In[∘]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\SL2Portfolio2"];
<< KnotTheory`;
<< "../Profile/Profile.m";
<< "Engine-Speedy.m";
<< "Objects.m";
<< "KT.m";
BeginProfile[];
```

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
This is Profile.m of http://www.drorbn.net/AcademicPensieve/Projects/Profile/.
This version: June 2018. Original version: July 1994.
```

» Warning: On Sep 4 2019 I swapped the operations ϵ and η. Some incompatibilities may arise in older notebooks.

In[∘]:=
```
$k = 0; ℏ = 1; γ = 1;
```

In[∘]:=

$$\mathrm{tr}_{k\_} := \mathbb{E}_{\{k\}\to\{\}}\left[\beta_k\, b_k,\ \frac{\mathcal{A}_k\,(1-\mathrm{B}_k)}{-1+\mathcal{A}_k}\,\xi_k\,\eta_k\,/\,\hbar,\ 1\right];$$

$$\mathrm{Simplify}\left[\left(dm_{i,j\to k}\ //\ \mathrm{tr}_k\right) \equiv \left(dm_{j,i\to k}\ //\ \mathrm{tr}_k\right)\right]$$

Out[∘]= True

In[∘]:=
```
Xp_{a_,b_} := Xp[a, b]; Xm_{a_,b_} := Xm[a, b];
```

In[∘]:=
```
SXForm[L_] := SXForm[
   Skeleton[L],
   Times @@ PD[L] /.
    X[i_, j_, k_, l_] :→ If[PositiveQ[X[i, j, k, l]], Xp[l, i], Xm[j, i]]
  ];
```

In[∘]:=
```
SXForm[Link[6, Alternating, 1]]
```

KnotTheory: Loading precomputed data in PD4Links`.

Out[∘]=
```
SXForm[{Loop[1, 2, 3, 4], Loop[5, 6, 7, 8, 9, 10, 11, 12]},
 Xm[1, 6] Xm[3, 10] Xm[5, 2] Xm[9, 4] Xp[7, 12] Xp[11, 8]]
```

In[∘]:=
```
Z[L_Link] := Module[{s, z},
   {s, z} = List @@ SXForm[L];
   z = z /. {Xp[i_, j_] :→ R_{i,j}, Xm[i_, j_] :→ R̄_{i,j}};
   Do[z = z // dm_{s⟦c,1⟧,s⟦c,k⟧→s⟦c,1⟧}, {c, Length[s]}, {k, 2, Length[s⟦c⟧]}];
   z
  ];
```

In[∘]:=
```
z = Z[Link[9, Alternating, 1]]
```

Out[∘]=
$$\mathbb{E}_{\{\}\to\{1,5\}}\Big[3\,a_5\,b_5,\ \frac{(2-7B_5+10B_5^2-7B_5^3+2B_5^4)\,x_1\,y_1}{B_1B_5-B_1^2B_5-4B_1B_5^2+3B_1^2B_5^2+4B_1B_5^3-4B_1^2B_5^3-2B_1B_5^4+2B_1^2B_5^4}+\frac{(-2B_5+2B_1B_5+5B_5^2-5B_1B_5^2-5B_5^3+5B_1B_5^3+2B_5^4-2B_1B_5^4)\,x_5\,y_1}{B_1-B_1^2-4B_1B_5+3B_1^2B_5+4B_1B_5^2-4B_1^2B_5^2-2B_1B_5^3+2B_1^2B_5^3}+\frac{(-2+2B_1+5B_5-5B_1B_5-5B_5^2+5B_1B_5^2+2B_5^3-2B_1B_5^3)\,x_1\,y_5}{B_1B_5-B_1^2B_5-4B_1B_5^2+3B_1^2B_5^2+4B_1B_5^3-4B_1^2B_5^3-2B_1B_5^4+2B_1^2B_5^4}+\Big((B_1-B_1^2+2B_5-7B_1B_5+4B_1^2B_5-3B_5^2+7B_1B_5^2-5B_1^2B_5^2+2B_5^3-6B_1B_5^3+3B_1^2B_5^3+2B_1B_5^4-2B_1^2B_5^4-2B_1B_5^5+2B_1^2B_5^5)\,x_5\,y_5\Big)\Big/\Big(B_1-B_1^2-4B_1B_5+3B_1^2B_5+4B_1B_5^2-4B_1^2B_5^2-2B_1B_5^3+2B_1^2B_5^3\Big),\ \frac{1}{-1+B_1+4B_5-3B_1B_5-4B_5^2+4B_1B_5^2+2B_5^3-2B_1B_5^3}+O[\epsilon]^1\Big]$$

In[∘]:=
```
z⟦0, 2, 2⟧
```

Out[∘]= {1, 5}

In[∘]:=
```
trZ[L_Link] := Module[{z, z1, comps},
  z = Echo@Z[L];
  comps = z⟦0, 2, 2⟧;
  z1 = z // (Times @@ (Rest[comps] /. i_Integer :→ tr_i));
  Echo@z1;
  (z1⟦3⟧ // Normal) /. Thread[(B_# & /@ comps) → (B_# & /@ Range@Length@comps)]
 ]
```

In[∘]:=
```
L = Link[7, Alternating, 2];
trZ[L]
```

» $$\mathbb{E}_{\{\}\to\{1,5\}}\Big[-2a_5b_1-2a_1b_5-3a_5b_5,\ \frac{(-1+3B_5-3B_5^2+2B_5^3-B_5^4)\,x_1\,y_1}{B_5^2-B_5^3+B_1B_5^4}+\frac{(-B_1-2B_5+2B_1B_5+2B_5^2-2B_1B_5^2-B_5^3)\,x_5\,y_1}{B_1B_5-B_1B_5^2+B_1^2B_5^3}+\frac{(-B_1-2B_5+2B_1B_5+2B_5^2-2B_1B_5^2-B_5^3)\,x_1\,y_5}{B_5^2-B_5^3+B_1B_5^4}+\frac{(-1-B_1B_5^2+B_1^2B_5^2-B_1^3B_5^2+B_1B_5^3-2B_1^2B_5^3+B_1^3B_5^3-B_1B_5^4+B_1^2B_5^4-B_1^3B_5^4)\,x_5\,y_5}{B_1^2B_5^3-B_1^2B_5^4+B_1^3B_5^5},\ \frac{B_1B_5^2}{1-B_5+B_1B_5^2}+O[\epsilon]^1\Big]$$

» $$\mathbb{E}_{\{\}\to\{1\}}\Big[-2a_1b_5,\ \frac{(1-B_5^2)\,x_1\,y_1}{-B_5^2+B_1B_5^2},\ \frac{-1+B_1^2B_5^3}{-B_1+B_1^2-2B_5+4B_1B_5-2B_1^2B_5+2B_5^2-4B_1B_5^2+2B_1^2B_5^2-B_5^3+B_1B_5^3}+O[\epsilon]^1\Big]$$

Out[∘]=
$$\frac{-1+B_1^2B_2^3}{-B_1+B_1^2-2B_2+4B_1B_2-2B_1^2B_2+2B_2^2-4B_1B_2^2+2B_1^2B_2^2-B_2^3+B_1B_2^3}$$

In[∘]:=
```
MultivariableAlexander[L][B] /. B[i_] :→ B_i
```

Out[∘]=
$$\frac{(-1+B_1)\,(-1+B_2)\,(1-B_2+B_2^2)}{\sqrt{B_1}\,B_2^{3/2}}$$

In[∘]:=
```
Simplify[trZ[L] * (MultivariableAlexander[L][B] /. B[i_] :→ B_i)]
```

» $\mathbb{E}_{\{\}\to\{1,5\}}\Big[-2\,a_5\,b_1-2\,a_1\,b_5-3\,a_5\,b_5,\ \frac{\left(-1+3\,B_5-3\,B_5^2+2\,B_5^3-B_5^4\right)\,x_1\,y_1}{B_5^2-B_5^3+B_1\,B_5^4}+\frac{\left(-B_1-2\,B_5+2\,B_1\,B_5+2\,B_5^2-2\,B_1\,B_5^2-B_5^3\right)\,x_5\,y_1}{B_1\,B_5-B_1\,B_5^2+B_1^2\,B_5^3}+\frac{\left(-B_1-2\,B_5+2\,B_1\,B_5+2\,B_5^2-2\,B_1\,B_5^2-B_5^3\right)\,x_1\,y_5}{B_5^2-B_5^3+B_1\,B_5^4}+\frac{\left(-1-B_1\,B_5^2+B_1^2\,B_5^2-B_1^3\,B_5^2+B_1\,B_5^3-2\,B_1^2\,B_5^3+B_1^3\,B_5^3-B_1\,B_5^4+B_1^2\,B_5^4-B_1^3\,B_5^4\right)\,x_5\,y_5}{B_1^2\,B_5^3-B_1^2\,B_5^4+B_1^3\,B_5^5},\ \frac{B_1\,B_5^2}{1-B_5+B_1\,B_5^2}+O[\epsilon]^1\Big]$

» $\mathbb{E}_{\{\}\to\{1\}}\Big[-2\,a_1\,b_5,\ \frac{\left(1-B_5^2\right)\,x_1\,y_1}{-B_5^2+B_1\,B_5^2},\ \frac{-1+B_1^2\,B_5^3}{-B_1+B_1^2-2\,B_5+4\,B_1\,B_5-2\,B_1^2\,B_5+2\,B_5^2-4\,B_1\,B_5^2+2\,B_1^2\,B_5^2-B_5^3+B_1\,B_5^3}+O[\epsilon]^1\Big]$

Out[•]= $\frac{1-B_1^2\,B_2^3}{\left(-1+B_1\right)\,\sqrt{B_1}\,B_2^{3/2}}$