (\partial_i v)^* + (\partial_i v) x_i (\partial_i u)^* \right)$$

see 24o117

$$\left(\begin{array}{c} \text{diagram 3} \end{array} \right) - \left(\begin{array}{c} \text{diagram 4} \end{array} \right) = \left(\begin{array}{c} \text{diagram 5} \end{array} \right) - \left(\begin{array}{c} \text{diagram 6} \end{array} \right) - \sum_i (\partial_i v) x_i (\partial_i u)^*$$

$$\begin{aligned}
 \rightsquigarrow [u, v]^{\text{sym}} &= \left[\begin{array}{c} \text{---} \text{---} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \text{---} \text{---} \end{array} \right] - \left[\begin{array}{c} \text{---} \text{---} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \text{---} \text{---} \end{array} \right] + \frac{1}{2} \sum_i \left((\partial_i u) x_i (\partial_i v)^* - (\partial_i v) x_i (\partial_i u)^* \right) \\
 &= [u^{\text{sym}}, v^{\text{sym}}] + \frac{1}{2} \sum_i \left((\partial_i u) x_i (\partial_i v)^* - (\partial_i v) x_i (\partial_i u)^* \right)
 \end{aligned}$$

$$\begin{aligned}
 [u^{\text{hor}} - R(u), v^{\text{hor}} - R(v)] &= [u^{\text{hor}}, v^{\text{hor}}] - [u, R(v)] - [R(u), v] \\
 &= [u, v]^{\text{hor}} - [u, R(v)] - [R(u), v]
 \end{aligned}$$

$$\rightsquigarrow \text{ get the formula for } R([u, v]) = [u, v]^{\text{hor}} - [u, v]^{\text{sym}}$$

//

I still don't know a closed formula....