$$G \,/\, m^{ij}_k = m^{ji}_k$$

In $\mathcal{U}(\mathrm{heis}(p,x))$ / commutators.

$$0 = [p, F(p,x)] = \partial_x F$$

$$\left(\begin{matrix} \alpha & \beta & \phi \\ \gamma & \delta & \psi \\ \theta & \epsilon & \end{matrix}\right) \to \left(\begin{matrix} \cdot & \cdot \\ & \cdot \end{matrix}\right)$$

$$pF - Fp \qquad F = \prod g_i$$

$$[p, F] = \sum \prod_{i<k} g_i \, [p, g_k] \prod_{i>k} g_i$$

$$0 = [x, F(p,x)] = -\partial_p F$$

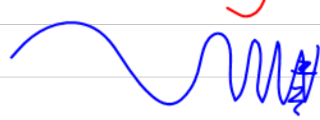
$$\begin{tikzcd} G_{2+s} \arrow[r, shift left, "m^{12}"] \arrow[r, shift right, "m^{21}"'] & G_{1+s} \arrow[r] \arrow[dr, "tr"'] & G_{1+s}/m^{12}=m^{21} \\ & & G_{0+s} \arrow[u, "mod"'] \arrow[r, hook] & G_{1+s} \end{tikzcd}$$

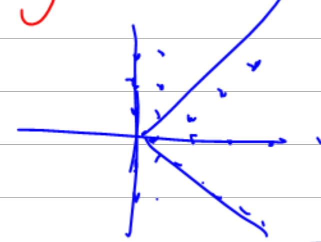
$\supset$ GDO represented by Gaussians in Greeks & Latins

$\not\supset$ GDO

Q: Can you get further "traces" using signatures?

$$\Lambda = (\lambda_{ab})$$





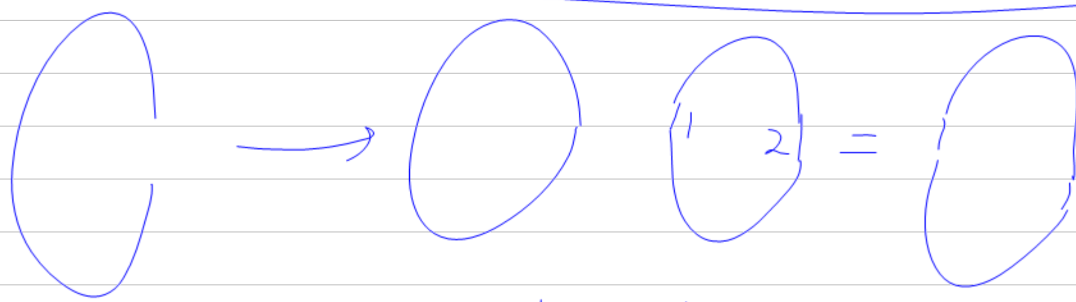
$$\int e^{-\frac{i}{2}\lambda_{ab}x^a x^b - \epsilon|x|^2} \xrightarrow[\epsilon\to 0]{} \frac{1}{(2\pi)^{n/2}\sqrt{\det(-i\Lambda+\epsilon I)}} = \frac{1}{(2\pi)^{n/2}\sqrt{\det\Lambda}} \cdot e^{i\frac{\pi}{4}\mathrm{sign}(\Lambda)}$$

July 29, 2020

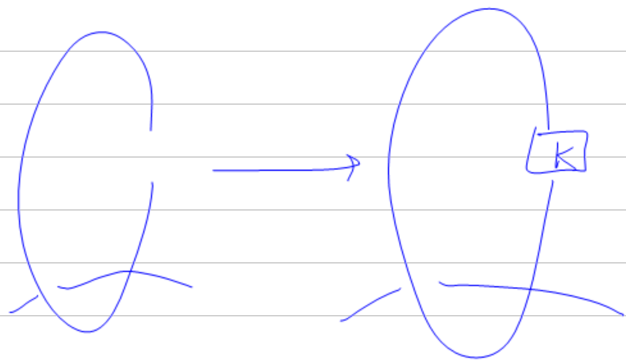
$$A^{\lambda} - 1 = e^{\alpha\lambda} - 1 = 1 + \alpha\lambda - 1 = \alpha\lambda$$

$$1 + \lambda\alpha - 1$$

Aug 6 2020

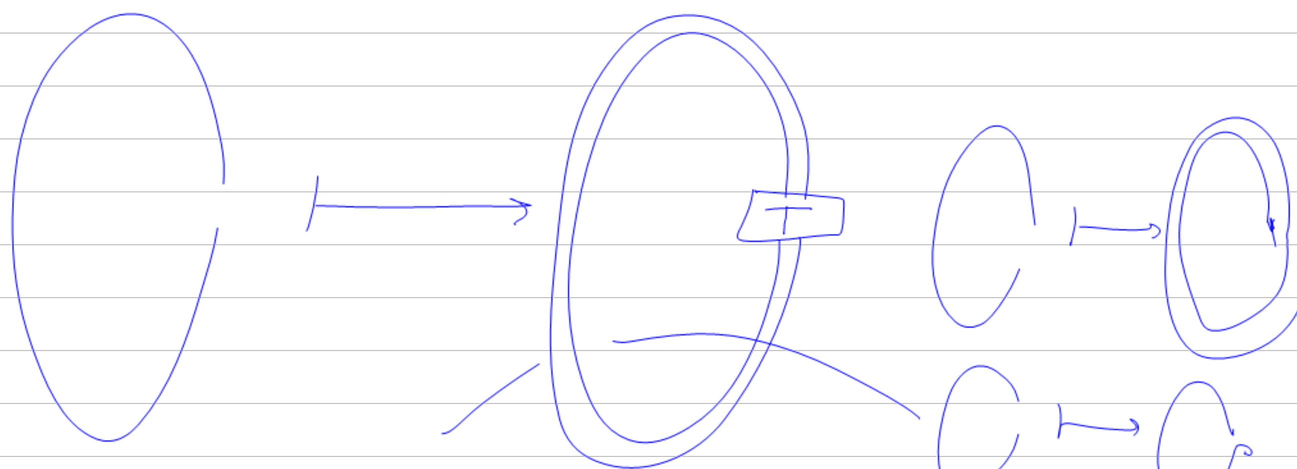


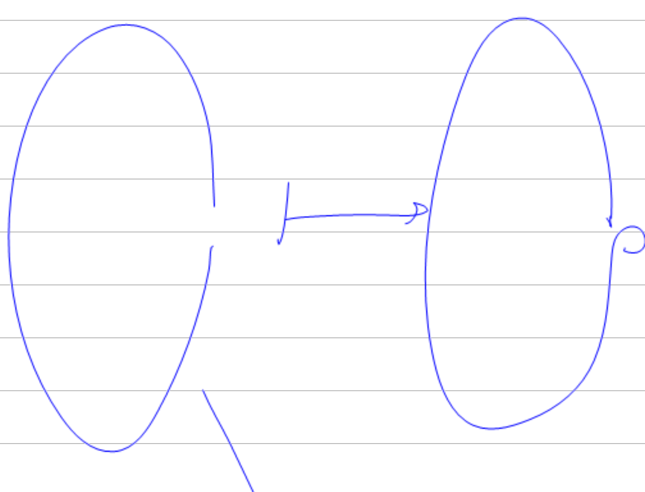
K is virtual



$$K = \underbrace{K_1}_{\text{classical maximal}} \# \underbrace{K_2}_{\text{virtual}}$$

Is K_1 unique?

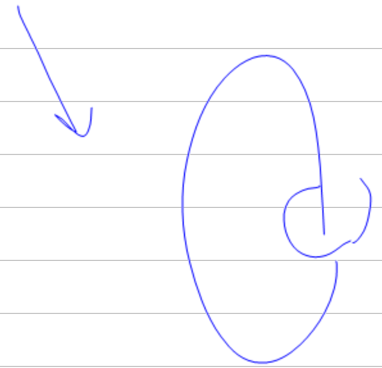


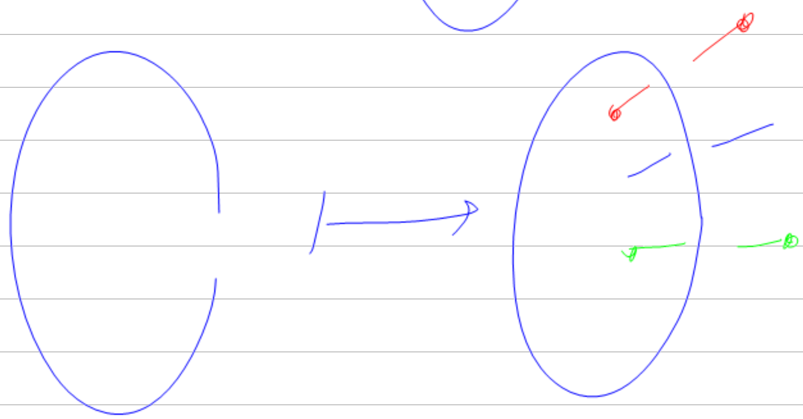


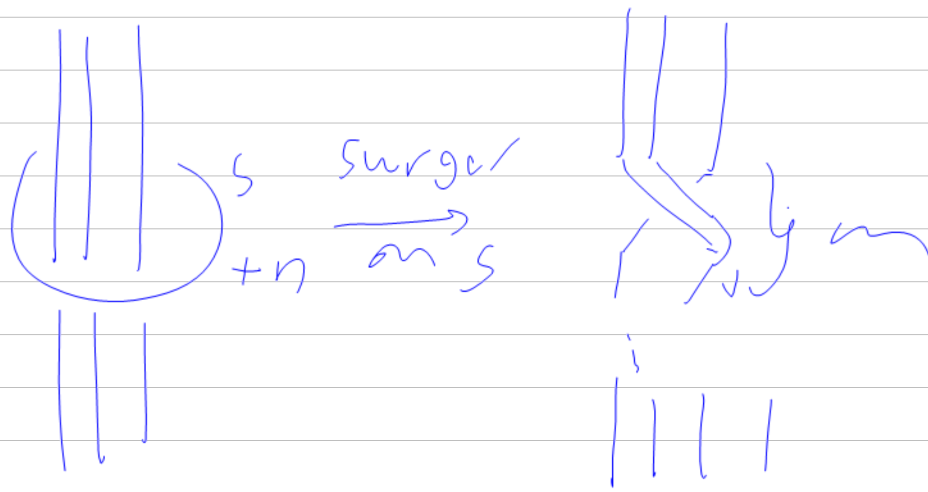
$$\begin{pmatrix} \alpha & \theta \\ \phi & \Xi \end{pmatrix} \xrightarrow{tr_H} (\;\;)$$

$$\downarrow \Delta$$

$$B_m \xrightarrow{tr_H} (\;\;)$$



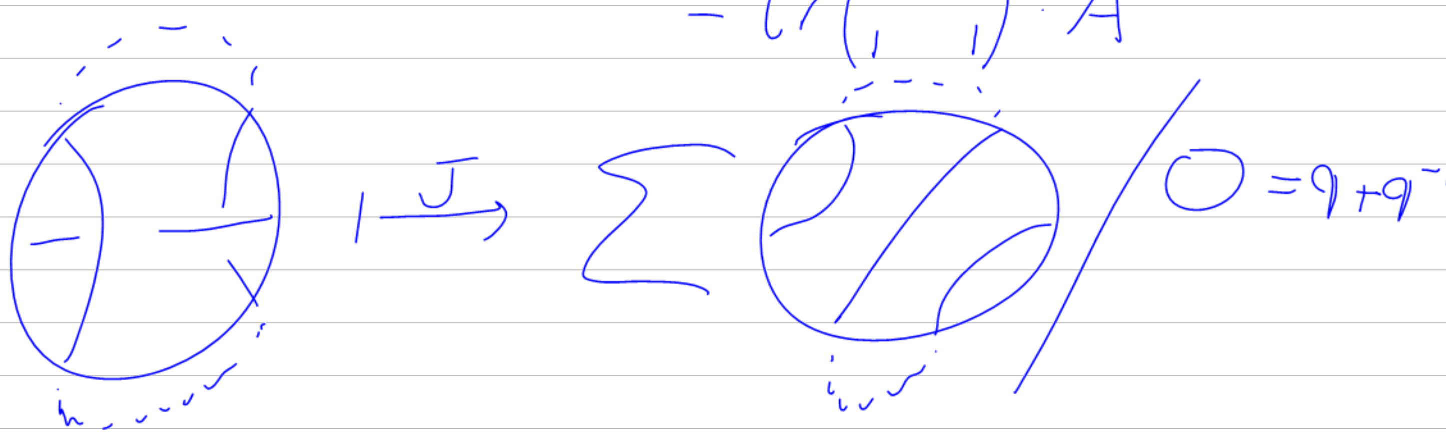




Aug 31, 2020

$$(a_{ij}) = A \qquad \sum a_{ij} = (1 \cdots 1) \cdot A \cdot \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$$

$$= \mathrm{tr} \begin{pmatrix} 1 & \cdots & 1 \\ 1 & & 1 \end{pmatrix} \cdot A$$



$$\circlearrowleft \partial s^* \qquad\qquad\qquad\qquad \circlearrowleft \partial s^*$$
$$GDO \longleftrightarrow \Gamma$$
$$GDO \xleftrightarrow{S^*} GDO' \longleftrightarrow \Gamma$$
$$GDO \underset{\text{easy}}{\longleftrightarrow} \Gamma' \longleftrightarrow \Gamma$$
$$\circlearrowleft tr'_H \qquad \circlearrowleft tr_H$$

Whirling (AP/2019-11)

$$W : \begin{array}{c} \begin{pmatrix} \Xi & \phi \\ \theta & \alpha \end{pmatrix} \\ \;\; n-1 \quad 1 \end{array} \longmapsto \frac{1}{\alpha} \begin{array}{c} \begin{pmatrix} 1 & -\theta \\ \phi & \alpha\Xi - \phi\theta \end{pmatrix} \\ 1 \qquad n-1 \end{array}$$

$$\xrightarrow{W'} \xrightarrow{\text{cyclic}}$$

<u>Thm</u> $W^n(\underset{n\times n}{A}) = A^{-1}$

$$W' : \begin{pmatrix} \Xi & \phi \\ \theta & \alpha \end{pmatrix} \longmapsto \frac{1}{\alpha} \begin{pmatrix} \alpha\Xi - \phi\theta & \phi \\ -\theta & 1 \end{pmatrix}$$

$\{x^n \wedge^k y^n\}$ $\quad \mathbb{Q}[x,y] \longrightarrow \mathbb{Q}[t]$

$$x^n y^m \longrightarrow \delta_{nm} t^n n!$$

$$e^{\xi x} e^{\eta y} \longrightarrow e^{\xi\eta t}$$

$$\| \qquad\qquad\qquad \|$$

$$\sum \frac{\xi^n x^n}{m!} \frac{\eta^m y^m}{n!} \longmapsto \sum_n \frac{\xi^n \eta^n}{(n!)^2} n! t^n$$

$$\mathbb{E}\left(r\alpha_m \beta_m / k + s\cancel{\beta_m b_m} + t\alpha_m a_m \frac{A_m^{1-r}\left(A_m' - \cancel{B_m^s}\right)}{A_m - 1}\xi_m \eta_m \right)$$

$$Q[b,a] \xrightarrow{P} Q[b,a]$$

$$b^k a^n \longmapsto \begin{cases} b^k a^n & k \le n \\ 0 & k > n \end{cases}$$

$$g(P) = \sum_{k \le n} \frac{(\beta b)^k (\alpha a)^n}{\cancel{k!}\, n!}$$

$$= \sum_{k \le n} (\beta b)^k \frac{(\alpha a)^n}{n!}$$

$$= \sum_n \frac{(\beta b)^{n+1} - 1}{\beta b - 1} \cdot \frac{(\alpha a)^n}{n!}$$

$$= \frac{\beta b}{\beta b - 1} e^{\beta b \alpha a} - \frac{e^{\alpha a}}{\beta b - 1}$$

$$(x^k)' = k x^{k-1}$$

$$\frac{\Delta F}{\Delta x} = F(x+1) - F(x)$$

$$x^{(k)} = x(x-1)\dots(x-k+1)$$

$$\frac{\delta x^{(k)}}{\delta x} = k x^{(k-1)}$$

$$L : x^k \longmapsto x^{(k)}$$

$$e^{x\log(1+\xi)}$$

$$g(L) = \sum \frac{\xi^k x^{(k)}}{k!} = (1+\xi)^x$$

$$= \sum \xi^k \binom{x}{k}$$

$$\frac{x^k}{k!} \qquad \binom{x}{k} \qquad \binom{x}{k-1} \quad \dots$$

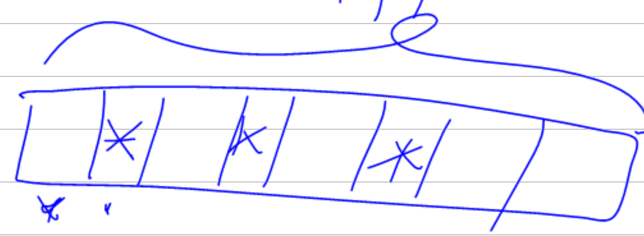
$$e^{\xi x} = \dots$$

$$1+2+3+\dots =$$

$$1^2+2^2+3^2+\dots$$

$$1^k+\dots+n^k = \dots$$

$n+1$



$$\binom{1}{k}+\binom{2}{k}+\dots+\binom{n}{k} = \binom{n+1}{k+1}$$

$$n^k \sim \binom{n}{k}$$

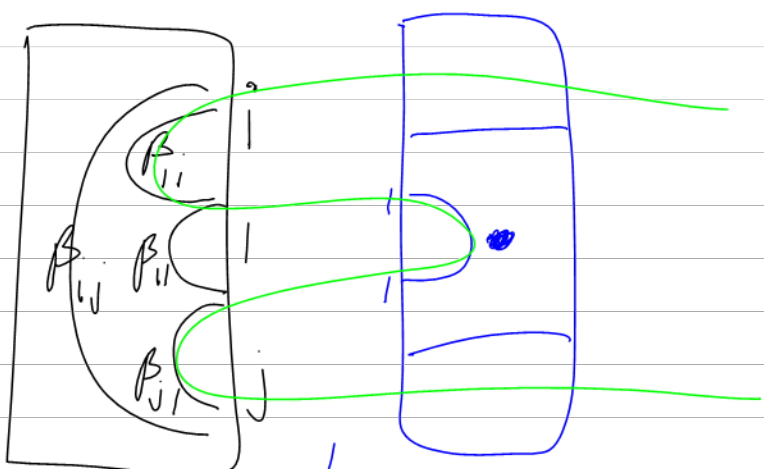
$$x^n \neq \sum_{k=0}^{n} \binom{n}{k} \binom{x}{k} k!$$

$$\cancel{x^2} = \underset{k=0}{1} + 2x + x(x-1) = x(x+1)$$

$$x^n = x^{(n)} + \underbrace{\binom{n}{2}} x^{(n-1)} + \cdots$$

$$+ \begin{pmatrix} \text{a poly} \\ \text{in } n \\ \text{of deg } 3 \end{pmatrix} x^{(n-2)}$$

$$+ \begin{pmatrix} \text{a poly in} \\ n \text{ of} \\ \text{deg } 4 \end{pmatrix} x^{(n-2)}$$



$$\frac{1}{1-\beta_{11}}$$

$$\bar{y} = A\bar{x}$$

$$\begin{pmatrix} x_1 \\ \vdots \\ x \end{pmatrix} \to \begin{pmatrix} x_1 \\ \vdots \\ x_{n-1} \\ y_n \end{pmatrix} \to \cdots \to \begin{pmatrix} x_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} \to \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$$

$$\begin{pmatrix} |\sigma_1|^{\pm 1} & & 0 \\ & \ddots & \\ 0 & & |\sigma_n|^{\pm 1} \end{pmatrix} \pm A$$

$$\underline{H_n(x) e^{-x^2/2}} \qquad L_n(x) e^{-x}$$

$$\chi_{[-1,1]}$$

$$\langle f, g \rangle = \int e^{-x^2/2} f g \, dx$$

$$y_1 = e_{11} x_1 + \dots + e_{1n} x_n$$

$$\vdots \qquad\qquad E = 1 + hF$$

$$y_n = e_{n1} x_1 + \dots + e_{nn} x_n$$

$$y' = A \underline{\underline{x'}} + B x'' \qquad \begin{pmatrix} x' \\ y'' \end{pmatrix} \mapsto \begin{pmatrix} y' \\ x'' \end{pmatrix}$$

$$\underline{\underline{y''}} = C x' + D x''$$

$$x'' = D^{-1}(y'' - C x')$$

$$\begin{pmatrix} B_1 & 0 \\ 0 & B_2 \end{pmatrix} - \begin{pmatrix} (1-\bar{B}_1)\beta_{11} & (1-\bar{B}_2)\beta_{12} \\ (1-\bar{B}_1)\beta_{21} & (1-\bar{B}_2)\beta_{22} \end{pmatrix}$$

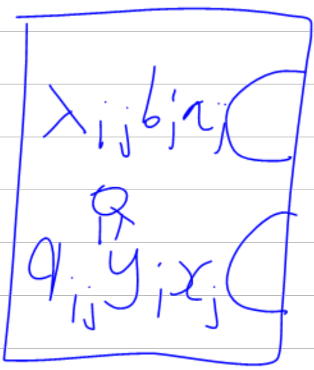
$$\begin{pmatrix} B_1^{-1} & 0 \\ 0 & B_2 \end{pmatrix} - \rho I \left(\begin{pmatrix} B_1 & 0 \\ 0 & B_2 \end{pmatrix} - \begin{pmatrix} \downarrow \end{pmatrix} \right)$$

$$\beta = \begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{pmatrix}$$

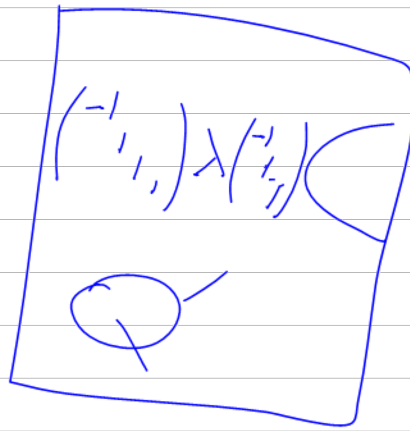
$$S_1\beta = \begin{pmatrix} \delta^{-1}(B_1^{-1}) & \\ & \delta(B_1) \end{pmatrix} \left[\begin{pmatrix} \gamma(B_1^{-1}) & \\ & \gamma(B_2) \end{pmatrix} \rho I_1 \left(\begin{pmatrix} \gamma(B_1) & 0 \\ 0 & \gamma(B_2) \end{pmatrix} - \begin{pmatrix} \delta(B_1) & \\ & \gamma(B_2) \end{pmatrix} \beta \begin{pmatrix} \epsilon(B_1) & \\ & \epsilon(B_2) \end{pmatrix} \right) \right] \begin{pmatrix} \epsilon^{-1}(\delta_1^{-1}) & \\ & \epsilon^{-1}(B_2) \end{pmatrix}$$

$$ds_1 = e^{-\alpha_1 a_1 - b_1\beta_1 - \frac{\eta_1 A_1}{B_1} y_1 - \xi_1 A_1 x_1 + \frac{1-B_1}{B_1} A_1 \eta_1 \xi_1}$$

$= e^{\alpha_1}$



// $ds_1 =$



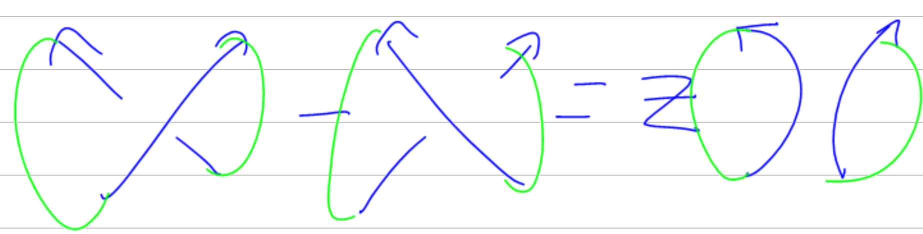
$$\tilde{Q} = (y_1 \ldots y_n) \begin{pmatrix} \tilde{\gamma} & \tilde{\theta} \\ \tilde{\phi} & \tilde{\Xi} \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

$$\Xi' = \Xi + \phi\theta \cdot \frac{(1-B_1)}{B_1} \prod_{j=1}^{n} B_1^{-\lambda_{j1}}$$

$$\sum_{i-}^{j-}$$

$$\underbrace{I \in g^* \otimes g}_{} \qquad g = \langle a, x \rangle$$

$$R_{ij} = e^{b_i a_j + x_i y_j} \qquad R_{ij},\ m^{ij}_k,$$



$S\ \Delta,\ t$

$$\mathbb{E}[\hbar \underset{\hat{\mathcal{Z}}}{\ell_{ij}} b_i a_j, \hbar q_{ij} y_j x_j] \quad \hat{F}(e^{\hbar b_i})$$

$xw = wx$

$$\rightarrow \sum \hbar^m \, CU^{\otimes n}_{\leq 2m}$$

$$Z = \sum \hbar^m Z_m \quad Z_m \in CU^{\otimes n}_{\leq 2m}$$

proj into coinv $\rightarrow \overline{Z}_m \in CoCU^{\sim}_{\leq 2m}$

$$\text{Coinv} = \langle y^k a^n x^k \rangle$$

$$\overset{\parallel}{a^n t^k}$$

$$CU_{\leq 2m} \xrightarrow{\pi} a^n t^k$$

$$g_{\leq 2m}(\pi) = \sum a^* t^* \overbrace{\eta_0 \beta_0 \alpha_0 \xi_0}^{\deg \leq 2m}$$

$$\Downarrow$$

$$tr^m = \mathbb{E}[0, 0, g_{\leq 2m}(\pi)]$$

$$cm_0^{ij} /\!/ tr_0^m - cm_0^{ji} /\!/ tr_0^m$$

Should vanish to deg 2m in greeks.

$\mathbb{Z} /\!/ tr_0^m$ mod out by having many gr0ks

is a link invt,

Either trivial or not.

$$x_i^{\,n} = x_i^{n}$$

$\{i, j, k, l\}$ = Exponent[word, #]&

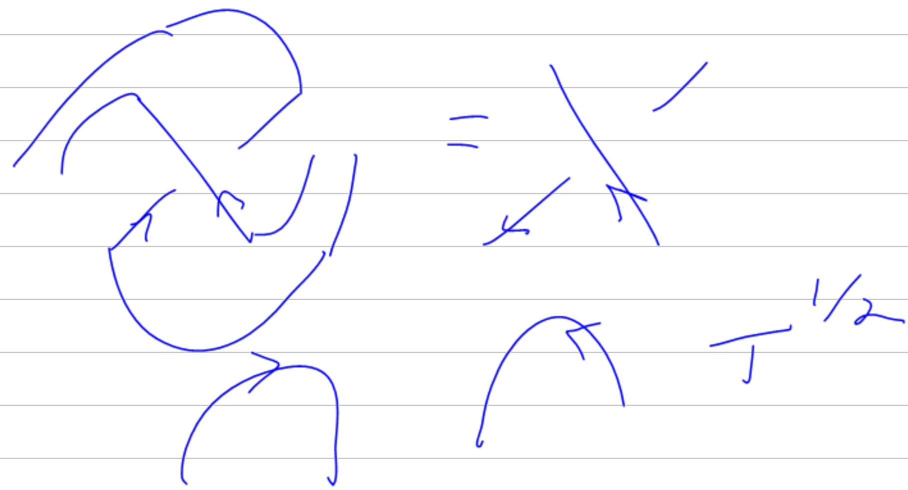
/@ {y, b, y, x}

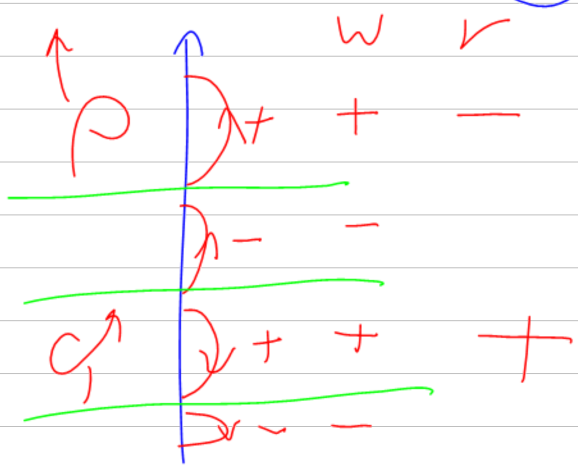
$$F_k U \uparrow U$$

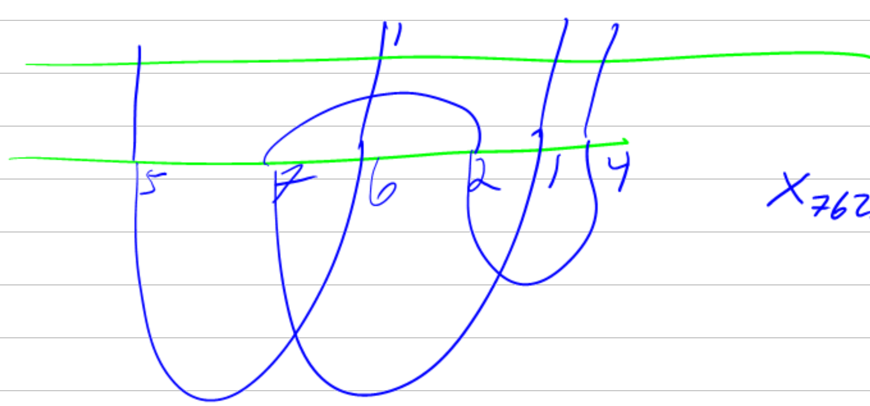
$$F_n(U^{\otimes 2}) = \sum_{j+k \le n} F_j U \otimes F_k U \xrightarrow{m} F_n(U)$$

$$m^{ij}_{k}$$

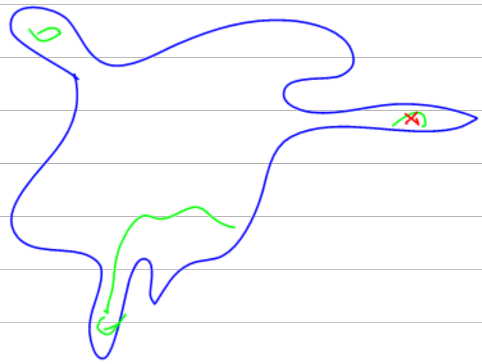
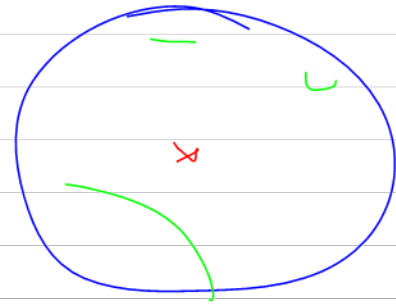
$$cR_{ij} = e^{b_i a_j} + \frac{e^{b_i}-1}{b_i} y_i x_j$$







X_{7623}

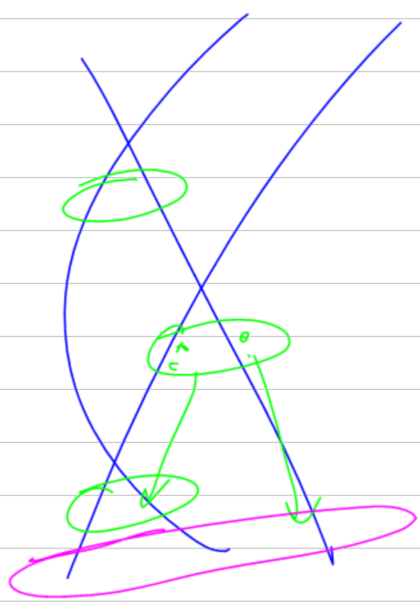
 

$$[p, x] = i$$

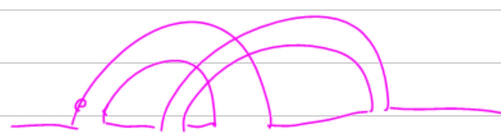
$$R_{ij} = e^{(e^t - 1)(p_i - p_j)x_j}$$

$$Z\begin{pmatrix}\text{loop} \\ \text{knot}\end{pmatrix} = W e^{\varphi}$$

Seifert In Bands



$$g_{ij}$$



$$(y,b,a,x) \mapsto (-tp, t, px, x)$$

$$e^{y_i x_j + b_i a_j} \mapsto e^{-tp_i x_j + tp_j x_j}$$

$$R_{ij} = e^{\pm t(p_i - p_j)x_j} = \mathbb{O}_{px}\left(e^{(e^t - 1)(p_i - p_j)x_j}\right)$$

$$R^{pxc}_{ij} = e^{-c_i p_i x_j + c_j p_j x_j}$$

$$= \mathbb{O}\left(e^{(e^{c_i} - 1)(p_i - p_j)x_j}\right)$$

$$A \twoheadrightarrow B \qquad \mu /\!/ \lambda = id.$$

$$\mu : (p, x) \mapsto \left(-\frac{y}{t}, x\right)$$

$$\begin{tikzcd}
CU \otimes CU \arrow[d, "cm"] & \mathbb{H} \otimes \mathbb{H} \arrow[l, "\mu \otimes \mu"'] \arrow[d, "hm"] \\
CU \arrow[r, "\lambda", "{\substack{ybax \mapsto \\ (-tp,\, t,\, px,\, x)}}"'] & \mathbb{H}
\end{tikzcd}$$

$$\mathcal{G}(\mu) = \exp\left(\frac{-\pi y}{t} + \xi x\right)$$

$$\mathcal{G}(\lambda) = \mathbb{O}_{px}^{-1} \lambda\left(e^{\eta y} e^{\beta b} e^{\alpha a} e^{\xi x}\right)$$

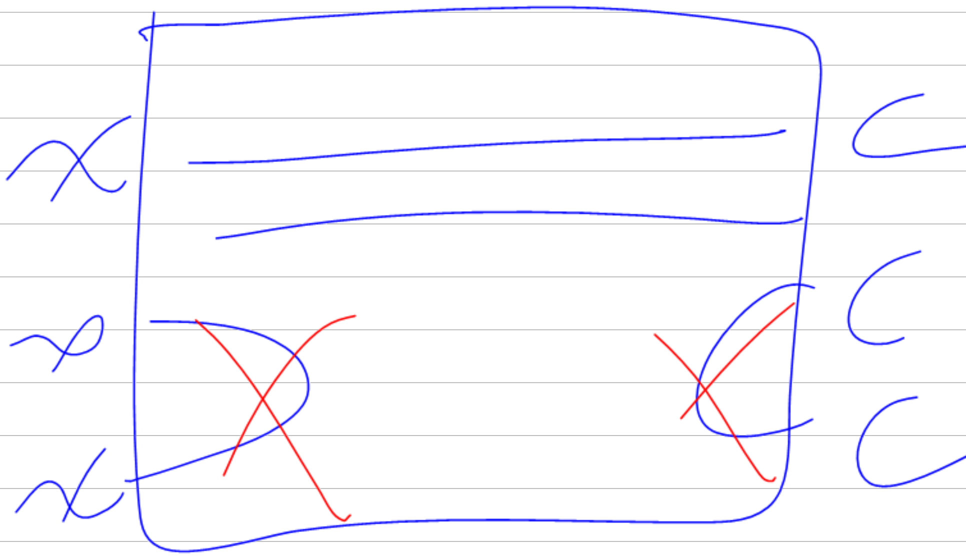
$$= \mathbb{O}_{px}^{-1}\left(e^{-t\eta \hat{p}} e^{\beta t} e^{\alpha \hat{p}\hat{x}} e^{\xi \hat{x}}\right) = \cancel{\mathbb{H}}$$

Lemma $e^{\alpha \hat{p}\hat{x}} = \mathcal{O}_{px}\left(e^{(1-e^{-\alpha})px}\right)$

$\# = e^{-t\eta p}\, e^{\beta t}\, e^{(1-e^{-\alpha})px}\, e^{\xi x}$

$ybx < ybnx$

$[y,x] = b$



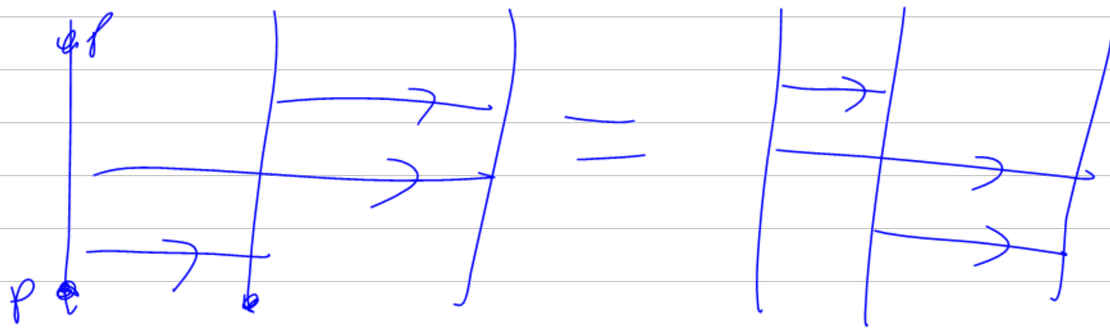
$m_k^{ij} = \overset{\text{old}}{m}{}_k^{ij} \,//\, c_i, c_j \to c_k$

$(x_i + x_j)c_k$

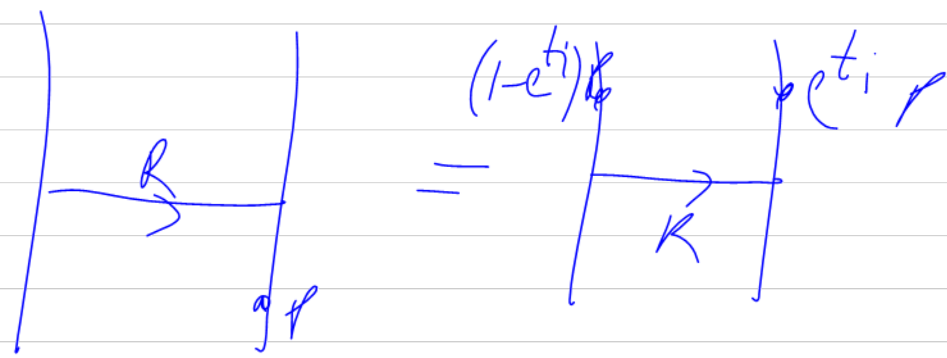
$$R_{ij} = e^{b_i a_j + y_i x_j}$$

$$R_{ij} = e^{t_i p_j x_j - t_i p_i x_j}$$

$$= e^{t_i (p_j - p_i) x_j}$$



$$[p, f(x)] = f'(x) \cdot t$$



$$[e^{px}, p] = e^a p - p e^a = e^a p - e^{a+t} p = e^a \underbrace{(1 - e^t)}\, p$$

$$[\underbrace{px}_{a}, p] = -tp$$

$$[a, p] = -tp$$

$$ap - pa = -tp$$

$$pa = ap + tp = (a+t)p$$

$$pa^{50} = (a+t)^{50} p$$

$$pe^a = e^{a+t} p$$

$$p_j e^{t(p_i - p_j)x_j}$$

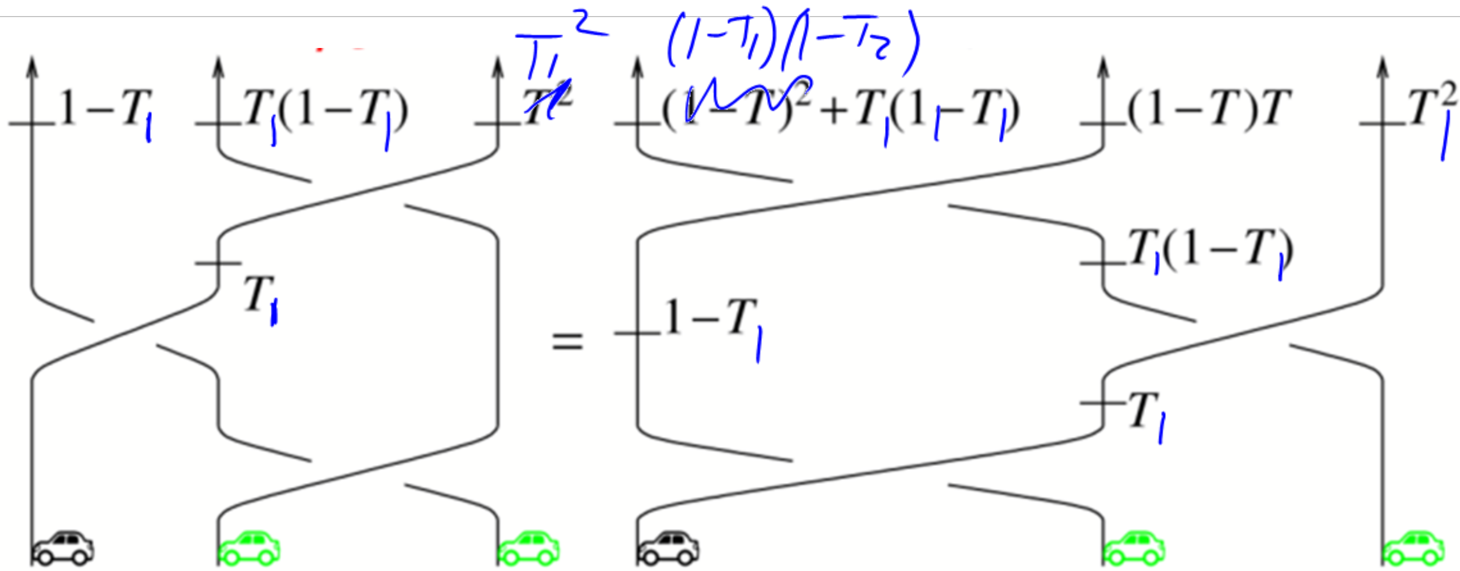
$$p_j (p_i - p_j) x_j = (p_i - p_j) x_j p_j + p_i - p_j$$

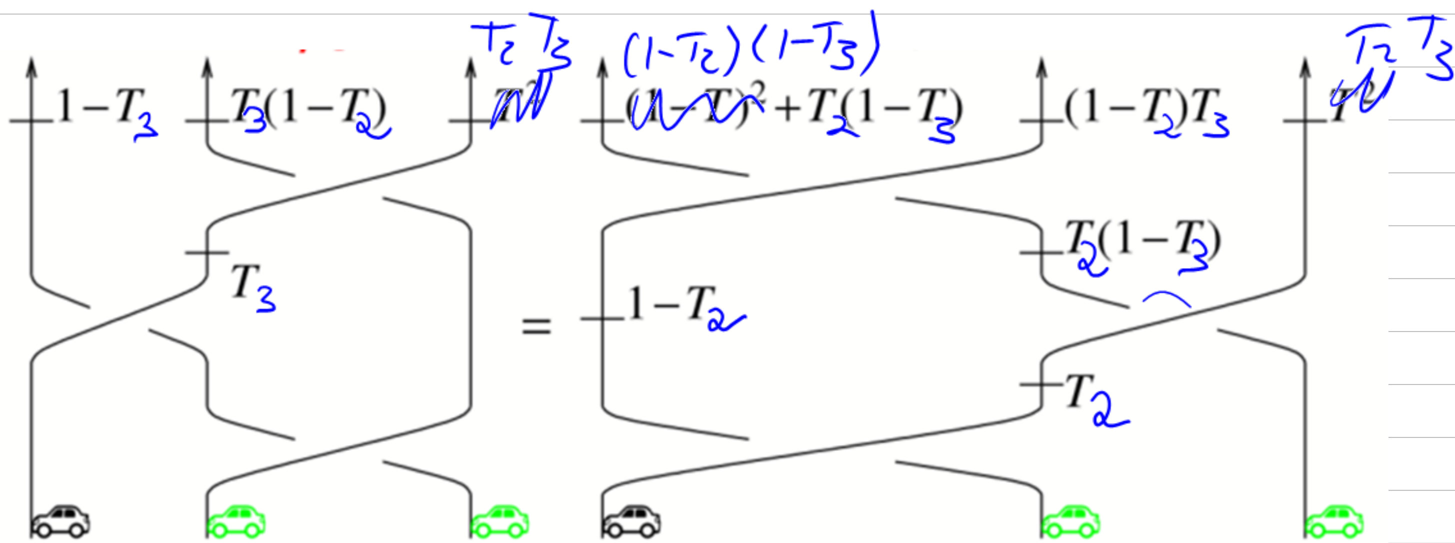
$$e^{-t(p_i - p_j)x_j} p_j e^{t(p_i - p_j)x_j}$$

$$e^{t\,\underbrace{\mathrm{ad}(p_i - p_j)x_j}_{A}} (p_j)$$

$$e^{tA} ($$

$$A\begin{pmatrix} p_j \\ 1 \end{pmatrix} = \begin{pmatrix} +q_j - p_i \\ - \quad - \end{pmatrix} = \begin{pmatrix} - & - \\ \ominus & - \end{pmatrix}($$





$$e^{(p_i-p_j)\underline{x_j}}$$

$$\partial_{px}\left(e^{\cancel{\partial\lambda}(p_i-p_j)x_j}\right) = e^{\lambda(p_i-p_j)x_j}$$

$$\underbrace{R_{12}R_{13}}\,R_{23} = \_\quad \_$$

$$\partial_{yx}\left(e^{xy}\right)y =$$

$$\underline{y\,\partial}_{yx}\left(e^{xy}\right) = \partial_{yx}\left(e^{xy}y\right)$$