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$PrePrint = If[MatrixQ@#, MatrixForm@#, #] &;
MΓ@Γ[ω_, λ_, L_] := Prepend[Prepend[λT, t# & /@ L]T, Prepend[h# & /@ L, ω]];
Format@Γ@i__ := MatrixForm@MΓ@Γ@i;
Γ /: Γ[ω1_, λ1_, L1_] Γ[ω2_, λ2_, L2_] := Module[
  {S = Ordering@Join[L1, L2]}, Γ[ω1 ω2, Join[Join[#, Table[0, {Length@λ2}]] & /@ λ1,
    Join[Table[0, {Length@λ1}], #] & /@ λ2][[S]]T[[S]]T, Union[L1, L2]];
P /: P[i_, j_] P[j_, k_] := P[i, k];
P /: P[i_, j_] P[i_, k_] := P[j, k];
P /: P[i_, j_] P[k_, j_] := P[i, k];
P /: P[i_, i_] := -t1/2 -  $\frac{1}{t^{1/2}}$ ;
P /: P[i__]2 := -t1/2 -  $\frac{1}{t^{1/2}}$ ;
Sa,b→c@K_ := S{a,b}→c@K;
S{a__}→c@K_ := If[Length@List@a == 1, K, MS{a}→c@M@K];
Ra,b→c@Γ[ω_, λ_, L_] :=
  Module[{i = FirstPosition[L, a][[1]], j = FirstPosition[L, b][[1]]},
    Module[{S = Join[{c}, Delete[L, {{i}, {j}}]}, α = λ[[i]][[i]], β = 1 - λ[[i]][[j]], γ = λ[[j]][[i]],
      δ = λ[[j]][[j]], θ = Delete[λ[[i]], {{i}, {j}}], ε = Delete[λ[[j]], {{i}, {j}}],
      φ = Delete[λT[[i]], {{i}, {j}}], ψ = Delete[λT[[j]], {{i}, {j}}],
      Ξ = If[Length@L == 2, {}, Delete[Delete[λ, {{i}, {j}}]T, {{i}, {j}}]T],
      Γ[Apart[β ω], Apart@Join[{Join[{γ +  $\frac{\alpha \delta}{\beta}$ ], ε +  $\frac{\delta \theta}{\beta}}$ ], Join[{φ +  $\frac{\alpha \psi}{\beta}$ ], If[Ξ == {},
        {}, (Ξ +  $\frac{\{\psi\}^T \cdot \{\theta\}}{\beta}$ )T]}]T][[Ordering@S]]T[[Ordering@S]]T, Union@S]]];
MS{a__}→c@Γ@i__ := Module[{x = (Γ@i /. Table[Tj → Tc, {j, {a}}]), j},
  For[j = 1, j < Length@{a}, j++, x = R{a}[[j]], {a}[[j+1]] => If[j+1 == Length@{a}, c, {a}[[j+1]]]@x];
  x];
MRra,b := Γ[1, ( $\begin{matrix} 1 & 1 - T_a \\ 0 & T_a \end{matrix}$ ), {a, b}];
MRla,b := MRra,b /. Ta →  $\frac{1}{T_a}$ ;
M@K_ := ToR@K /. {Rri → MRri, Rli → MRli};
ToX@K_ := Module[{x = If[Count[ToR@K, (Rr | Rl)_] == 0, 2, 2 Length@K]},
  ToR@K /. {Rra,b → Xa,Mod[b,x]+1,Mod[a,x]+1,b, Rla,b → Xa,b,Mod[a,x]+1,Mod[b,x]+1};
ToR@K_ := K /. {Rr@i__ → Rri, Rl@i__ → Rli, X@i__ → Xi,
  Xi,j,k,l → If[(j == l + 1 || j == 1 && l ≠ 2 || k == l) && j ≠ k, Rri,l, Rli,j];
MA@K_ := Module[{q = Apart@Module[
  {p = Module[{x = If[MatrixQ@K, K, MΓ@K]}, If[Length@x == 2 && Length@x[[1]] == 2,
    x[[1]][[1]] /. T_ → t, 0]}], p t $\frac{\text{Exponent}[p, t] - \text{Exponent}[p, t]}{2}$ ]}],
  If[IntegerQ[q /. t → 1] && PolynomialQ[q /. ti_Integer → t, t],
    q If[Sign@Coefficient[q, t, 0] ≥ 0, 1, -1], 0]];

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B@K_ := SModule[{V={}}, ToR@K/. (Rr|Rl)_a_ -> (V=Union[V, {a}];); V] -> 1 @K;
B@Gamma[i_, L_] := S_{L-1} @Gamma[i, L];
A@K_ := MA@B@ToR@K;
A@X_ := 1;
A@X[_] := 1;
A@Gamma@i_ := MA@B@Gamma@i;
A@0 := 0;

J@K_ := Module[{p = Apart[

$$\frac{t^{\frac{3}{4}(\text{Count}[\text{ToR@K}, \text{Rr}_-] - \text{Count}[\text{ToR@K}, \text{Rl}_-])}{t^{\frac{1}{2} + \frac{1}{\frac{1}{t^2}}}}$$

}],
Module[{V={}}, Module[{k = ToX@K, r = X_{i,j,k,l} -> (V = Union[V, {i, j, k, l}];

$$\frac{P[i, j] P[k, l]}{t^{\frac{1}{4}}} + t^{\frac{1}{4}} P[i, l] P[j, k]$$

)], Module[{x = (k[[1]] /. r)  $\frac{k}{k[[1]]}$ },
Do[{x = Module[{v = x[[1]] x[[1 + Ordering[Length[Complement[#, V]]] & /@
Module[{S={}], AppendTo[S, List@@#[[2] ;;]]; & /@ x[[2] ;;]];
S]][[1]]}], Expand[v /. r]  $\frac{x}{v}$ ], {Length@x - 2}];
Expand[x /. r]]]]], If[IntegerQ[p /. {t -> 1, P@_ -> Infinity}] &&
PolynomialQ[p /. t^{i_Integer} -> t, t], If[Sign@Coefficient[p, t, 0] >= 0, 1, -1] p /.
t -> t^{If[Sign@Differences[Module[{i=0}, While[!PolynomialQ[Apart[#t^i], t], i++]; i] & /@ {p, p /. t -> 1/t}][[1]] >= 0, 1, -1]
0}];
J@X_ := 1;
J@X[_] := 1;
J@0 := 0;
K@_ := 0;
V :=
Prepend[Module[{R = {61, 88, 89, 820, (*927, *)941, 946, 103, 1022, 1035, (*1042, *)
1048, (*1075, *)1087, (*1099, *)10123, 10129, 10137, 10140, 10153, 10155}},
{#, KnotData[List@@#, "AlexanderPolynomial"]@t === A@K@#,
MemberQ[Table[KnotData[List@@#, "JonesPolynomial"]@i, {i, {t, 1/t}}]},
J@K@#}] & /@ R], {"", "Alexander", "Jones"}]^T;

K@61 = X_{1,8,2,9} X_{4,16,5,15} X_{5,2,6,3} X_{6,12,7,11} X_{9,16,10,1} X_{12,8,13,7} X_{13,10,14,11} X_{14,4,15,3};
K@88 = X_{1,14,2,15} X_{3,26,4,27} X_{6,16,7,15} X_{8,30,9,29} X_{9,20,10,21} X_{10,6,11,5} X_{11,2,12,3} X_{12,24,13,23}
X_{17,30,18,31} X_{19,16,20,17} X_{22,26,23,25} X_{24,14,25,13} X_{27,4,28,5} X_{28,22,29,21} X_{31,18,32,19} X_{32,8,1,7};
K@89 = X_{1,27,2,26} X_{2,12,3,11} X_{4,27,5,28} X_{7,16,8,17} X_{8,31,9,32} X_{12,4,13,3} X_{13,24,14,25} X_{14,22,15,21} X_{15,6,16,7}
X_{19,33,20,32} X_{20,18,21,17} X_{23,29,24,28} X_{25,10,26,11} X_{29,23,30,22} X_{30,5,31,6} X_{33,19,34,18} X_{34,9,1,10};
K@820 = X_{2,15,3,16} X_{3,10,4,11} X_{5,18,6,19} X_{7,4,8,5} X_{8,14,9,13} X_{12,18,13,17}
X_{14,10,15,9} X_{16,1,17,2} X_{19,6,20,7} X_{20,12,1,11};
(*K@927=X_{5,21,6,20} X_{7,4,8,5} X_{10,29,11,30} X_{12,48,13,47} X_{13,27,14,26} X_{16,2,17,1} X_{17,31,18,30} X_{18,41,19,42}

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$X_{19,7,20,6} X_{22,9,23,10} X_{23,39,24,38} X_{24,33,25,34} X_{25,15,26,14} X_{31,2,32,3} X_{34,27,35,28} X_{35,48,36,49} X_{37,29,38,28}$   
 $X_{40,4,41,3} X_{42,21,43,22} X_{43,8,44,9} X_{44,40,45,39} X_{45,32,46,33} X_{46,16,47,15} X_{49,36,50,37} X_{50,12,1,11} ; *$   
 $(*K@927=X_{1,14,2,15} X_{3,21,4,20} X_{5,10,6,11} X_{8,13,9,14} X_{9,17,10,16} X_{11,6,12,7}$   
 $X_{12,17,13,18} X_{15,2,16,3} X_{18,8,19,7} X_{19,1,20,22} X_{21,5,22,4} ; *)$   
 $K@941 = X_{2,18,3,17} X_{3,42,4,43} X_{6,14,7,13} X_{8,40,9,39} X_{9,20,10,21} X_{11,38,12,39} X_{15,32,16,33} X_{16,6,17,5}$   
 $X_{21,10,22,11} X_{22,28,23,27} X_{23,36,24,37} X_{24,2,25,1} X_{26,38,27,37} X_{28,20,29,19} X_{29,40,30,41} X_{31,14,32,15}$   
 $X_{34,42,35,41} X_{35,18,36,19} X_{43,4,44,5} X_{44,34,45,33} X_{45,30,46,31} X_{46,8,47,7} X_{47,12,48,13} X_{48,26,1,25} ;$   
 $K@946 = X_{3,18,4,19} X_{4,8,5,7} X_{5,12,6,13} X_{8,2,9,1} X_{10,15,11,16} X_{13,6,14,7}$   
 $X_{14,20,15,19} X_{16,9,17,10} X_{17,2,18,3} X_{20,12,1,11} ;$   
 $K@103 = X_{2,31,3,32} X_{4,29,5,30} X_{5,20,6,21} X_{8,36,9,35} X_{9,16,10,17} X_{12,40,13,39} X_{13,10,14,11}$   
 $X_{14,24,15,23} X_{17,6,18,7} X_{18,28,19,27} X_{21,40,22,1} X_{24,16,25,15} X_{25,36,26,37}$   
 $X_{28,20,29,19} X_{30,3,31,4} X_{32,1,33,2} X_{33,26,34,27} X_{34,8,35,7} X_{37,22,38,23} X_{38,12,39,11} ;$   
 $K@1022 = X_{1,24,2,25} X_{2,46,3,45} X_{9,20,10,21} X_{10,50,11,49} X_{11,28,12,29} X_{12,42,13,41} X_{16,6,17,5}$   
 $X_{18,32,19,31} X_{19,8,20,9} X_{21,30,22,31} X_{23,4,24,5} X_{26,40,27,39} X_{27,14,28,15} X_{33,52,34,53}$   
 $X_{34,18,35,17} X_{35,22,36,23} X_{36,48,37,47} X_{37,44,38,45} X_{38,26,39,25} X_{42,14,43,13}$   
 $X_{43,40,44,41} X_{46,4,47,3} X_{48,30,49,29} X_{50,8,51,7} X_{51,32,52,33} X_{53,6,54,7} X_{54,16,1,15} ;$   
 $K@1035 = X_{3,24,4,25} X_{4,38,5,37} X_{7,42,8,43} X_{8,20,9,19} X_{9,16,10,17} X_{10,46,11,45} X_{14,30,15,29} X_{15,12,16,13}$   
 $X_{17,32,18,33} X_{20,2,21,1} X_{22,36,23,35} X_{23,6,24,7} X_{25,40,26,41} X_{26,22,27,21} X_{27,48,28,1} X_{28,14,29,13}$   
 $X_{33,18,34,19} X_{34,44,35,43} X_{38,6,39,5} X_{39,36,40,37} X_{41,2,42,3} X_{44,32,45,31} X_{46,12,47,11} X_{47,30,48,31} ;$   
 $(*K@1042=X_{1,16,2,17} X_{2,50,3,49} X_{3,63,4,62} X_{5,61,6,60} X_{7,59,8,58} X_{9,36,10,37} X_{12,66,13,65} X_{14,30,15,29}$   
 $X_{15,68,16,69} X_{17,38,18,39} X_{19,70,20,71} X_{22,40,23,39} X_{24,74,25,73} X_{25,44,26,45} X_{26,22,27,21}$   
 $X_{27,18,28,19} X_{28,48,29,47} X_{31,52,32,53} X_{32,14,33,13} X_{33,10,34,11} X_{34,56,35,55} X_{41,74,42,75} X_{43,40,44,41}$   
 $X_{46,70,47,69} X_{48,38,49,37} X_{50,68,51,67} X_{51,30,52,31} X_{53,66,54,67} X_{56,36,57,35} X_{57,9,58,8} X_{59,7,60,6}$   
 $X_{61,5,62,4} X_{63,54,64,55} X_{64,12,65,11} X_{71,20,72,21} X_{72,46,73,45} X_{75,42,76,43} X_{76,24,1,23} ; *)$   
 $(*K@1042=X_{1,19,2,18} X_{4,24,5,23} X_{6,22,7,21} X_{7,10,8,11} X_{9,17,10,16} X_{12,3,13,4} X_{13,1,14,28}$   
 $X_{15,26,16,27} X_{17,9,18,8} X_{19,3,20,2} X_{20,12,21,11} X_{22,6,23,5} X_{25,14,26,15} X_{27,24,28,25} ; *)$   
 $K@1048 = X_{2,22,3,21} X_{5,1,6,50} X_{7,16,8,17} X_{8,47,9,48} X_{11,14,12,15} X_{12,45,13,46} X_{18,34,19,33}$   
 $X_{19,6,20,7} X_{22,4,23,3} X_{23,36,24,37} X_{24,30,25,29} X_{25,10,26,11} X_{27,47,28,46}$   
 $X_{28,16,29,15} X_{31,49,32,48} X_{32,18,33,17} X_{34,49,35,50} X_{37,20,38,21} X_{38,1,39,2}$   
 $X_{39,5,40,4} X_{40,35,41,36} X_{41,31,42,30} X_{42,9,43,10} X_{43,27,44,26} X_{44,13,45,14} ;$   
 $(*K@1075=X_{2,30,3,29} X_{4,52,5,51} X_{5,28,6,29} X_{8,56,9,55} X_{10,22,11,21} X_{11,58,12,59} X_{13,20,14,21} X_{16,64,17,63}$   
 $X_{17,14,18,15} X_{18,36,19,35} X_{23,40,24,41} X_{24,10,25,9} X_{25,6,26,7} X_{26,44,27,43} X_{31,48,32,49} X_{32,2,33,1}$   
 $X_{33,64,34,1} X_{36,20,37,19} X_{38,58,39,57} X_{39,22,40,23} X_{41,56,42,57} X_{44,28,45,27} X_{45,52,46,53} X_{47,30,48,31}$   
 $X_{49,46,50,47} X_{50,4,51,3} X_{53,42,54,43} X_{54,8,55,7} X_{59,12,60,13} X_{60,38,61,37} X_{61,34,62,35} X_{62,16,63,15} ; *)$   
 $(*K@1075=X_{2,24,3,23} X_{4,30,5,29} X_{5,34,6,35} X_{7,37,8,36} X_{8,16,9,15} X_{10,31,11,32} X_{11,27,12,26}$   
 $X_{13,20,14,21} X_{16,34,17,33} X_{17,30,18,31} X_{19,24,20,25} X_{21,14,22,15} X_{22,2,23,1}$   
 $X_{25,13,26,12} X_{27,18,28,19} X_{28,4,29,3} X_{32,9,33,10} X_{35,38,36,1} X_{37,7,38,6} ; *)$   
 $K@1087 = X_{1,55,2,54} X_{2,25,3,26} X_{4,59,5,60} X_{6,35,7,36} X_{9,59,10,58} X_{11,25,12,24} X_{12,55,13,56}$   
 $X_{15,31,16,30} X_{17,1,18,70} X_{18,27,19,28} X_{19,53,20,52} X_{20,3,21,4} X_{21,11,22,10} X_{22,45,23,46}$   
 $X_{31,15,32,14} X_{32,41,33,42} X_{33,49,34,48} X_{34,7,35,8} X_{37,29,38,28} X_{38,69,39,70} X_{40,29,41,30}$   
 $X_{43,57,44,56} X_{44,23,45,24} X_{46,57,47,58} X_{49,37,50,36} X_{51,61,52,60} X_{53,27,54,26} X_{61,51,62,50}$   
 $X_{62,5,63,6} X_{63,9,64,8} X_{64,47,65,48} X_{65,43,66,42} X_{66,13,67,14} X_{67,17,68,16} X_{68,39,69,40} ;$   
 $(*K@1099=X_{1,26,2,27} X_{3,30,4,31} X_{4,24,5,23} X_{5,58,6,59} X_{7,22,8,23} X_{10,72,11,71} X_{12,38,13,37}$

$X_{13,66,14,67} X_{15,36,16,37} X_{17,70,18,71} X_{19,8,20,9} X_{20,50,21,49} X_{27,2,28,3} X_{28,56,29,55} X_{31,46,32,47}$   
 $X_{32,12,33,11} X_{33,16,34,17} X_{34,42,35,41} X_{40,70,41,69} X_{42,36,43,35} X_{44,66,45,65} X_{45,38,46,39}$   
 $X_{47,72,48,1} X_{50,22,51,21} X_{52,58,53,57} X_{53,24,54,25} X_{54,30,55,29} X_{56,26,57,25} X_{59,6,60,7}$   
 $X_{60,52,61,51} X_{61,48,62,49} X_{62,10,63,9} X_{63,18,64,19} X_{64,40,65,39} X_{67,14,68,15} X_{68,44,69,43} ; *$   
 $( *K@1099 = X_{3,16,4,17} X_{4,28,5,27} X_{6,26,7,25} X_{10,19,11,20} X_{12,31,13,32} X_{13,22,14,23} X_{15,2,16,3} X_{17,14,18,15}$   
 $X_{18,30,19,29} X_{20,11,21,12} X_{23,8,24,9} X_{24,6,25,5} X_{26,8,27,7} X_{28,2,29,1} X_{30,22,31,21} X_{32,9,1,10} ; * )$   
**K@10123 =**  $X_{2,71,3,72} X_{3,28,4,29} X_{8,80,9,79} X_{10,5,11,6} X_{11,69,12,68} X_{12,46,13,45} X_{14,82,15,81}$   
 $X_{15,40,16,41} X_{17,88,18,89} X_{20,30,21,29} X_{22,86,23,85} X_{24,38,25,37} X_{25,84,26,85} X_{31,54,32,55} X_{32,20,33,19}$   
 $X_{33,16,34,17} X_{34,58,35,57} X_{35,50,36,51} X_{36,24,37,23} X_{41,64,42,65} X_{42,10,43,9} X_{43,6,44,7} X_{44,68,45,67}$   
 $X_{48,84,49,83} X_{49,38,50,39} X_{51,86,52,87} X_{53,30,54,31} X_{56,88,57,87} X_{58,40,59,39} X_{59,82,60,83}$   
 $X_{61,46,62,47} X_{62,69,63,70} X_{63,5,64,4} X_{65,80,66,81} X_{70,28,71,27} X_{72,1,73,2} X_{73,26,74,27} X_{74,48,75,47}$   
 $X_{75,60,76,61} X_{76,14,77,13} X_{77,66,78,67} X_{78,8,79,7} X_{89,18,90,19} X_{90,56,91,55} X_{91,52,92,53} X_{92,22,1,21} ;$   
**K@10129 =**  $X_{2,27,3,28} X_{3,18,4,19} X_{6,32,7,31} X_{7,14,8,15} X_{9,34,10,35} X_{11,8,12,9}$   
 $X_{12,22,13,21} X_{15,4,16,5} X_{16,26,17,25} X_{20,34,21,33} X_{22,14,23,13} X_{23,32,24,33}$   
 $X_{26,18,27,17} X_{28,1,29,2} X_{29,24,30,25} X_{30,6,31,5} X_{35,10,36,11} X_{36,20,1,19} ;$   
**K@10137 =**  $X_{2,27,3,28} X_{3,18,4,19} X_{6,32,7,31} X_{8,12,9,11} X_{9,34,10,35} X_{13,22,14,23}$   
 $X_{14,8,15,7} X_{15,4,16,5} X_{16,26,17,25} X_{20,34,21,33} X_{21,12,22,13} X_{23,32,24,33}$   
 $X_{26,18,27,17} X_{28,1,29,2} X_{29,24,30,25} X_{30,6,31,5} X_{35,10,36,11} X_{36,20,1,19} ;$   
**K@10140 =**  $X_{1,19,2,18} X_{3,17,4,16} X_{4,10,5,9} X_{7,1,8,22} X_{10,6,11,5} X_{11,14,12,15}$   
 $X_{12,21,13,22} X_{15,8,16,9} X_{17,3,18,2} X_{19,7,20,6} X_{20,13,21,14} ;$   
**K@10153 =**  $X_{1,14,2,15} X_{3,10,4,11} X_{5,22,6,23} X_{7,4,8,5} X_{8,18,9,17} X_{11,2,12,3}$   
 $X_{12,20,13,19} X_{16,22,17,21} X_{18,10,19,9} X_{20,14,21,13} X_{23,6,24,7} X_{24,16,1,15} ;$   
**K@10155 =**  $X_{1,29,2,28} X_{3,16,4,17} X_{6,32,7,31} X_{8,18,9,17} X_{10,34,11,33} X_{11,22,12,23}$   
 $X_{12,8,13,7} X_{13,4,14,5} X_{14,26,15,25} X_{19,34,20,35} X_{21,18,22,19} X_{23,32,24,33}$   
 $X_{26,16,27,15} X_{27,3,28,2} X_{29,24,30,25} X_{30,6,31,5} X_{35,20,36,21} X_{36,10,1,9} ;$