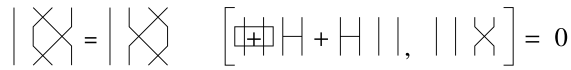
Cheat Sheet CD_a (with Travis Ens)

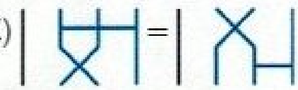
Notation. $id_1 = \mathsf{II}$, $id_2 = \mathsf{III}$, $d_0 D = \mathsf{II} \vdash D$, $d_\infty D = D \vdash \mathsf{II}$.

Theorem. As strict monoidal categories enriched over graded algebras, $CD_a = SMC\langle \mathsf{H}, \mathsf{IX} \mid \mathcal{R}\rangle$[1], where $\deg(\mathsf{H}, \mathsf{IX}) = (1, 0)$ and where $\mathcal{R}$ is generated by:

- Idempotency: $(\mathsf{IX})^2 = id_2$.
- Flat R3: $(\mathsf{IX} \vdash \mathsf{II})(\mathsf{II} \vdash \mathsf{IX})(\mathsf{IX} \vdash \mathsf{II}) = (\mathsf{II} \vdash \mathsf{IX})(\mathsf{IX} \vdash \mathsf{II})(\mathsf{II} \vdash \mathsf{IX})$
- Chord Flip: $[d_0\mathsf{H} + \mathsf{IH}, \mathsf{IX}] = 0$.



Flat R3 Chord Flip

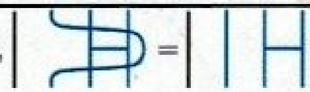
- *Chord slide:* $d_\infty(\mathsf{IX}) \cdot d_0(\mathsf{IX}) \cdot \left(d_0\mathsf{H} - \mathsf{IX}\cdot\mathsf{HI}\cdot\mathsf{IX}\right) \cdot d_0(\mathsf{IX}) = \left(d_0^2(\mathsf{H}) - d_0\mathsf{IX}\cdot d_0\mathsf{H}\cdot d_0\mathsf{IX}\right)\cdot d_\infty(\mathsf{IX})$ 

Corollary 1. $\mathbf{GRT}_a = \mathrm{Aut}(\widehat{\mathcal{CD}_a})$ *is all grouplike, invertible elements* $\Gamma \in \widehat{\mathcal{U}\mathfrak{t}_3}$ *which satisfy the equations*

$$\Gamma^{-1} = \Gamma^{0,2,1}$$

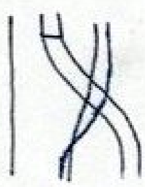
$$\Gamma^{0,1,2}\Gamma^{02,1,3}\Gamma^{0,2,3} = \Gamma^{01,2,3}\Gamma^{0,1,3}\Gamma^{03,1,2}$$

$$\Gamma^{0,1,2}\Gamma^{02,1,3}\left(t_{03} + t_{23} - \Gamma^{0,2,3}t_{03}\Gamma^{0,3,2}\right)\Gamma^{02,3,1} = \left(t_{03} + t_{13} + t_{23} - \Gamma^{01,2,3}(t_{03} + t_{13})\Gamma^{01,3,2}\right)\Gamma^{0,1,2}$$

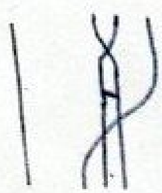
Modulo other relations,  implies the chord slide relation. Let E be the error in this relation.

Syzygies:

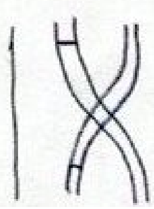
a) $E^{\cancel{0},2,3,4} + E^{\cancel{0}2,1,3,4} - E^{\cancel{0},1,3,4} - E^{\cancel{0}1,2,3,4}$



b) $E^{\cancel{0},1,2,3} = E^{\cancel{0},1,3,2}$



c) $[t_{12}, E^{\cancel{0},1,3,4} + E^{\cancel{0}1,2,3,4}] = [t_{34}, E^{\cancel{0},3,1,4} + E^{\cancel{0}3,4,1,2}]$



Computations:

Degree	1	2	3	4	5	6	7	8	9	10
All Syzygies.	3	0	1	2	3	4				
Syzygy a)	4	1	3	5	11	17	35	59	111	
Free Lie	2	1	2	3	6	9	18	30	56	99
Syzygy b)	4	1	5	9	27	60				
Syzygy c)	6	4	10	21	54	125				
Syzygy ab)	3	0	2	2	6	8	18	28		
Syzygy ac)	4	1	2	4	6	9				
Syzygy ~~ab~~) abc	3	0	1	2	3	4				

[1] IX is \mathrm{\mathsf{IX}}.