FONDS FRANCE CANADA POUR LA RECHERCHE
FRANCE CANADA RESEARCH FUND

Programme 2018 « Nouvelles collaborations de recherche »

Formulaire de candidature

(also available in English)

Pre-screening

Veuillez répondre aux questions suivantes objectivement.

Questions d'éligibilité :

1/ Avez-vous déjà obtenu un financement du FFCR avec votre partenaire ou l'un des membres de l'équipe partenaire ?

☐OUI
☒NON

2/ Avez-vous déjà bénéficié d'un contrat doctoral ou postdoctoral au sein de l'équipe partenaire ?

☐OUI
☒NON

3/ Votre projet consiste-t-il en l'organisation d'un symposium conjoint ?

☐OUI
☒NON

Si une des réponses aux trois premières questions est OUI, votre candidature n'est pas éligible

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Questions complémentaires :

1/ Des doctorants font-ils partie de votre équipe et seront-ils associés au projet?

☒OUI
☐NON

2/Des jeunes chercheurs (5 ans d'expérience) sont-ils associés au projet ?

☒OUI
☐NON

3/ Souhaitez-vous soumettre des candidatures à l'un ou l'autre des programmes de financement complémentaires du FFCR (cf l'annexe 1 du guide)

☐OUI **Si oui, lequel/lesquels ?**
☒NON

4/ Veuillez cocher la ou les cases correspondante(s) au(x) domaine(s) de rattachement de votre projet (plusieurs cases peuvent être cochées) :

☐ Bio/Santé
☐ Environnement/ Biodiversité/ Agro
☐ STIC/ Sécurité/ Communications
☐ Energie/ Ressources/ Transports
☐ Humanités/ Diversité culturelle/ Francophonie/ Sociétés
☒ Autres

Titre du projet : Knot polynomials which can be calculated in polynomial time

Domaine(s) scientifique(s) :

- ☒ *Maths/ Maths appliquées*
- ☐ *Physique*
- ☐ *Sciences de la Terre, de l'Univers, de l'Espace*
- ☐ *Chimie*
- ☐ *Sciences de l'ingénieur*
- ☐ *Biologie, Santé, Médecine*
- ☐ *Sciences humaines*
- ☐ *Sciences sociales*
- ☐ *STIC*
- ☐ *Agronomie, production Végétale et Animale*

1. Partenaires

	Équipe française	***Équipe canadienne***
Chercheur principal		
Nom, prénom	Fiedler, Thomas	Bar-Natan Dror
Fonction / grade	professeur CE2	Full Professor
Téléphone	33-561556789	1-416-946-5438
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Courriel	fiedler@math.univ-toulouse.fr	drorbn@math.toronto.edu
Laboratoire		
Intitulé	Institut de Mathématiques de Toulouse	Department of Mathematics
No. d'identification	UMR 5219	s/o
Directeur (nom, prénom)	Guedj Vincent	Quastel, Jeremy
Établissement de rattachement	FSI, Université Paul Sabatier Toulouse III	University of Toronto
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Courriel	d.dallariva@math.univ-toulouse.fr	chair@math.toronto.edu

Nom des directeurs des unités de recherche **SIGNATURE**	**Vincent Guedj**	**Jeremy Quastel**

(Un maximum de 6 pages est alloué pour les sections 2 à 7)

2. Description du projet

Problématique

Knot polynomials which can be calculated in polynomial time

Knots are as old as mankind. The first mathematical paper which has mentioned knots is « Remarques sur les problèmes de situation » by A.T. Vandermonde in 1771. Since then, mathematicians try to solve the main problem in knot theory: for two given knots in 3-space, decide in an effective way whether or not they are isotopic, i.e. can be deformed one into the other in a continuous way?

The main tool to achieve this are knot invariants. There is an enormous number of known knot invariants, but either they are not calculable, i.e. they do not reduce to numbers which can be calculated by an algorithm in polynomial time, or they evidently fail to distinguish all knots.

Hence the problem is still very far from its solution.

There were two main advances in knot theory in the last decades: the discovery of quantum knot polynomials (and their upgradings to Topological Quantum Field Theories and categorifications), which come from representations of Lie algebras and more generally quantum groups, on one hand, and the discovery of finite type invariants (and the Kontsevich integral as their universal expression), which come from singularity theory on the other hand. The singularity theory under question was Vassiliev's study of the discriminant of all singular knots in 3-space. But it disappeared again rapidly because of Dror Bar-Natans paper « On the Vassiliev knot invariants » which has transformed the subject back into representation theory of Lie algebras.

Can history happen twice?

There is a new approach which constructs knot polynomials by using singularity theory [1]. This time, the singularity theory is the study of the discriminant of all singular projections of a (non-singular) knot into the plane. The result are new knot polynomials, let us call them 1-cocycle polynomials. They are graded by natural integer's n. Each of the polynomials can be calculated in at most quartic time and they can distinguish the orientations of knots. Greg Kuperberg had observed that if finite type invariants (and hence quantum polynomials) do not distinguish the orientations of knots then they fail to distinguish unoriented knots as well.

There are very many 1-cocycle polynomials for each fixed n and so there is at least a chance that they distinguish perhaps all knots in 3-space. But for the moment they are not related to representation theory at all.

On the other hand there is a new approach which constructs knot polynomials by using solvable Lie algebras [2]. Each of the new polynomials is calculable in polynomial time. They are also graded by natural integer's n. It is conjectured that these polynomials are slices of the colored Jones polynomial and that in a similar way each colored quantum polynomial can be sliced into knot polynomials which are calculable in polynomial time.

For a very long time (in fact since 1923) only one polynomial time knot polynomial was known, the Alexander polynomial. Unexpectedly, new polynomial time knot polynomials have arised from the very different new approaches mentioned above. We cannot expect that the two approaches are equivalent, but it could well be that there are relations between them. The Alexander polynomial is a very particular special case of the polynomials constructed from the solvable Lie algebra approach (in fact this was a highly non-trivial conjecture for some time). Hence, a first reasonable question would be :

Is the Alexander polynomial determined by 1-cocycle polynomials too?

It is known that the Alexander polynomial is calculable in quartic time exactly as the 1-cocycle polynomials. Hence we could expect a finite linear relation between them, if there is any at all.

[1] Th. Fiedler: Knot polynomials from 1-cocycles, http://arxiv.org/abs/1709.10332

[2] D. Bar-Natan, R. van der Veen: A polynomial time knot polynomial, https://arxiv.org/abs/1708.04853

Méthodologie

Using computer programs in order to formulate conjectures and proving them by combining methods from representation theory with methods from singularity theory and combinatorics.

3. Présentation des équipes (fournir CV en annexe, maximum de 4 CV)

Composition des équipes impliquées dans le projet *(nom, prénom, fonction, grade, % d'implication dans le projet)*
France :
Chercheurs Fiedler, Thomas, professeur, CE2, 100% Florens, Vincent, MCF, Mortier, Arnaud, Assistant Professor,
Techniciens/Assistants de recherche
Étudiants *(codirections, cotutelles éventuelles)* Hok, Jean-Marc, thésard, directeur : Th. Fiedler, 100%
Canada :
Chercheurs Bar-Natan, Dror, Full Professor, 25%
Techniciens/Assistants de recherche
Étudiants *(codirections, cotutelles éventuelles)* Gaudreau, Robin, PhD-student, advisor : D. Bar-Natan, 25%

4. Complémentarité entre les équipes de recherche française et canadienne

Unfortunately, there is not yet a computer program in order to calculate any 1-cocycle polynomials. The group working on the 1-cocycle approach is not enough qualified for doing this. So it's difficult to get an idea what is going on and to make precise conjectures. On the other hand, the group working on the solvable Lie algebra approach is highly qualified to write such computer programs, as they have proven in the past many times. Hence, the natural project is to bring these two very different working groups together, to learn from each other, to share ideas, to write computer programs together and perhaps to make connections between these two approaches, which are the only ones which produce knot polynomials calculable in polynomial time.

5. Moyens disponibles pour la réalisation du projet moyens de terrain *(p.ex., moyens de terrain; analytiques ; ressources bibliographiques ; banques de données)*

En France :
Office space , computers, calculation capacities
Au Canada :
Office space , computers, calculation capacities

6. Perspectives et viabilité de la collaboration *(p.ex. formation par la recherche, collaborations académiques, publications, communications, organisation de colloques, valorisation économique, sociale, industrielle)*

If it works then it could turn out to be a major step towards a solution of the knot problem!
It will create a new direction in low-dimensional topology, which implies common publications, workshops, conferences, new PhD-students and so on.

7. Calendrier prévisionnel des travaux

Starting with a first meeting in Toronto in August 2018 "to get the stone rolling". Followed by regular meetings once a semester of parts of the groups in Toronto and in Toulouse up to August 2020.

8. Demande budgétaire

(1 page maximum)

Rappel : Montant des fonds demandé : entre 8 000 $ et 15 000$ Durée d'utilisation des fonds : 2 ans

Type de dépenses éligibles	Détails des dépenses (nombre de personnes concernées, nombre déplacements)	coût	%
Déplacements vers la France			
Hébergement (France)			
Déplacements vers le Canada			
Hébergement (Canada)			
Autres (sauf salaires)			
Total			**100**

Justification du budget

Le candidat doit justifier chaque item planifié au budget.

ANNEXE - CV détaillés des membres de l'équipe

(Chercheur principal de chaque équipe et autres collaborateurs, 4 CV maximum)