

$f[0] = f[1] = 1;$

$f[n_] := f[n] = f[n-1] + f[n-2];$

$f[10]$

89

$f[20]$

10946

$f[30]$

1346269

$f[40]$

165580141

$f[100]$

573147844013817084101

$f_{1,0,k}[\{e_\_ \}] := \frac{1}{e}; f_{0,1,k}[\{e_\_ \}] := \frac{(-1)^{k+1}}{e};$

$f_{-1,-,[_]} = 0; f_{-, -1,[_]} = 0;$

$f_{a,b,k}[e\_List] := f_{a,b,k}[e] = \text{Factor} \left[ \frac{1}{\text{Total}[e]} \text{Plus} \left[ \begin{aligned} &\text{Sum} \left[ (-1)^{j+1} f_{a-1,b,k-1}[\text{Delete}[e, j]], \{j, 1, k\} \right], \\ &\text{Sum} \left[ (-1)^{j+1} f_{a-1,b,k}[\text{Delete}[e, j]], \{j, k+1, a+b\} \right], \\ &\text{Sum} \left[ (-1)^{j+a+b} f_{a,b-1,k-1}[\text{Delete}[e, j]], \{j, 1, k\} \right], \\ &\text{Sum} \left[ (-1)^{j+a+b+1} f_{a,b-1,k}[\text{Delete}[e, j]], \{j, k+1, a+b\} \right] \end{aligned} \right] \right];$

$f_{a,b,k}[e\_\_] := f_{a,b,k}[\{e\}];$

$g_{a,b}[s\_] := f_{a,b, \text{If}[\text{EvenQ}[a+b], (a+b)/2, (a+b+1)/2]}[s + \text{Range}[a+b]];$

$f_{1,1,1}[e1, e2]$

$-\frac{2}{e1 e2}$

$f_{2,2,2}[e1, e2, e3, e4]$

$-\left( \left( 2 (e1 - e2) (e3 - e4) \right. \right. \\ \left. \left. (e1^2 e2^2 + 3 e1^2 e2 e3 + 3 e1 e2^2 e3 + e1 e2 e3^2 + 3 e1^2 e2 e4 + 3 e1 e2^2 e4 + e1^2 e3 e4 + 2 e1 e2 e3 e4 + \right. \right. \\ \left. \left. e2^2 e3 e4 + 3 e1 e3^2 e4 + 3 e2 e3^2 e4 + e1 e2 e4^2 + 3 e1 e3 e4^2 + 3 e2 e3 e4^2 + e3^2 e4^2) \right) \right) / \\ (e1 e2 (e1 + e2) e3 (e1 + e3) (e2 + e3) e4 (e1 + e4) (e2 + e4) (e3 + e4))$

$f_{4,0,2}[e1, e2, e3, e4]$

$\left( (e1 - e2) (e1 - e3) (e2 - e3) (e1 - e4) (e2 - e4) (e3 - e4) \right) / \\ (e1 e2 (e1 + e2) e3 (e1 + e3) (e2 + e3) e4 (e1 + e4) (e2 + e4) (e3 + e4))$

**f<sub>6,0,2</sub>[e1, e2, e3, e4, e5, e6]**

$$\frac{(e1 - e2)(e1 - e3)(e2 - e3)(e1 - e4)(e2 - e4)(e3 - e4)(e1 - e5)(e2 - e5)(e3 - e5)(e4 - e5)(e1 - e6)(e2 - e6)(e3 - e6)(e4 - e6)(e5 - e6)}{(e1 e2 (e1 + e2) e3 (e1 + e3) (e2 + e3) e4 (e1 + e4) (e2 + e4) (e3 + e4) e5 (e1 + e5) (e2 + e5) (e3 + e5) (e4 + e5) e6 (e1 + e6) (e2 + e6) (e3 + e6) (e4 + e6) (e5 + e6)}$$

**g<sub>2,0</sub>[s] // Factor**

$$-\frac{1}{(1+s)(2+s)(3+2s)}$$

**Table[g<sub>2-j,j</sub>[s], {j, 0, 2}] // Simplify**

$$\left\{ -\frac{1}{6+13s+9s^2+2s^3}, -\frac{2}{2+3s+s^2}, -\frac{1}{6+13s+9s^2+2s^3} \right\}$$

**Sum[g<sub>4-j,j</sub>[s], {j, 0, 4}] // Simplify**

$$-\frac{4}{(1+s)(2+s)^2(3+s)(5+2s)^2}$$

**{f<sub>2,0,1</sub>[e1, e2], f<sub>1,1,1</sub>[e1, e2], f<sub>0,2,1</sub>[e1, e2]} // Factor**

$$\left\{ \frac{e1 - e2}{e1 e2 (e1 + e2)}, -\frac{2}{e1 e2}, \frac{e1 - e2}{e1 e2 (e1 + e2)} \right\}$$

**g<sub>2,2</sub>[s] // Factor**

$$-\frac{(2(11+10s+2s^2)(27+20s+4s^2))}{((1+s)(2+s)^2(3+s)^2(4+s)(3+2s)(5+2s)^2(7+2s))}$$

**Roots[(11 + 10 s + 2 s<sup>2</sup>) (27 + 20 s + 4 s<sup>2</sup>) == 0, s]**

$$s = \frac{1}{2}(-5 - \sqrt{3}) \quad || \quad s = \frac{1}{2}(-5 + \sqrt{3}) \quad || \quad s = \frac{1}{2}(-5 - i\sqrt{2}) \quad || \quad s = \frac{1}{2}(-5 + i\sqrt{2})$$

**g<sub>3,3</sub>[s]**

$$\frac{(8(6498 + 11886s + 8803s^2 + 3402s^3 + 733s^4 + 84s^5 + 4s^6))}{((1+s)(2+s)^2(3+s)^3(4+s)^3(5+s)^2(6+s)(3+2s)(5+2s)(7+2s)^2(9+2s)(11+2s))}$$

**g<sub>4,4</sub>[s]**

$$\frac{(144(4372404765750 + 16848330326100s + 30226607375760s^2 + 33485975037198s^3 + 25627514400759s^4 + 14365596714390s^5 + 6102539665580s^6 + 2004987787236s^7 + 515257013513s^8 + 104006124720s^9 + 16449977360s^{10} + 2019000384s^{11} + 188661472s^{12} + 12983040s^{13} + 620800s^{14} + 18432s^{15} + 256s^{16}))}{((1+s)(2+s)^2(3+s)^3(4+s)^4(5+s)^4(6+s)^3(7+s)^2(8+s)(3+2s)(5+2s)^2(7+2s)^3(9+2s)^4(11+2s)^3(13+2s)^2(15+2s))}$$

$g_{5,5}[s]$

$$- \left( (10368 (80959559679180000 + 317466561988422000s + 591592563497170800s^2 + 695441699816220720s^3 + 577440929968994286s^4 + 359447959310152350s^5 + 173834632743961735s^6 + 66820854214489430s^7 + 20723204742629761s^8 + 5235404373851850s^9 + 1083486544761945s^{10} + 184088304487890s^{11} + 25649872746001s^{12} + 2917336994000s^{13} + 268444616080s^{14} + 19694188800s^{15} + 1125849696s^{16} + 48364800s^{17} + 1469440s^{18} + 28160s^{19} + 256s^{20})) / \right. \\ \left. \left( (1+s)(2+s)^2(3+s)^3(4+s)^4(5+s)^5(6+s)^5(7+s)^4(8+s)^3(9+s)^2(10+s)(3+2s)(5+2s)^2(7+2s)^2(9+2s)^3(11+2s)^4(13+2s)^3(15+2s)^2(17+2s)^2(19+2s) \right) \right)$$

Solve[ $g_{4,4}[s] = 0, s$ ]

$$\left\{ \left\{ s \rightarrow \frac{1}{2} \left( -9 - \sqrt{(81 + 4 \text{Root}[4372404765750 + 1872036702900\#1 + 350056428060\#1^2 + 37290753542\#1^3 + 2471544713\#1^4 + 104179952\#1^5 + 2721760\#1^6 + 40192\#1^7 + 256\#1^8 \&, 1] )} \right) \right\}, \right. \\ \left\{ s \rightarrow \frac{1}{2} \left( -9 + \sqrt{(81 + 4 \text{Root}[4372404765750 + 1872036702900\#1 + 350056428060\#1^2 + 37290753542\#1^3 + 2471544713\#1^4 + 104179952\#1^5 + 2721760\#1^6 + 40192\#1^7 + 256\#1^8 \&, 1] )} \right) \right\}, \left\{ s \rightarrow \frac{1}{2} \left( -9 - \sqrt{(81 + 4 \text{Root}[4372404765750 + 1872036702900\#1 + 350056428060\#1^2 + 37290753542\#1^3 + 2471544713\#1^4 + 104179952\#1^5 + 2721760\#1^6 + 40192\#1^7 + 256\#1^8 \&, 2] )} \right) \right\}, \right. \\ \left\{ s \rightarrow \frac{1}{2} \left( -9 + \sqrt{(81 + 4 \text{Root}[4372404765750 + 1872036702900\#1 + 350056428060\#1^2 + 37290753542\#1^3 + 2471544713\#1^4 + 104179952\#1^5 + 2721760\#1^6 + 40192\#1^7 + 256\#1^8 \&, 2] )} \right) \right\}, \right. \\ \left\{ s \rightarrow \frac{1}{2} \left( -9 - \sqrt{(81 + 4 \text{Root}[4372404765750 + 1872036702900\#1 + 350056428060\#1^2 + 37290753542\#1^3 + 2471544713\#1^4 + 104179952\#1^5 + 2721760\#1^6 + 40192\#1^7 + 256\#1^8 \&, 3] )} \right) \right\}, \right. \\ \left\{ s \rightarrow \frac{1}{2} \left( -9 + \sqrt{(81 + 4 \text{Root}[4372404765750 + 1872036702900\#1 + 350056428060\#1^2 + 37290753542\#1^3 + 2471544713\#1^4 + 104179952\#1^5 + 2721760\#1^6 + 40192\#1^7 + 256\#1^8 \&, 3] )} \right) \right\}, \right. \\ \left\{ s \rightarrow \frac{1}{2} \left( -9 - \sqrt{(81 + 4 \text{Root}[4372404765750 + 1872036702900\#1 + 350056428060\#1^2 + 37290753542\#1^3 + 2471544713\#1^4 + 104179952\#1^5 + 2721760\#1^6 + 40192\#1^7 + 256\#1^8 \&, 4] )} \right) \right\}, \right. \\ \left\{ s \rightarrow \frac{1}{2} \left( -9 + \sqrt{(81 + 4 \text{Root}[4372404765750 + 1872036702900\#1 + 350056428060\#1^2 + 37290753542\#1^3 + 2471544713\#1^4 + 104179952\#1^5 + 2721760\#1^6 + 40192\#1^7 + 256\#1^8 \&, 4] )} \right) \right\}, \right. \\ \left\{ s \rightarrow \frac{1}{2} \left( -9 - \sqrt{(81 + 4 \text{Root}[4372404765750 + 1872036702900\#1 + 350056428060\#1^2 + 37290753542\#1^3 + 2471544713\#1^4 + 104179952\#1^5 + 2721760\#1^6 + 40192\#1^7 + 256\#1^8 \&, 5] )} \right) \right\}, \right. \\ \left\{ s \rightarrow \frac{1}{2} \left( -9 + \sqrt{(81 + 4 \text{Root}[4372404765750 + 1872036702900\#1 + 350056428060\#1^2 + 37290753542\#1^3 + 2471544713\#1^4 + 104179952\#1^5 + 2721760\#1^6 + 40192\#1^7 + 256\#1^8 \&, 5] )} \right) \right\}, \right. \\ \left\{ s \rightarrow \frac{1}{2} \left( -9 - \sqrt{(81 + 4 \text{Root}[4372404765750 + 1872036702900\#1 + 350056428060\#1^2 + 37290753542\#1^3 + 2471544713\#1^4 + 104179952\#1^5 + 2721760\#1^6 + 40192\#1^7 + 256\#1^8 \&, 5] )} \right) \right\} \right\}$$

$$\begin{aligned}
 & 37\,290\,753\,542 \, \#1^3 + 2\,471\,544\,713 \, \#1^4 + 104\,179\,952 \, \#1^5 + \\
 & 2\,721\,760 \, \#1^6 + 40\,192 \, \#1^7 + 256 \, \#1^8 \, \& , 6 ] ] ) \} , \\
 \{ s \rightarrow \frac{1}{2} ( -9 + \sqrt{ ( 81 + 4 \operatorname{Root} [ 4\,372\,404\,765\,750 + 1\,872\,036\,702\,900 \, \#1 + 350\,056\,428\,060 \, \#1^2 + \\
 & 37\,290\,753\,542 \, \#1^3 + 2\,471\,544\,713 \, \#1^4 + 104\,179\,952 \, \#1^5 + \\
 & 2\,721\,760 \, \#1^6 + 40\,192 \, \#1^7 + 256 \, \#1^8 \, \& , 6 ] ] ) \} , \\
 \{ s \rightarrow \frac{1}{2} ( -9 - \sqrt{ ( 81 + 4 \operatorname{Root} [ 4\,372\,404\,765\,750 + 1\,872\,036\,702\,900 \, \#1 + 350\,056\,428\,060 \, \#1^2 + \\
 & 37\,290\,753\,542 \, \#1^3 + 2\,471\,544\,713 \, \#1^4 + 104\,179\,952 \, \#1^5 + \\
 & 2\,721\,760 \, \#1^6 + 40\,192 \, \#1^7 + 256 \, \#1^8 \, \& , 7 ] ] ) \} , \\
 \{ s \rightarrow \frac{1}{2} ( -9 + \sqrt{ ( 81 + 4 \operatorname{Root} [ 4\,372\,404\,765\,750 + 1\,872\,036\,702\,900 \, \#1 + 350\,056\,428\,060 \, \#1^2 + \\
 & 37\,290\,753\,542 \, \#1^3 + 2\,471\,544\,713 \, \#1^4 + 104\,179\,952 \, \#1^5 + \\
 & 2\,721\,760 \, \#1^6 + 40\,192 \, \#1^7 + 256 \, \#1^8 \, \& , 7 ] ] ) \} , \\
 \{ s \rightarrow \frac{1}{2} ( -9 - \sqrt{ ( 81 + 4 \operatorname{Root} [ 4\,372\,404\,765\,750 + 1\,872\,036\,702\,900 \, \#1 + 350\,056\,428\,060 \, \#1^2 + \\
 & 37\,290\,753\,542 \, \#1^3 + 2\,471\,544\,713 \, \#1^4 + 104\,179\,952 \, \#1^5 + \\
 & 2\,721\,760 \, \#1^6 + 40\,192 \, \#1^7 + 256 \, \#1^8 \, \& , 8 ] ] ) \} , \\
 \{ s \rightarrow \frac{1}{2} ( -9 + \sqrt{ ( 81 + 4 \operatorname{Root} [ 4\,372\,404\,765\,750 + 1\,872\,036\,702\,900 \, \#1 + 350\,056\,428\,060 \, \#1^2 + \\
 & 37\,290\,753\,542 \, \#1^3 + 2\,471\,544\,713 \, \#1^4 + 104\,179\,952 \, \#1^5 + \\
 & 2\,721\,760 \, \#1^6 + 40\,192 \, \#1^7 + 256 \, \#1^8 \, \& , 8 ] ] ) \} \}
 \end{aligned}$$

**Solve**[g<sub>5,5</sub>[s] == 0, s]

$$\begin{aligned}
 & \{ \{ s \rightarrow \frac{1}{2} ( -11 - \\
 & \quad \sqrt{ ( 121 + 4 \operatorname{Root} [ 80\,959\,559\,679\,180\,000 + 28\,860\,596\,544\,402\,000 \, \#1 + 4\,650\,677\,412\,832\,800 \, \#1^2 + \\
 & \quad 445\,624\,941\,197\,520 \, \#1^3 + 28\,073\,792\,698\,686 \, \#1^4 + 1\,212\,519\,412\,626 \, \#1^5 + \\
 & \quad 36\,264\,599\,809 \, \#1^6 + 739\,166\,992 \, \#1^7 + 9\,784\,416 \, \#1^8 + 75\,520 \, \#1^9 + 256 \, \#1^{10} \, \& , 1 ] ] ) \} , \\
 \{ s \rightarrow \frac{1}{2} ( -11 + \sqrt{ ( 121 + 4 \operatorname{Root} [ 80\,959\,559\,679\,180\,000 + 28\,860\,596\,544\,402\,000 \, \#1 + 4\,650\,677\,412\,832\,800 \\
 & \quad \#1^2 + 445\,624\,941\,197\,520 \, \#1^3 + 28\,073\,792\,698\,686 \, \#1^4 + 1\,212\,519\,412\,626 \, \#1^5 + \\
 & \quad 36\,264\,599\,809 \, \#1^6 + 739\,166\,992 \, \#1^7 + 9\,784\,416 \, \#1^8 + 75\,520 \, \#1^9 + 256 \, \#1^{10} \, \& , 1 ] ] ) \} , \\
 \{ s \rightarrow \frac{1}{2} ( -11 - \sqrt{ ( 121 + 4 \operatorname{Root} [ 80\,959\,559\,679\,180\,000 + 28\,860\,596\,544\,402\,000 \, \#1 + 4\,650\,677\,412\,832\,800 \\
 & \quad \#1^2 + 445\,624\,941\,197\,520 \, \#1^3 + 28\,073\,792\,698\,686 \, \#1^4 + 1\,212\,519\,412\,626 \, \#1^5 + \\
 & \quad 36\,264\,599\,809 \, \#1^6 + 739\,166\,992 \, \#1^7 + 9\,784\,416 \, \#1^8 + 75\,520 \, \#1^9 + 256 \, \#1^{10} \, \& , 2 ] ] ) \} , \\
 \{ s \rightarrow \frac{1}{2} ( -11 + \sqrt{ ( 121 + 4 \operatorname{Root} [ 80\,959\,559\,679\,180\,000 + 28\,860\,596\,544\,402\,000 \, \#1 + 4\,650\,677\,412\,832\,800 \\
 & \quad \#1^2 + 445\,624\,941\,197\,520 \, \#1^3 + 28\,073\,792\,698\,686 \, \#1^4 + 1\,212\,519\,412\,626 \, \#1^5 + \\
 & \quad 36\,264\,599\,809 \, \#1^6 + 739\,166\,992 \, \#1^7 + 9\,784\,416 \, \#1^8 + 75\,520 \, \#1^9 + 256 \, \#1^{10} \, \& , 2 ] ] ) \} , \\
 \{ s \rightarrow \frac{1}{2} ( -11 - \sqrt{ ( 121 + 4 \operatorname{Root} [ 80\,959\,559\,679\,180\,000 + 28\,860\,596\,544\,402\,000 \, \#1 + 4\,650\,677\,412\,832\,800 \\
 & \quad \#1^2 + 445\,624\,941\,197\,520 \, \#1^3 + 28\,073\,792\,698\,686 \, \#1^4 + 1\,212\,519\,412\,626 \, \#1^5 + \\
 & \quad 36\,264\,599\,809 \, \#1^6 + 739\,166\,992 \, \#1^7 + 9\,784\,416 \, \#1^8 + 75\,520 \, \#1^9 + 256 \, \#1^{10} \, \& , 3 ] ] ) \} , \\
 \{ s \rightarrow \frac{1}{2} ( -11 + \sqrt{ ( 121 + 4 \operatorname{Root} [ 80\,959\,559\,679\,180\,000 + 28\,860\,596\,544\,402\,000 \, \#1 + 4\,650\,677\,412\,832\,800 \\
 & \quad \#1^2 + 445\,624\,941\,197\,520 \, \#1^3 + 28\,073\,792\,698\,686 \, \#1^4 + 1\,212\,519\,412\,626 \, \#1^5 + \\
 & \quad 36\,264\,599\,809 \, \#1^6 + 739\,166\,992 \, \#1^7 + 9\,784\,416 \, \#1^8 + 75\,520 \, \#1^9 + 256 \, \#1^{10} \, \& , 3 ] ] ) \} , \\
 \{ s \rightarrow \frac{1}{2} ( -11 - \sqrt{ ( 121 + 4 \operatorname{Root} [ 80\,959\,559\,679\,180\,000 + 28\,860\,596\,544\,402\,000 \, \#1 + 4\,650\,677\,412\,832\,800 \\
 & \quad \#1^2 + 445\,624\,941\,197\,520 \, \#1^3 + 28\,073\,792\,698\,686 \, \#1^4 + 1\,212\,519\,412\,626 \, \#1^5 + \\
 & \quad 36\,264\,599\,809 \, \#1^6 + 739\,166\,992 \, \#1^7 + 9\,784\,416 \, \#1^8 + 75\,520 \, \#1^9 + 256 \, \#1^{10} \, \& , 4 ] ] ) \} ,
 \end{aligned}$$

