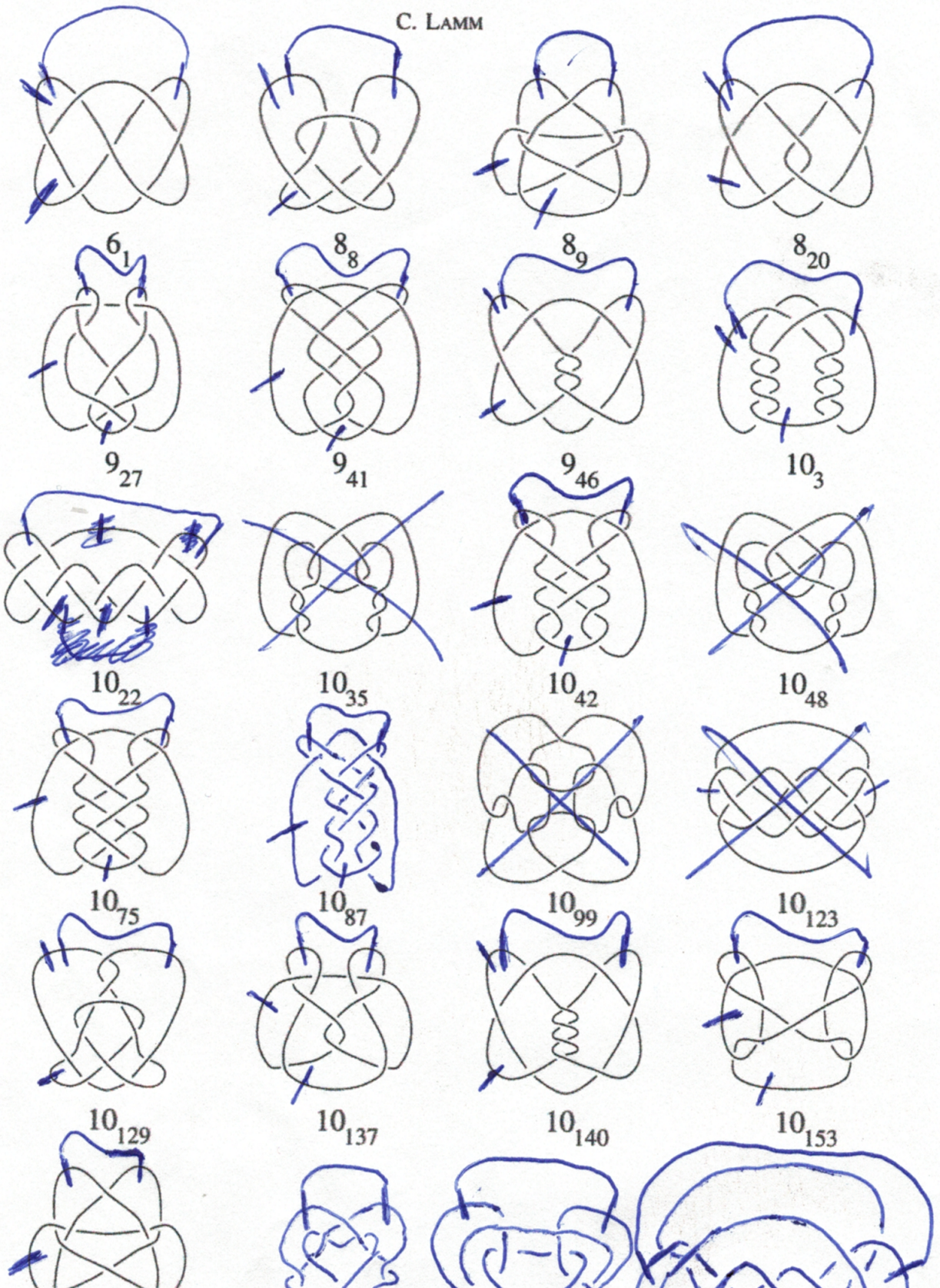
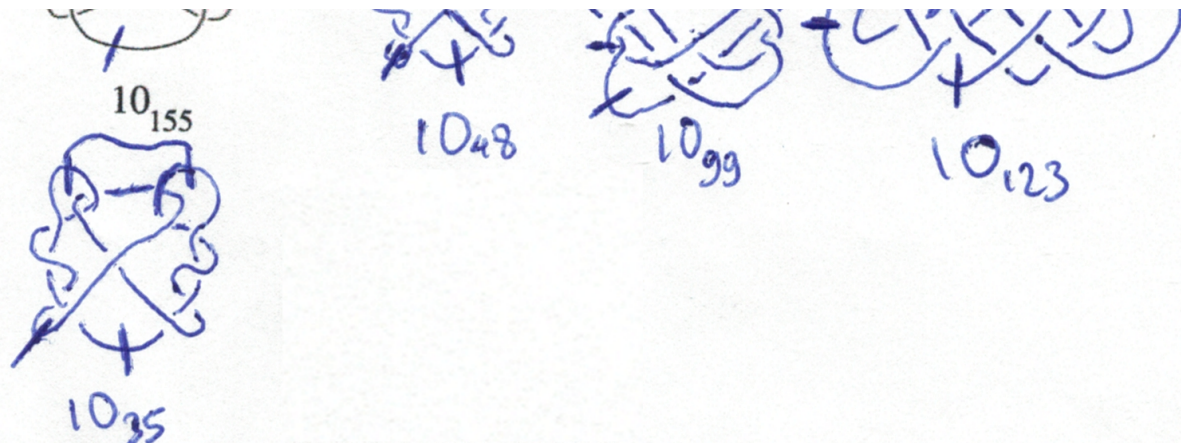


C. LAMM





$K@6_1 = X_{1,7,2,6} X_{2,9,3,10} X_{4,11,5,12} X_{7,1,8,14} X_{10,5,11,6} X_{12,3,13,4} X_{13,9,14,8};$
 $T@6_1 = \{\{1, 2, 3, 4, 5, 6, 7\}, \{10, 9, 8\}, \{11, 12, 13, 14\}, \{\}\};$
 $K@8_8 = X_{1,9,2,8} X_{3,1,4,18} X_{5,11,6,10} X_{6,13,7,14} X_{9,3,10,2} X_{12,15,13,16} X_{14,7,15,8} X_{16,11,17,12} X_{17,5,18,4};$
 $T@8_8 = \{\{1, 2, 3\}, \{14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4\}, \{15, 16, 17, 18\}, \{\}\};$
 $K@8_9 = X_{1,8,2,9} X_{3,11,4,10} X_{6,19,7,20} X_{9,16,10,17}$
 $X_{11,19,12,18} X_{12,6,13,5} X_{14,7,15,8} X_{15,3,16,2} X_{17,4,18,5} X_{20,14,1,13};$
 $T@8_9 = \{\{1, 2, 3, 4\}, \{8, 7, 6, 5\}, \{9, 10, 11, 12\}, \{20, 19, 18, 17, 16, 15, 14, 13\}\};$
 $K@8_{20} = X_{1,6,2,7} X_{2,11,3,12} X_{5,16,6,1} X_{8,14,9,13} X_{10,3,11,4} X_{12,8,13,7} X_{14,10,15,9} X_{15,4,16,5};$
 $T@8_{20} = \{\{1, 2, 3, 4\}, \{12, 11, 10, 9, 8, 7, 6, 5\}, \{13, 14, 15, 16\}, \{\}\};$
 $K@9_{27} = X_{2,24,3,23} X_{5,20,6,21} X_{7,15,8,14} X_{8,1,9,2} X_{10,20,11,19}$
 $X_{11,5,12,4} X_{13,16,14,17} X_{15,7,16,6} X_{18,3,19,4} X_{21,13,22,12} X_{22,17,23,18} X_{24,9,1,10};$
 $T@9_{27} = \{\{1, 2, 3, 4, 5, 6, 7, 8, 9\}, \{14, 13, 12, 11, 10\},$
 $\{15, 16, 17, 18, 19\}, \{24, 23, 22, 21, 20\}\};$
 $K@9_{41} = X_{2,17,3,18} X_{3,21,4,20} X_{6,13,7,14} X_{7,24,8,25} X_{9,19,10,18} X_{10,1,11,2} X_{12,24,13,23}$
 $X_{14,5,15,6} X_{16,27,17,28} X_{19,9,20,8} X_{21,27,22,26} X_{22,16,23,15} X_{25,4,26,5} X_{28,11,1,12};$
 $T@9_{41} = \{\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}, \{18, 17, 16, 15, 14, 13, 12\},$
 $\{19, 20, 21, 22, 23\}, \{28, 27, 26, 25, 24\}\};$
 $K@9_{46} = X_{1,9,2,8} X_{2,13,3,14} X_{4,11,5,12} X_{6,15,7,16} X_{9,1,10,18} X_{12,3,13,4} X_{14,7,15,8} X_{16,5,17,6} X_{17,11,18,10};$
 $T@9_{46} = \{\{1, 2, 3, 4, 5, 6, 7, 8, 9\}, \{14, 13, 12, 11, 10\}, \{15, 16, 17, 18\}, \{\}\};$
 $K@10_3 = X_{2,11,3,12} X_{3,16,4,17} X_{5,14,6,15} X_{8,22,9,21}$
 $X_{10,20,11,19} X_{13,6,14,7} X_{15,4,16,5} X_{17,13,18,12} X_{18,1,19,2} X_{20,10,21,9} X_{22,8,1,7};$
 $T@10_3 = \{\{1, 2, 3, 4, 5, 6\}, \{12, 11, 10, 9, 8, 7\},$
 $\{13, 14, 15, 16, 17, 18, 19, 20, 21, 22\}, \{\}\};$
 $K@10_{22} = X_{2,12,3,11} X_{5,18,6,19} X_{7,16,8,17} X_{9,5,10,4} X_{12,22,13,21} X_{14,2,15,1}$
 $X_{15,6,16,7} X_{17,8,18,9} X_{19,11,20,10} X_{20,3,21,4} X_{22,14,1,13};$
 $T@10_{22} = \{\{1, 2, 3\}, \{6, 5, 4\}, \{7, 8, 9, 10, 11, 12, 13,$
 $14, 15, 16, 17, 18, 19, 20\}, \{22, 21\}\};$
 $K@10_{35} = X_{2,11,3,12} X_{3,1,4,26} X_{5,25,6,24} X_{9,19,10,18} X_{10,1,11,2} X_{7,13,8,12} X_{14,21,15,22}$
 $X_{16,19,17,20} X_{17,9,18,8} X_{20,15,21,16} X_{22,13,23,14} X_{23,7,24,6} X_{25,5,26,4};$
 $T@10_{35} = \{\{1, 2, 3, 4, 5\}, \{18, 17, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6\},$
 $\{19, 20, 21, 22\}, \{26, 25, 24, 23\}\};$


```

K@1042 = X2,28,3,27 X4,9,5,10 X5,24,6,25 X8,18,9,17 X11,14,12,15 X13,22,14,23 X15,27,16,26
        X16,3,17,4 X18,8,19,7 X20,2,21,1 X21,12,22,13 X23,6,24,7 X25,11,26,10 X28,20,1,19;
T@1042 = {{1, 2, 3, 4, 5, 6}, {12, 11, 10, 9, 8, 7}, {13, 14, 15, 16, 17, 18},
        {28, 27, 26, 25, 24, 23, 22, 21, 20, 19}};
K@1048 = X3,13,4,12 X4,22,5,21 X6,24,7,23 X8,13,9,14 X10,19,11,20 X11,3,12,2 X15,26,16,27
        X17,28,18,1 X18,9,19,10 X20,2,21,1 X22,6,23,5 X24,8,25,7 X25,14,26,15 X27,16,28,17;
T@1048 = {{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14},
        {20, 19, 18, 17, 16, 15}, {21, 22, 23, 24}, {28, 27, 26, 25}};
K@1075 = X2,30,3,29 X4,9,5,10 X5,26,6,27 X8,18,9,17 X11,14,12,15 X13,21,14,20 X15,29,16,28
        X16,3,17,4 X19,7,20,6 X21,13,22,12 X22,1,23,2 X24,7,25,8 X25,19,26,18 X27,11,28,10 X30,23,1,24;
T@1075 = {{1, 2, 3, 4, 5, 6}, {12, 11, 10, 9, 8, 7},
        {13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23}, {30, 29, 28, 27, 26, 25, 24}};
K@1087 = X3,17,4,16 X5,11,6,10 X6,19,7,20 X9,27,10,26 X12,29,13,30 X13,3,14,2 X15,24,16,25
        X18,12,19,11 X20,7,21,8 X22,2,23,1 X23,14,24,15 X25,9,26,8 X27,4,28,5 X28,17,29,18 X30,22,1,21;
T@1087 = {{1, 2, 3, 4, 5, 6, 7}, {14, 13, 12, 11, 10, 9, 8},
        {15, 16, 17, 18, 19, 20}, {30, 29, 28, 27, 26, 25, 24, 23, 22, 21}};
K@1099 = X1,26,2,27 X3,12,4,13 X6,31,7,32 X10,20,11,19 X11,2,12,3 X13,8,14,9 X15,23,16,22 X18,10,19,9
        X20,28,21,27 X21,17,22,16 X24,8,25,7 X25,4,26,5 X28,18,29,17 X29,14,30,15 X30,24,31,23 X32,5,1,6;
T@1099 = {{1, 2, 3, 4, 5}, {20, 19, 18, 17, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6},
        {21, 22, 23, 24, 25, 26, 27, 28, 29, 30}, {32, 31}};
K@10123 = X2,25,3,26 X3,23,4,22 X5,14,6,15 X6,19,7,20 X8,28,9,27 X9,19,10,18 X10,1,11,2 X11,24,12,25
        X13,1,14,32 X16,29,17,30 X17,27,18,26 X20,16,21,15 X23,12,24,13 X28,8,29,7 X30,22,31,21 X31,4,32,5;
T@10123 = {{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13}, {18, 17, 16, 15, 14},
        {19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32}, {}};
K@10129 = X1,18,2,19 X3,11,4,10 X5,1,6,22 X7,13,8,12 X8,15,9,16 X11,5,12,4
        X14,19,15,20 X16,9,17,10 X17,2,18,3 X20,13,21,14 X21,7,22,6;
T@10129 = {{1, 2, 3, 4, 5}, {18, 17, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6},
        {19, 20, 21, 22}, {}};
K@10137 = X2,20,3,19 X5,8,6,9 X7,13,8,12 X10,17,11,18 X11,4,12,5 X13,7,14,6
        X14,1,15,2 X16,4,17,3 X18,9,19,10 X20,15,1,16;
T@10137 = {{1, 2, 3}, {6, 5, 4}, {7, 8, 9, 10, 11, 12, 13, 14, 15}, {20, 19, 18, 17, 16}};
K@10140 = X1,8,2,9 X2,15,3,16 X4,13,5,14 X7,20,8,1
        X10,18,11,17 X12,5,13,6 X14,3,15,4 X16,10,17,9 X18,12,19,11 X19,6,20,7;
T@10140 = {{1, 2, 3, 4, 5, 6}, {16, 15, 14, 13, 12, 11, 10, 9, 8, 7}, {17, 18, 19, 20}, {}};
K@10153 = X2,22,3,21 X3,17,4,16 X6,2,7,1 X7,12,8,13
        X10,19,11,20 X11,14,12,15 X13,8,14,9 X17,5,18,4 X18,9,19,10 X20,15,21,16 X22,6,1,5;
T@10153 = {{1, 2, 3, 4}, {12, 11, 10, 9, 8, 7, 6, 5},
        {13, 14, 15, 16, 17}, {22, 21, 20, 19, 18}};
K@10155 = X3,11,4,10 X6,19,7,20 X8,2,9,1 X9,16,10,17 X11,19,12,18
        X12,6,13,5 X14,7,15,8 X15,3,16,2 X17,4,18,5 X20,14,1,13;
T@10155 = {{1, 2, 3, 4}, {8, 7, 6, 5}, {9, 10, 11, 12}, {20, 19, 18, 17, 16, 15, 14, 13}};

$PrePrint = If[MatrixQ@#, MatrixForm@#, #] &;
SetAttributes[P, Orderless];

```

```

P /: P[i_, j_] P[j_, k_] := P[i, k];
P /: P[i_, i_] := -√t - 1/√t;
P /: P[___]^2 := -√t - 1/√t;
Γ /: Γ[ω1_, λ1_, L1_] Γ[ω2_, λ2_, L2_] := Module[
  {S = Ordering@Join[L1, L2]}, Γ[ω1 ω2, Join[Join[#, Table[0, {Length@λ2}]] & /@ λ1,
    Join[Table[0, {Length@λ1}], #] & /@ λ2][[S, S], Union[L1, L2]]];
MΓ@Γ[ω_, λ_, L_] := Prepend[Prepend[λ^T, t# & /@ L]^T, Prepend[h# & /@ L, ω]];
Format@Γ@i__ := MatrixForm@MΓ@Γ@i;
K@_ = 0;
T@_ = 0;
PD@K_ := Module[{x = 2 Length[X K] - 2},
  XS@K /. {Xpa,b → Xb,Mod[a,x]+1,Mod[b,x]+1,a, Xma,b → Xb,a,Mod[b,x]+1,Mod[a,x]+1};
XS@K_ := K //. {Xp@i__ → Xpi, Xm@i__ → Xmi, X@i__ → Xi,
  Xi,j,k,l → If[j == l + 1 || j == 1 && l ≠ 2 || k == l && j ≠ k, Xpl,i, Xmj,i]}];
Rra,b := Γ[1, ( 1 1 - Ta ) / 0 Ta ], {a, b}];
Rla,b := Rra,b / . Ta → 1/Ta;
M@Γ@i__ := Γ@i;
M@K_ := XS@K /. {Xpi → Rri, Xmi → Rli};
Sa→c@K_ := S{a}→c@K;
S{a}→c@K_ := K / . Ta → Tc;
Sa,b→c[Γ[ω_, λ_, L_], f___] :=
Module[{i = FirstPosition[L, a][[1]], j = FirstPosition[L, b][[1]], n, o,
  d, α, β, γ, δ, θ, ε, φ, ψ, Ξ}, n = Prepend[Delete[L, {{i}, {j}}], c];
o = Ordering@n;
d = Delete[Range@Length@L, {{i}, {j}}];
( α β θ ) = ( λ[[i, i]] 1/(1-λ[[i, j]]) λ[[i, d]] )
( γ δ ε ) = ( λ[[j, i]] λ[[j, j]] λ[[j, d]] )
( φ ψ Ξ ) = ( λ[[d, i]] λ[[d, j]] λ[[d, d]] )
Γ[Apart@ω/β, If[f === BlankNullSequence, Apart, FullSimplify]@
  Join[{Join[{γ + β α δ}, ε + β δ θ]}, Join[{φ + β α ψ},
    If[Ξ == {}, {Ξ}, (Ξ + β {ψ}^T . {θ})^T]^T][[o, o], n[[o]]] / . Ta|b → Tc];
Sa,b→c@K_ := Sa,b→c@M@K;
S{a,b,l}→c@Γ@i__ := S{c,l}→c@Sa,b→c[Γ@i / . Table[Tj → Tc, {j, {l}}]];
S{a,b,l}→c@K_ := S{a,b,l}→c@M@K;
G{c}→c@K_ := K;
G{a,b,l}→c@Γ@i__ := G{c,l}→c@Sa,b→c[Γ@i, 0];
B@Γ[i_, L_] := SL→L[[1]]@Γ[i, L];
B@i_j :=
Module[{l = T@ij, r}, r = Quiet@SJoin[Reverse@l[[1], l[[2]] → "1" @ SJoin[Reverse@l[[3], l[[4]]] → "2" @ K@ij;

```



```

FullSimplify@If[r[[2]][[1, 1]] == Indeterminate,
  GJoin[Reverse@l[[3], l[[4]]] -> "2" @ GJoin[Reverse@l[[1], l[[2]]] -> "1" @ M@K@i_j, r]];
B@K_ := SUnion@Flatten[List@@#[2 ;;] &/@ (List@@PD[X K]) [2 ;;] -> "1" @ K;
A@0 = 0;
A@i_j_ := If[T@i_j == 0, 0, Module[{R = MΓ@B@i_j, p}, p = R[[1, 1]] /. T_ -> t;
  If[Length@R == 3 && Length@R[[1]] == 3 &&
    Apart[p t  $\frac{\text{Exponent}[p, \frac{1}{t}] - \text{Exponent}[p, t]}{2}$ ] == 1 && R[[2 ;;, 2 ;;]] ==  $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ , 1, 0]]];
A@K_ := Module[{x = If[MatrixQ@K, K, MΓ@B@K], p, q},
  p = If[Length@x == 2 && Length@x[[1]] == 2, x[[1]][[1]] /. T_ -> t, 0];
  q = Apart[p t  $\frac{\text{Exponent}[p, \frac{1}{t}] - \text{Exponent}[p, t]}{2}$ ];
  If[IntegerQ[q /. t -> 1] && PolynomialQ[q /. ti_Integer -> t, t], q, 0]];
J@0 = 0;
J@i_j_ := Module[{k = PD@K@i_j, e, l, v}, e = T[i_j];
  l = {e[[1]], Reverse@e[[2]], e[[3]], Reverse@e[[4]]};
  v = {Flatten[l][[1]], Last@Flatten@{l[[3 ;;], l[[ ; 2]]},
    Flatten[{l[[3 ;;], l[[ ; 2]]}[[1]], Last@Flatten@l];
  l = l /. {v[[1]] -> v[[3]], v[[3]] -> v[[1]]};
  J[k /. X_s_ -> If[Complement[{v[[2]], v[[3]]}, {s}] == {}, X_s /. v[[3]] -> v[[1]],
    If[Complement[{v[[1]], v[[4]]}, {s}] == {}, X_s /. v[[1]] -> v[[3]], X_s]] // X_s_ ->
  If[Complement[{Last@Flatten@{l[[2 ;;], l[[1]]}, Flatten[{l[[2 ;;], l[[1]]}[[1]]],
    {s}] == {}, X_s /. Flatten[{l[[2 ;;], l[[1]]}[[1]]] -> a, If[Complement[
    {Last@Flatten@{l[[2 ;;], l[[1]]}, Flatten[{l[[2 ;;], l[[1]]}[[1]]}, {s}] ==
    {Last@Flatten@{l[[2 ;;], l[[1]]}, X_s /. Flatten[{l[[2 ;;], l[[1]]}[[1]]] -> b,
  If[Complement[{Last@Flatten@{l[[4]], l[[ ; 3]]}, Flatten[{l[[4]], l[[ ; 3]]}[[1]]],
    {s}] == {}, X_s /. Flatten[{l[[4]], l[[ ; 3]]}[[1]] -> c, If[Complement[
    {Last@Flatten@{l[[4]], l[[ ; 3]]}, Flatten[{l[[4]], l[[ ; 3]]}[[1]]}, {s}] ==
    {Last@Flatten@{l[[4]], l[[ ; 3]]}, X_s /. Flatten[{l[[4]], l[[ ; 3]]}[[1]] -> d,
  X_s]]], Count[XS@k, Xp_] - Count[XS@k, Xm_]]];
J[K_, f___] := If[Length[X K] == 2, 1, Module[
  {k = If[f == BlankNullSequence, PD, Identity]@K, d = {}, r, x, v, p},
  r = Xi,j,k,l -> (d = Union[d, {i, j, k, l}];

$$\frac{P[i, j] P[k, l]}{\sqrt{t}} + \sqrt[4]{t} P[i, l] P[j, k]$$
);
  x =  $\frac{k(k[[1]] /. r)}{k[[1]}}$ ;
  Do[x *= (v = x[[1]] x[[1 + Ordering[Length@Complement[#, d] &/@
    List@@#[2 ;;] &/@ List@@(K@6_1[[2 ;;])][[1]]];

$$\frac{\text{Expand}[v /. r]}{v}$$
), {Length@x - 2}];

```

```

p = Apart@FullSimplify[ (t3/4 If[f===BlankNullSequence, Count[XS@K, Xp_] - Count[XS@K, Xm_], f]
Expand[x /. r] ) / If[f=== BlankNullSequence, sqrt(t) + 1/sqrt(t), 1]];
If[!(f=== BlankNullSequence) || IntegerQ[p /. {t -> 1, P@_ -> infinity}] &&
PolynomialQ[p /. ti-Integer -> t, t],
If[Sign@Coefficient[p /. P@_ -> 1, t, 0] >= 0, 1, -1] p, 0]];
V := Join[{"Knot Index", "Alexander Knot", "Jones Knot",
"Alexander Tangle", "Jones Tangle"}],
{#, A@K@# === KnotData[List@@#, "AlexanderPolynomial"]@t, J@K@# ===
KnotData[List@@#, "JonesPolynomial"]@t, A@# == 1, J@# == P[a, b] P[c, d]} & /@
{61, 88, 89, 820, 927, 941, 946, 103, 1022, 1035, 1042, 1048, 1075, 1087,
1099, 10123, 10129, 10137, 10140, 10153, 10155}]^T;

```

V

Knot Index	61	88	89	820	927	941	946	103	1022	1035	1042	1048	1
Alexander Knot	True	True	True	True	True	True	True	True	True	True	True	True	True
Jones Knot	True	True	True	True	True	True	True	True	True	True	True	True	True
Alexander Tangle	True	True	True	True	True	True	True	True	True	True	True	True	True
Jones Tangle	True	True	True	True	True	True	True	True	True	True	True	True	True