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$PrePrint = If[MatrixQ@#, MatrixForm@#, #] &;
MΓ@Γ[ω_, λ_, L_] := Prepend[Prepend[λT, t# & /@ L]T, Prepend[h# & /@ L, ω]];
Format@Γ@i__ := MatrixForm@MΓ@Γ@i;
Γ /: Γ[ω1_, λ1_, L1_] Γ[ω2_, λ2_, L2_] := Module[
  {S = Ordering@Join[L1, L2]}, Γ[ω1 ω2, Join[Join[#, Table[0, {Length@λ2}]] & /@ λ1,
    Join[Table[0, {Length@λ1}], #] & /@ λ2][[S, S], Union[L1, L2]]];
P /: P[i_, j_] P[j_, k_] := P[i, k];
P /: P[i_, j_] P[i_, k_] := P[j, k];
P /: P[i_, j_] P[k_, j_] := P[i, k];
P /: P[i_, i_] := -√t - 1/√t;
P /: P[i__]^2 := -√t - 1/√t;
Sa,b→c@K_ := S{a,b}→c@M@K;
S{a}→c@K_ := K /. a → c;
Sa,b→c@Γ[ω_, λ_, L_] :=
  Module[{i = FirstPosition[L, a][[1]], j = FirstPosition[L, b][[1]], n, o,
    d, α, β, γ, δ, θ, ε, φ, ψ, Ξ}, n = Prepend[Delete[L, {{i}, {j}}], c];
  o = Ordering@n;
  d = Delete[Table[k, {k, Length@L}], {{i}, {j}}];
  
$$\begin{pmatrix} \alpha & \beta & \theta \\ \gamma & \delta & \epsilon \\ \phi & \psi & \xi \end{pmatrix} = \begin{pmatrix} \lambda[[i, i]] & \frac{1}{1-\lambda[[i, j]]} & \lambda[[i, d]] \\ \lambda[[j, i]] & \lambda[[j, j]] & \lambda[[j, d]] \\ \lambda[[d, i]] & \lambda[[d, j]] & \lambda[[d, d]] \end{pmatrix};$$

  Γ[Apart[ω/β], Apart@Join[{Join[{γ + β α δ}, ε + β δ θ]},
    Join[{φ + β α ψ}, If[Ξ == {}, {}, (Ξ + β {ψ}^T . {θ})^T]}]]][[o, o], n[[o]]];
S{a,b,l}→c@Γ@i__ := S{c,l}→c@Sa,b→c@(Γ@i /. Table[Tj → Tc, {j, {a, b, l}}]);
Rra,b := Γ[1, (1 1 - Ta); {a, b}];
Rla,b := Rra,b /. Ta → 1/Ta;
M@K_ := XS@K /. {Xpi → Rri, Xmi → Rli};
PD@K_ := Module[{x = 2 Length[X K] - 2},
  XS@K /. {Xpa,b → Xb,Mod[a,x]+1,Mod[b,x]+1,a, Xma,b → Xb,a,Mod[b,x]+1,Mod[a,x]+1};
XS@K_ := K //. {Xp@i__ → Xpi, Xm@i__ → Xmi, X@i__ → Xi,
  Xi,j,k,l → If[(j == l + 1 || j == 1 && l ≠ 2 || k == l) && j ≠ k, Xpl,i, Xmj,i];
B@K_ := SUnion@Flatten[List@@#][2;] & /@ List@@(PD@K) → 1 @M@K;
B@Γ[i__, L_] := SL→1@Γ[i, L];
A@K_ := Module[{x = If[MatrixQ@K, K, MΓ@B@XS@K], p, q},
  p = If[Length@x == 2 && Length@x[[1]] == 2, x[[1]][[1]] /. T_ → t, 0];
  q = Apart[p t1/2 (Exponent[p, t] - Exponent[p, t])];
  If[IntegerQ[q /. t → 1] && PolynomialQ[q /. ti-Integer → t, t], q, 0];
A@0 = 0;

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J@K_ := If[Length[X K] == 2, 1,
Module[{k = PD@K, d = {}, r, x, v, p}, r = Xi,j,k,l => (d = Union[d, {i, j, k, l}];

$$\frac{P[i, j] P[k, l]}{\sqrt{t}} + \sqrt[4]{t} P[i, l] P[j, k]$$
);
x = (k[[1]] /. r)  $\frac{k}{k[[1]]}$ ;
Do[x *= (v = x[[1]] x[[1 + Ordering[Length@Complement[#, d] & /@
List@@#[2 ;;] & /@List@@(K@61[[2 ;;])] ][[1]]];

$$\frac{\text{Expand}[v /. r]}{v}$$
), {Length@x - 2}];
p = Apart[ $\frac{t^{\frac{3}{4}} (\text{Count}[X_{S@K}, X_{P\_}] - \text{Count}[X_{S@K}, X_{M\_}])}{\sqrt{t} + \frac{1}{\sqrt{t}}}$  Expand[x /. r]];
If[IntegerQ[p /. {t -> 1, P@_ -> ∞}] && PolynomialQ[p /. ti_Integer -> t, t],
If[Sign@Coefficient[p, t, 0] ≥ 0, 1, -1] p, 0]]];
J@0 = 0;
K@_ = 0;
V := Prepend[{#, KnotData[List@@#, "AlexanderPolynomial"]@t == A@K@#,
KnotData[List@@#, "JonesPolynomial"]@t == J@K@#} & /@
{61, 88, 89, 820, 927, 941, 946, 103, 1022, 1035, 1042, 1048, 1075, 1087, 1099,
10123, 10129, 10137, 10140, 10153, 10155}, {"", "Alexander", "Jones"}]†;
(*K@61=X1,7,2,6X4,11,5,12X7,1,8,14X9,3,10,2X10,5,11,6X12,3,13,4X13,9,14,8*)
K@61 = X1,7,2,6 X2,9,3,10 X4,11,5,12 X7,1,8,14 X10,5,11,6 X12,3,13,4 X13,9,14,8;
K@88 = X1,9,2,8 X3,1,4,18 X5,11,6,10 X6,13,7,14 X9,3,10,2 X12,15,13,16 X14,7,15,8 X16,11,17,12 X17,5,18,4;
K@89 = X1,8,2,9 X3,11,4,10 X6,19,7,20 X9,16,10,17
X11,19,12,18 X12,6,13,5 X14,7,15,8 X15,3,16,2 X17,4,18,5 X20,14,1,13;
(*K@820=X1,6,2,7X3,11,4,10X5,16,6,1X8,14,9,13X11,3,12,2X12,8,13,7X14,10,15,9X15,4,16,5*)
K@820 = X1,6,2,7 X2,11,3,12 X5,16,6,1 X8,14,9,13 X10,3,11,4 X12,8,13,7 X14,10,15,9 X15,4,16,5;
K@927 = X2,24,3,23 X5,20,6,21 X7,15,8,14 X8,1,9,2 X10,20,11,19
X11,5,12,4 X13,16,14,17 X15,7,16,6 X18,3,19,4 X21,13,22,12 X22,17,23,18 X24,9,1,10;
K@941 = X2,17,3,18 X3,21,4,20 X6,13,7,14 X7,24,8,25 X9,19,10,18 X10,1,11,2 X12,24,13,23
X14,5,15,6 X16,27,17,28 X19,9,20,8 X21,27,22,26 X22,16,23,15 X25,4,26,5 X28,11,1,12;
(*K@946=X1,9,2,8X3,13,4,12X6,15,7,16X9,1,10,18X11,5,12,4X13,3,14,2X14,7,15,8X16,5,17,6X17,11,18,10*)
K@946 = X1,9,2,8 X2,13,3,14 X4,11,5,12 X6,15,7,16 X9,1,10,18 X12,3,13,4 X14,7,15,8 X16,5,17,6 X17,11,18,10;
K@103 = X2,11,3,12 X3,16,4,17 X5,14,6,15 X8,22,9,21
X10,20,11,19 X13,6,14,7 X15,4,16,5 X17,13,18,12 X18,1,19,2 X20,10,21,9 X22,8,1,7;
(*K@1022=X2,12,3,11X5,18,6,19X7,16,8,17X9,5,10,4X10,19,11,20X12,22,13,21
X14,2,15,1X15,6,16,7X17,8,18,9X20,3,21,4X22,14,1,13*)
K@1022 = X2,12,3,11 X5,18,6,19 X7,16,8,17 X9,5,10,4 X12,22,13,21 X14,2,15,1
X15,6,16,7 X17,8,18,9 X19,11,20,10 X20,3,21,4 X22,14,1,13;
(*K@1035=X1,6,2,7X3,11,4,10X4,19,5,20X7,14,8,15X9,12,10,13X11,3,12,2

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$X_{13,8,14,9} X_{16,22,17,21} X_{18,5,19,6} X_{20,18,21,17} X_{22,16,1,15} ; *$
K@1035 = $X_3,11,4,10 X_4,19,5,20 X_6,2,7,1 X_7,14,8,15 X_9,12,10,13 X_{11,3,12,2}$
 $X_{13,8,14,9} X_{16,22,17,21} X_{18,5,19,6} X_{20,18,21,17} X_{22,16,1,15} ;$
K@1042 = $X_2,28,3,27 X_4,9,5,10 X_5,24,6,25 X_8,18,9,17 X_{11,14,12,15} X_{13,22,14,23} X_{15,27,16,26}$
 $X_{16,3,17,4} X_{18,8,19,7} X_{20,2,21,1} X_{21,12,22,13} X_{23,6,24,7} X_{25,11,26,10} X_{28,20,1,19} ;$
K@1048 = $X_1,6,2,7 X_3,11,4,10 X_4,21,5,22 X_7,17,8,16 X_9,19,10,18 X_{12,6,13,5}$
 $X_{14,23,15,24} X_{17,9,18,8} X_{19,2,20,3} X_{20,12,21,11} X_{22,13,23,14} X_{24,15,1,16} ;$
K@1075 = $X_2,30,3,29 X_4,9,5,10 X_5,26,6,27 X_8,18,9,17 X_{11,14,12,15} X_{13,21,14,20} X_{15,29,16,28}$
 $X_{16,3,17,4} X_{19,7,20,6} X_{21,13,22,12} X_{22,1,23,2} X_{24,7,25,8} X_{25,19,26,18} X_{27,11,28,10} X_{30,23,1,24} ;$
 $(* K@1087 = X_2,13,3,14 X_3,17,4,16 X_6,19,7,20 X_9,27,10,26 X_{10,5,11,6} X_{12,29,13,30} X_{15,24,16,25}$
 $X_{17,29,18,28} X_{18,12,19,11} X_{20,7,21,8} X_{22,2,23,1} X_{23,14,24,15} X_{25,9,26,8} X_{27,4,28,5} X_{30,22,1,21} ; *)$
K@1087 = $X_3,17,4,16 X_5,11,6,10 X_6,19,7,20 X_9,27,10,26 X_{12,29,13,30} X_{13,3,14,2} X_{15,24,16,25}$
 $X_{18,12,19,11} X_{20,7,21,8} X_{22,2,23,1} X_{23,14,24,15} X_{25,9,26,8} X_{27,4,28,5} X_{28,17,29,18} X_{30,22,1,21} ;$
K@1099 = $X_2,10,3,9 X_5,26,6,27 X_6,2,7,1 X_8,14,9,13 X_{10,4,11,3} X_{12,20,13,19} X_{14,8,15,7}$
 $X_{15,20,16,21} X_{17,24,18,25} X_{18,12,19,11} X_{21,28,22,1} X_{23,16,24,17} X_{25,4,26,5} X_{27,22,28,23} ;$
K@10123 = $X_2,20,3,19 X_4,11,5,12 X_5,16,6,17 X_7,15,8,14 X_8,1,9,2 X_{10,28,11,27} X_{13,24,14,25}$
 $X_{15,23,16,22} X_{17,13,18,12} X_{20,9,21,10} X_{23,7,24,6} X_{25,19,26,18} X_{26,3,27,4} X_{28,21,1,22} ;$
K@10129 = $X_1,18,2,19 X_3,11,4,10 X_5,1,6,22 X_7,13,8,12 X_8,15,9,16 X_{11,5,12,4}$
 $X_{14,19,15,20} X_{16,9,17,10} X_{17,2,18,3} X_{20,13,21,14} X_{21,7,22,6} ;$
 $(* K@10137 = X_2,20,3,19 X_5,8,6,9 X_7,13,8,12 X_9,19,10,18 X_{11,4,12,5} X_{13,7,14,6}$
 $X_{14,1,15,2} X_{16,4,17,3} X_{17,11,18,10} X_{20,15,1,16} ; *)$
K@10137 = $X_2,20,3,19 X_5,8,6,9 X_7,13,8,12 X_{10,17,11,18} X_{11,4,12,5} X_{13,7,14,6}$
 $X_{14,1,15,2} X_{16,4,17,3} X_{18,9,19,10} X_{20,15,1,16} ;$
 $(* K@10140 = X_1,8,2,9 X_3,15,4,14 X_5,13,6,12 X_7,20,8,1 X_{10,18,11,17} X_{13,5,14,4}$
 $X_{15,3,16,2} X_{16,10,17,9} X_{18,12,19,11} X_{19,6,20,7} ; *)$
K@10140 = $X_1,8,2,9 X_2,15,3,16 X_4,13,5,14 X_7,20,8,1 X_{10,18,11,17} X_{12,5,13,6}$
 $X_{14,3,15,4} X_{16,10,17,9} X_{18,12,19,11} X_{19,6,20,7} ;$
K@10153 = $X_2,22,3,21 X_3,17,4,16 X_6,2,7,1 X_7,12,8,13 X_{10,19,11,20} X_{11,14,12,15}$
 $X_{13,8,14,9} X_{17,5,18,4} X_{18,9,19,10} X_{20,15,21,16} X_{22,6,1,5} ;$
K@10155 = $X_3,11,4,10 X_6,19,7,20 X_8,2,9,1 X_9,16,10,17 X_{11,19,12,18}$
 $X_{12,6,13,5} X_{14,7,15,8} X_{15,3,16,2} X_{17,4,18,5} X_{20,14,1,13} ;$