What are the building blocks?

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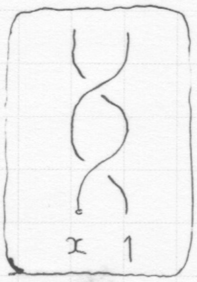
2023.11.10



$$\overset{Z}{\longmapsto} e^{x} \xrightarrow{\Delta_{x\to x,y}} e^{x+y}$$

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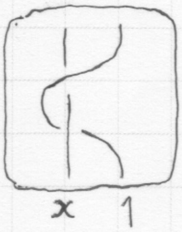
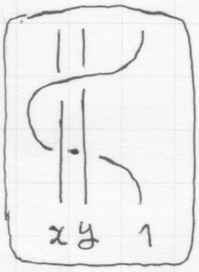


$$\downarrow \Delta_{x\to x,y}$$



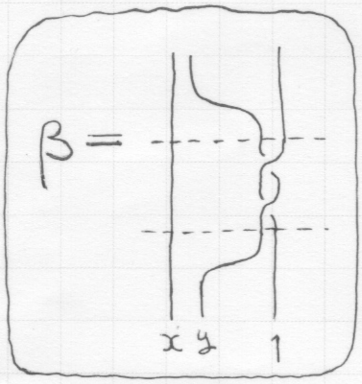
=

$$\boxed{Z(\alpha)\,Z(\beta) = e^{x+y}}$$

6-gon for α & β

$$\Delta_{1\to 1,2}\big(Z(\beta)\big) = Z\big(\Delta_{1\to 1,2}(\beta)\big)$$

$$\Delta_{1\to 1,2}\big(Z(\alpha)\big) = Z\big(\Delta_{1\to 1,2}(\beta)\big)$$



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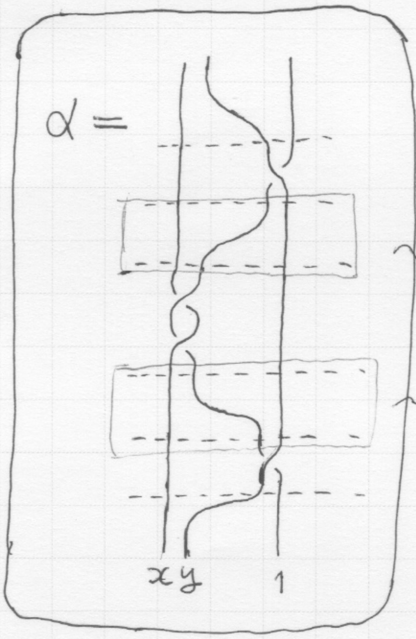
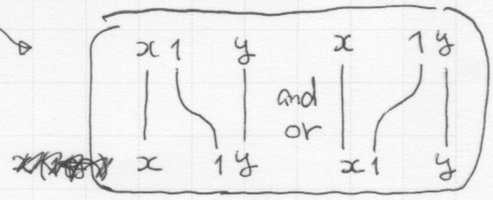
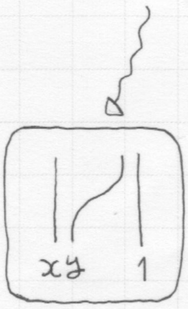
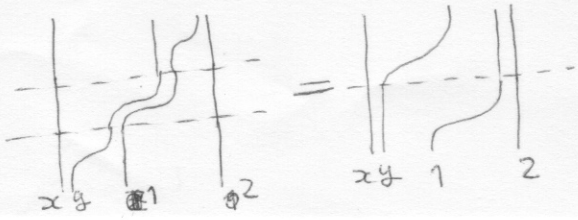
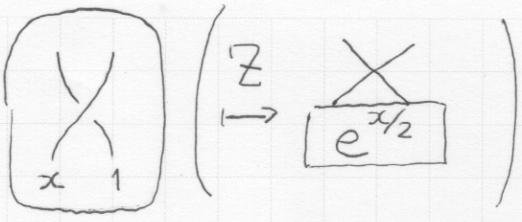




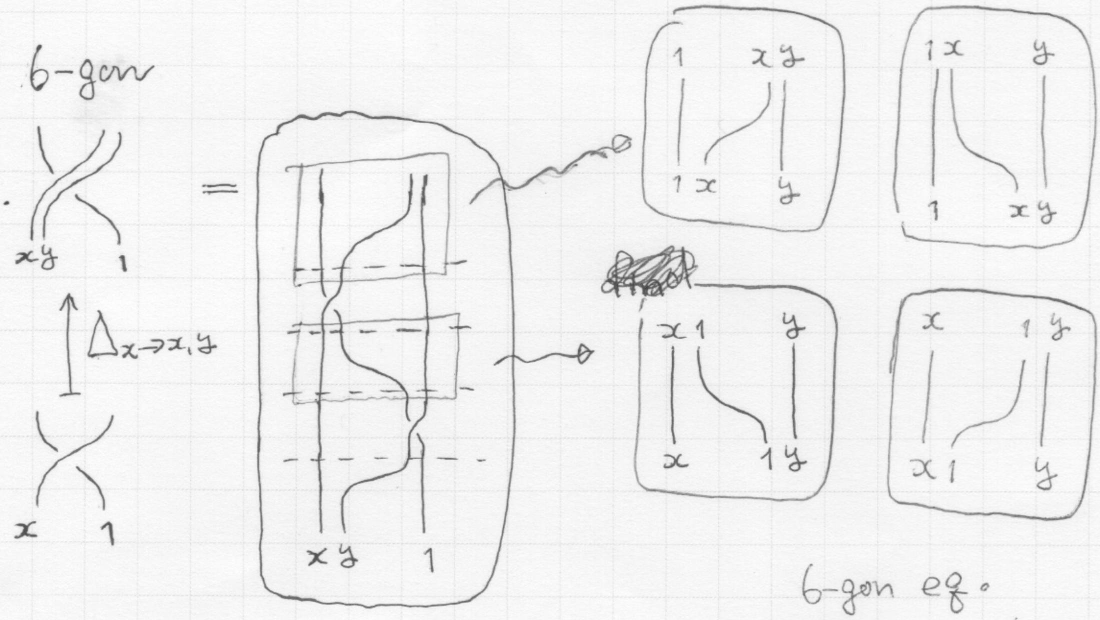


Pentagon eqn



Want : 

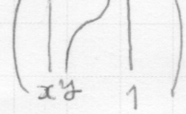
6-gon



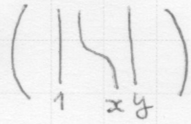
6-gon eq.

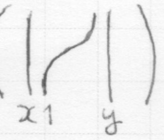
$$e^{(x+y)/2} = \Phi(x,y)\, e^{y/2}\, \hat{\Phi}(x,y)^{-1}\, e^{x/2}\, \Phi(y,x)^{-1}$$

$$\hat{\Phi}(x,y) = e^{x/2}\, \Phi(y,x)^{-1}\, e^{-(x+y)/2}\, \Phi(x,y)\, e^{y/2}$$

Z() $=: \Phi(x,y) = \Phi_{em}$

Assumption(?):

Z() $= \Phi(y,x)$

Z() $=: \hat{\Phi}(x,y)$

$$Z(\beta) = \Phi(x,y)\, e^{y}\, \Phi(x,y)^{-1}$$

$$Z(\alpha) = \Phi(x,y)\, e^{y/2}\, \hat{\Phi}(x,y)^{-1}\, e^{x}\, \hat{\Phi}(x,y)\, e^{-y/2}\, \Phi(x,y)^{-1}$$

$$= \cdots$$

$$= e^{(x+y)/2}\, \Phi(y,x)\, e^{x}\, \Phi(y,x)^{-1}\, e^{-(x+y)/2}$$

$$\boxed{Z(\alpha)Z(\beta) = e^{x+y}} \iff \boxed{e^{(x+y)/2}\, \Phi(y,x) e^{x} \Phi(y,x)^{-1} e^{-(x+y)/2}\, \Phi(x,y) e^{y} \Phi(x,y)^{-1} = e^{x+y}}$$

Rather random questions

(1) Does $\Phi_{\mathrm{em}} = \Phi_{\mathrm{pps}}$ determine $Z(\alpha)$ and $Z(\beta)$? More concretely, should it be the following?

$$\begin{aligned} Z(\alpha) =& e^{\frac{x+y}{2}} \Phi_{\mathrm{em}}(y,x)\, e^x\, \Phi_{\mathrm{em}}(y,x)^{-1} e^{-\frac{x+y}{2}}, \\ Z(\beta) =& \Phi_{\mathrm{em}}(x,y)\, e^y\, \Phi_{\mathrm{em}}(x,y)^{-1}. \end{aligned}$$

(2) Does the pps-pentagon (plus normalization $\Phi(x,y) = 1 + \frac{1}{24}[x,y] + \cdots$) imply the equation $Z(\alpha)Z(\beta) = e^{x+y}$?

(3) On the pps-hexagon. We want to have $\Delta_{1\to 1,2}(Z(\alpha)) = Z(\Delta_{1\to 1,2}(\alpha))$ and $\Delta_{1\to 1,2}(Z(\beta)) = Z(\Delta_{1\to 1,2}(\beta))$. Does any of the two follow from the other? (Do we need $Z(\alpha)Z(\beta) = e^{x+y}$?)

(4) If Φ_{em} satisfies the pps-pentagon (with the normalization) and the pps-hexagon, does it induce an expansion Z which respects "adding/deleting poles/strands" and "pole/strand doubling"?

(5) For questions above, does the argument in [AET] work?

(6) What about the HOMFLY-PT quotient? For instance, will the set of solutions to the pps-pentagon be different?