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$PrePrint = If[MatrixQ@#, MatrixForm@#, #] &;
SetAttributes[P, Orderless];
P /: P[i_, j_] P[j_, k_] := P[i, k];
P /: P[i_, i_] := -sqrt[t] - 1/sqrt[t];
P /: P[___]^2 := -sqrt[t] - 1/sqrt[t];
Gamma /: Gamma[omega1_, lambda1_, L1_] Gamma[omega2_, lambda2_, L2_] := Module[
  {S = Ordering@Join[L1, L2]}, Gamma[omega1 omega2, Join[Join[#, Table[0, {Length@lambda2}]] & /@ lambda1,
    Join[Table[0, {Length@lambda1}], #] & /@ lambda2][[S, S], Union[L1, L2]]];
MG@Gamma[omega_, lambda_, L_] := Prepend[Prepend[lambda^T, t# & /@ L]^T, Prepend[h# & /@ L, omega]];
Format@Gamma@i___ := MatrixForm@MG@Gamma@i;
K@_ = 0;
T@_ = 0;
PD@K_ := Module[{x = 2 Length[XK] - 2},
  XS@K /. {Xp_a_, b_ -> Xb, Mod[a, x] + 1, Mod[b, x] + 1, a, Xm_a_, b_ -> Xb, a, Mod[b, x] + 1, Mod[a, x] + 1};
XS@K_ := K /. {Xp@i___ -> Xp_i, Xm@i___ -> Xm_i, X@i___ -> X_i,
  Xi_, j_, k_, l_ -> If[j == l + 1 || j == 1 && l != 2 || k == l && j != k, Xp_l, i, Xm_j, i]};
Rr_a_, b_ := Gamma[1, (1 1 - T_a / 0 T_a), {a, b}];
Rl_a_, b_ := Rr_a_, b / . T_a -> 1/T_a;
M@Gamma@i___ := Gamma@i;
M@K_ := XS@K /. {Xp_i___ -> Rr_i, Xm_i___ -> Rl_i};
S_a_ -> c_ @K_ := S_{a} -> c @K;
S_{a} -> c_ @K_ := K / . T_a -> T_c;
S_a_, b_ -> c_ [Gamma[omega_, lambda_, L_], f___] :=
  Module[{i = FirstPosition[L, a][[1]], j = FirstPosition[L, b][[1]], n, o,
    d, alpha, beta, gamma, delta, theta, epsilon, phi, psi, xi}, n = Prepend[Delete[L, {{i}, {j}}], c];
  o = Ordering@n;
  d = Delete[Range@Length@L, {{i}, {j}}];
  (alpha beta theta / gamma delta epsilon / phi psi xi) = (lambda[[i, i]] 1/(1 - lambda[[i, j]]) lambda[[i, d]] / lambda[[j, i]] lambda[[j, j]] lambda[[j, d]] / lambda[[d, i]] lambda[[d, j]] lambda[[d, d]]);
  Gamma[Apart@omega/beta, If[f === BlankNullSequence, Apart, FullSimplify]@
    Join[{Join[{gamma + beta alpha delta}, epsilon + beta delta theta]}, Join[{phi + beta alpha psi},
      If[xi == {}, {xi}, (xi + beta {psi}^T . {theta})^T]]][[o, o], n[[o]]] / . T_a|b -> T_c];
S_a_, b_ -> c_ @K_ := S_a_, b_ -> c_ @M@K;
S_{a, b, l___} -> c_ @Gamma@i___ := S_{c, l} -> c @S_a_, b_ -> c [Gamma@i / . Table[T_j -> T_c, {j, {l}}]];
S_{a, b, l___} -> c_ @K_ := S_{a, b, l} -> c @M@K;
G_{c} -> c_ @K_ := K;
G_{a, b, l___} -> c_ @Gamma@i___ := G_{c, l} -> c @S_a_, b_ -> c [Gamma@i, 0];

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B@Γ[i_., L_] := SL→L[1]@Γ[i, L];
B@i_._ :=
Module[{l = T@ij, r}, r = Quiet@SJoin[Reverse@l[[1], l[[2]]] → "1" @ SJoin[Reverse@l[[3], l[[4]]] → "2" @ K@ij;
FullSimplify@If[r[[2]][[1, 1]] == Indeterminate,
GJoin[Reverse@l[[3], l[[4]]] → "2" @ GJoin[Reverse@l[[1], l[[2]]] → "1" @ M@K@ij, r]];
B@K_ := SUnion@Flatten[List@@#[[2, ;;]] &/@ (List@PD[X K] [[2, ;;]] → "1" @ K;
A@0 = 0;
A@i_._ := If[T@ij == 0, 0, Module[{R = MΓ@B@ij, p}, p = R[[1, 1]] /. T_ → t;
If[Length@R == 3 && Length@R[[1]] == 3 &&
Apart[p t Exponent[p, t] - Exponent[p, t] / 2] == 1 && R[[2, ;;, 2, ;;]] == (  $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ , 1, 0]]];
A@K_ := Module[{x = If[MatrixQ@K, K, MΓ@B@K], p, q},
p = If[Length@x == 2 && Length@x[[1]] == 2, x[[1]][[1]] /. T_ → t, 0];
q = Apart[p t Exponent[p, t] - Exponent[p, t] / 2];
If[IntegerQ[q /. t → 1] && PolynomialQ[q /. ti-Integer → t, t], q, 0]];
J@0 = 0;
J@i_._ := Module[{k = PD@K@ij, e, l, v}, e = T[ij];
l = {e[[1]], Reverse@e[[2]], e[[3]], Reverse@e[[4]]};
v = {Flatten[l] [[1]], Last@Flatten@{l[[3, ;;]], l[[ ; ; 2]]},
Flatten[{l[[3, ;;]], l[[ ; ; 2]]} [[1]], Last@Flatten@l};
l = l /. {v[[1]] → v[[3]], v[[3]] → v[[1]]};
J[k /. Xs → If[Complement[{v[[2]], v[[3]]}, {s}] == {}, Xs /. v[[3]] → v[[1]],
If[Complement[{v[[1]], v[[4]]}, {s}] == {}, Xs /. v[[1]] → v[[3]], Xs]] // . Xs →
If[Complement[{Last@Flatten@{l[[2, ;;]], l[[1]]}, Flatten[{l[[2, ;;]], l[[1]]} [[1]]},
{s}] == {}, Xs /. Flatten[{l[[2, ;;]], l[[1]]} [[1]] → a, If[Complement[
{Last@Flatten@{l[[2, ;;]], l[[1]]}, Flatten[{l[[2, ;;]], l[[1]]} [[1]]}, {s}] ==
{Last@Flatten@{l[[2, ;;]], l[[1]]}, Xs /. Flatten[{l[[2, ;;]], l[[1]]} [[1]] → b,
If[Complement[{Last@Flatten@{l[[4]], l[[ ; ; 3]]}, Flatten[{l[[4]], l[[ ; ; 3]]} [[1]]},
{s}] == {}, Xs /. Flatten[{l[[4]], l[[ ; ; 3]]} [[1]] → c, If[Complement[
{Last@Flatten@{l[[4]], l[[ ; ; 3]]}, Flatten[{l[[4]], l[[ ; ; 3]]} [[1]]}, {s}] ==
{Last@Flatten@{l[[4]], l[[ ; ; 3]]}, Xs /. Flatten[{l[[4]], l[[ ; ; 3]]} [[1]] → d,
Xs]]], Count[Xs@k, Xp_] - Count[Xs@k, Xm_]]];
J[K_, f_...] := If[Length[X K] == 2, 1, Module[
{k = If[f == BlankNullSequence, PD, Identity]@K, d = {}, r, x, v, p},
r = Xi, j, k, l → ( d = Union[d, {i, j, k, l}];

$$\frac{P[i, j] P[k, l]}{\sqrt{t}} + \sqrt[4]{t} P[i, l] P[j, k] );
x =  $\frac{k (k[[1]] /. r)}{k[[1]}}$ ;
Do[x *= ( v = x[[1]] x[[1 + Ordering[Length@Complement[#, d] &/@$$

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List@@#[2 ;;] & /@List@@(K@61[[2 ;;]])[[1]];
Expand[v /. r]
v
), {Length@x - 2}];
p = Apart@FullSimplify[(t^(3/4) If[f===BlankNullSequence, Count[XS@K, Xp_] - Count[XS@K, Xm_], f]
Expand[x /. r]) / If[f===BlankNullSequence, sqrt[t] + 1/sqrt[t], 1]];
If[!(f===BlankNullSequence) || IntegerQ[p /. {t -> 1, P@_ -> infinity}] &&
PolynomialQ[p /. t^i_Integer -> t, t],
If[Sign@Coefficient[p /. P@_ -> 1, t, 0] >= 0, 1, -1] p, 0]];
V := Join[{"Knot Index", "Alexander Knot", "Jones Knot",
"Alexander Tangle", "Jones Tangle"},
{#, A@K@# === KnotData[List@@#, "AlexanderPolynomial"]@t, J@K@# ===
KnotData[List@@#, "JonesPolynomial"]@t, A@# == 1, J@# == P[a, b] P[c, d]} & /@
{61, 88, 89, 820, 927, 941, 946, 103, 1022, 1035, 1042, 1048, 1075, 1087,
1099, 10123, 10129, 10137, 10140, 10153, 10155}]^T;

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V

	61	88	89	820	927	941	946	103	1022	1035	1042	1048	1
Knot Index													
Alexander Knot	True	True	True	True	True	True	True	True	True	True	True	True	True
Jones Knot	True	True	True	True	True	True	True	True	True	True	True	True	True
Alexander Tangle	True	True	True	True	True	True	True	True	True	True	True	True	True
Jones Tangle	True	True	True	True	True	True	True	True	True	True	True	True	True

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K@61 = X1,7,2,6 X2,9,3,10 X4,11,5,12 X7,1,8,14 X10,5,11,6 X12,3,13,4 X13,9,14,8;
T@61 = {{1, 2, 3, 4, 5, 6, 7}, {10, 9, 8}, {11, 12, 13, 14}, {}};
K@88 = X1,9,2,8 X3,1,4,18 X5,11,6,10 X6,13,7,14 X9,3,10,2 X12,15,13,16 X14,7,15,8 X16,11,17,12 X17,5,18,4;
T@88 = {{1, 2, 3}, {14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4}, {15, 16, 17, 18}, {}};
K@89 = X1,8,2,9 X3,11,4,10 X6,19,7,20 X9,16,10,17
X11,19,12,18 X12,6,13,5 X14,7,15,8 X15,3,16,2 X17,4,18,5 X20,14,1,13;
T@89 = {{1, 2, 3, 4}, {8, 7, 6, 5}, {9, 10, 11, 12}, {20, 19, 18, 17, 16, 15, 14, 13}};
K@820 = X1,6,2,7 X2,11,3,12 X5,16,6,1 X8,14,9,13 X10,3,11,4 X12,8,13,7 X14,10,15,9 X15,4,16,5;
T@820 = {{1, 2, 3, 4}, {12, 11, 10, 9, 8, 7, 6, 5}, {13, 14, 15, 16}, {}};
K@927 = X2,24,3,23 X5,20,6,21 X7,15,8,14 X8,1,9,2 X10,20,11,19
X11,5,12,4 X13,16,14,17 X15,7,16,6 X18,3,19,4 X21,13,22,12 X22,17,23,18 X24,9,1,10;
T@927 = {{1, 2, 3, 4, 5, 6, 7, 8, 9}, {14, 13, 12, 11, 10},
{15, 16, 17, 18, 19}, {24, 23, 22, 21, 20}};
K@941 = X2,17,3,18 X3,21,4,20 X6,13,7,14 X7,24,8,25 X9,19,10,18 X10,1,11,2 X12,24,13,23
X14,5,15,6 X16,27,17,28 X19,9,20,8 X21,27,22,26 X22,16,23,15 X25,4,26,5 X28,11,1,12;
T@941 = {{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}, {18, 17, 16, 15, 14, 13, 12},
{19, 20, 21, 22, 23}, {28, 27, 26, 25, 24}};
K@946 = X1,9,2,8 X2,13,3,14 X4,11,5,12 X6,15,7,16 X9,1,10,18 X12,3,13,4 X14,7,15,8 X16,5,17,6 X17,11,18,10;
T@946 = {{1, 2, 3, 4, 5, 6, 7, 8, 9}, {14, 13, 12, 11, 10}, {15, 16, 17, 18}, {}};
K@103 = X2,11,3,12 X3,16,4,17 X5,14,6,15 X8,22,9,21
X10,20,11,19 X13,6,14,7 X15,4,16,5 X17,13,18,12 X18,1,19,2 X20,10,21,9 X22,8,1,7;
T@103 = {{1, 2, 3, 4, 5, 6}, {12, 11, 10, 9, 8, 7},

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{13, 14, 15, 16, 17, 18, 19, 20, 21, 22}, {}};
K@1022 = X2,12,3,11 X5,18,6,19 X7,16,8,17 X9,5,10,4 X12,22,13,21 X14,2,15,1
X15,6,16,7 X17,8,18,9 X19,11,20,10 X20,3,21,4 X22,14,1,13;
T@1022 = {{1, 2, 3}, {6, 5, 4}, {7, 8, 9, 10, 11, 12, 13,
14, 15, 16, 17, 18, 19, 20}, {22, 21}}};
K@1035 = X2,11,3,12 X3,1,4,26 X5,25,6,24 X9,19,10,18 X10,1,11,2 X7,13,8,12 X14,21,15,22
X16,19,17,20 X17,9,18,8 X20,15,21,16 X22,13,23,14 X23,7,24,6 X25,5,26,4;
T@1035 = {{1, 2, 3, 4, 5}, {18, 17, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6},
{19, 20, 21, 22}, {26, 25, 24, 23}}};
K@1042 = X2,28,3,27 X4,9,5,10 X5,24,6,25 X8,18,9,17 X11,14,12,15 X13,22,14,23 X15,27,16,26
X16,3,17,4 X18,8,19,7 X20,2,21,1 X21,12,22,13 X23,6,24,7 X25,11,26,10 X28,20,1,19;
T@1042 = {{1, 2, 3, 4, 5, 6}, {12, 11, 10, 9, 8, 7}, {13, 14, 15, 16, 17, 18},
{28, 27, 26, 25, 24, 23, 22, 21, 20, 19}}};
K@1048 = X3,13,4,12 X4,22,5,21 X6,24,7,23 X8,13,9,14 X10,19,11,20 X11,3,12,2 X15,26,16,27
X17,28,18,1 X18,9,19,10 X20,2,21,1 X22,6,23,5 X24,8,25,7 X25,14,26,15 X27,16,28,17;
T@1048 = {{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14},
{20, 19, 18, 17, 16, 15}, {21, 22, 23, 24}, {28, 27, 26, 25}}};
K@1075 = X2,30,3,29 X4,9,5,10 X5,26,6,27 X8,18,9,17 X11,14,12,15 X13,21,14,20 X15,29,16,28
X16,3,17,4 X19,7,20,6 X21,13,22,12 X22,1,23,2 X24,7,25,8 X25,19,26,18 X27,11,28,10 X30,23,1,24;
T@1075 = {{1, 2, 3, 4, 5, 6}, {12, 11, 10, 9, 8, 7},
{13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23}, {30, 29, 28, 27, 26, 25, 24}}};
K@1087 = X3,17,4,16 X5,11,6,10 X6,19,7,20 X9,27,10,26 X12,29,13,30 X13,3,14,2 X15,24,16,25
X18,12,19,11 X20,7,21,8 X22,2,23,1 X23,14,24,15 X25,9,26,8 X27,4,28,5 X28,17,29,18 X30,22,1,21;
T@1087 = {{1, 2, 3, 4, 5, 6, 7}, {14, 13, 12, 11, 10, 9, 8},
{15, 16, 17, 18, 19, 20}, {30, 29, 28, 27, 26, 25, 24, 23, 22, 21}}};
K@1099 = X1,26,2,27 X3,12,4,13 X6,31,7,32 X10,20,11,19 X11,2,12,3 X13,8,14,9 X15,23,16,22 X18,10,19,9
X20,28,21,27 X21,17,22,16 X24,8,25,7 X25,4,26,5 X28,18,29,17 X29,14,30,15 X30,24,31,23 X32,5,1,6;
T@1099 = {{1, 2, 3, 4, 5}, {20, 19, 18, 17, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6},
{21, 22, 23, 24, 25, 26, 27, 28, 29, 30}, {32, 31}}};
K@10123 = X2,25,3,26 X3,23,4,22 X5,14,6,15 X6,19,7,20 X8,28,9,27 X9,19,10,18 X10,1,11,2 X11,24,12,25
X13,1,14,32 X16,29,17,30 X17,27,18,26 X20,16,21,15 X23,12,24,13 X28,8,29,7 X30,22,31,21 X31,4,32,5;
T@10123 = {{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13}, {18, 17, 16, 15, 14},
{19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32}, {}}};
K@10129 = X1,18,2,19 X3,11,4,10 X5,1,6,22 X7,13,8,12 X8,15,9,16 X11,5,12,4
X14,19,15,20 X16,9,17,10 X17,2,18,3 X20,13,21,14 X21,7,22,6;
T@10129 = {{1, 2, 3, 4, 5}, {18, 17, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6},
{19, 20, 21, 22}, {}}};
K@10137 = X2,20,3,19 X5,8,6,9 X7,13,8,12 X10,17,11,18 X11,4,12,5 X13,7,14,6
X14,1,15,2 X16,4,17,3 X18,9,19,10 X20,15,1,16;
T@10137 = {{1, 2, 3}, {6, 5, 4}, {7, 8, 9, 10, 11, 12, 13, 14, 15}, {20, 19, 18, 17, 16}}};
K@10140 = X1,8,2,9 X2,15,3,16 X4,13,5,14 X7,20,8,1
X10,18,11,17 X12,5,13,6 X14,3,15,4 X16,10,17,9 X18,12,19,11 X19,6,20,7;
T@10140 = {{1, 2, 3, 4, 5, 6}, {16, 15, 14, 13, 12, 11, 10, 9, 8, 7}, {17, 18, 19, 20}, {}}};

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$K@10_{153} = X_{2,22,3,21} X_{3,17,4,16} X_{6,2,7,1} X_{7,12,8,13}$
 $X_{10,19,11,20} X_{11,14,12,15} X_{13,8,14,9} X_{17,5,18,4} X_{18,9,19,10} X_{20,15,21,16} X_{22,6,1,5};$
 $T@10_{153} = \{\{1, 2, 3, 4\}, \{12, 11, 10, 9, 8, 7, 6, 5\},$
 $\{13, 14, 15, 16, 17\}, \{22, 21, 20, 19, 18\}\};$
 $K@10_{155} = X_{3,11,4,10} X_{6,19,7,20} X_{8,2,9,1} X_{9,16,10,17} X_{11,19,12,18}$
 $X_{12,6,13,5} X_{14,7,15,8} X_{15,3,16,2} X_{17,4,18,5} X_{20,14,1,13};$
 $T@10_{155} = \{\{1, 2, 3, 4\}, \{8, 7, 6, 5\}, \{9, 10, 11, 12\}, \{20, 19, 18, 17, 16, 15, 14, 13\}\};$