Initialization

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\People\\Ens\\171205"];
<< FreeLie.m;
$SeriesShowDegree = 1;
$IterationLimit = 2^20;
Arb = Arbitrator → (Replace[#, _ :> RandomInteger[{-100, 100}], 1] &);
```

```
FreeLie` implements / extends
 {*, +, **, $SeriesShowDegree, ⟨⟩, ∫, ≡, ad, Ad, adSeries, AllCyclicWords, AllLyndonWords,
  AllWords, Arbitrator, ASeries, AW, b, BCH, BooleanSequence, BracketForm, BS, CC, Crop, cw,
  CW, CWS, CWSeries, D, Deg, DegreeScale, DerivationSeries, div, DK, DKS, DKSeries, EulerE,
  Exp, Inverse, j, J, JA, LieDerivation, LieMorphism, LieSeries, LS, LW, LyndonFactorization,
  Morphism, New, RandomCWSeries, Randomizer, RandomLieSeries, RC, SeriesSolve, Support,
  t, tb, TopBracketForm, tr, UndeterminedCoefficients, αMap, Γ, ι, Λ, σ, ħ, ⎼, ⏜}.

FreeLie` is in the public domain. Dror Bar-Natan is committed
  to support it within reason until July 15, 2022. This is version 150814.
```

Testing The Reidemeister III Syzygy

```
FF = DKS[4, FFs];
R3Info = SeriesSolve[FF,
    (FF ≡ FF^σ[1,2,4,3]) ∧ (FF^σ[1,3,4,5] + FF^σ[13,2,4,5] - FF^σ[1,2,4,5] - FF^σ[12,3,4,5] ≡ DKS[0]), Arb];

Do[FF@{k}; Print[Length[Last[#]] & /@ Read[R3Info]], {k, 9}]
```

```
{3, 0, 2, 2, 6, 8, 18, 28}

{3, 0, 2, 2, 6, 8, 18, 28}

{3, 0, 2, 2, 6, 8, 18, 28}

{3, 0, 2, 2, 6, 8, 18, 28}

{3, 0, 2, 2, 6, 8, 18, 28}

{3, 0, 2, 2, 6, 8, 18, 28}

{3, 0, 2, 2, 6, 8, 18, 28}

{3, 0, 2, 2, 6, 8, 18, 28}

SeriesSolve: In degree 9 arbitrarily setting
    {FFs[4, 1, 1, 1, 1, 1, 1, 1, 2, 3] → 97, FFs[4, 1, 1, 1, 1, 1, 2, 3, 3, 3] → 16, FFs[4, 1, 1, 1, 1, 1, 3, 2, 3, 3] → 81, FFs[4, 1, 1, 1, 1, 1, 3, 3, 2, 3
    ] → -56, FFs[4, 1, 1, 1, 1, 3, 1, 2, 3, 3] → -61, FFs[4, 1, 1, 1, 1, 3, 1, 3, 2, 3] → 73, FFs[4, 1, 1, 1, 1, 3, 3, 1, 2, 3] → -84, FFs[4, 1, 1
    , 1, 2, 3, 3, 3, 3, 3] → -87, FFs[4, 1, 1, 1, 3, 1, 1, 2, 3, 3] → 59, ≪33≫, FFs[4, 2, 2, 2, 3, 3, 3, 3, 2, 3] → 62, FFs[4, 2, 2, 3, 2, 2, 3, 3
    , 3, 3] → 78, FFs[4, 2, 2, 3, 2, 3, 2, 3, 3, 3] → -41, FFs[4, 2, 2, 3, 2, 3, 3, 2, 3, 3] → 48, FFs[4, 2, 2, 3, 2, 3, 3, 3, 2, 3] → -60, FFs[4,
    2, 2, 3, 3, 2, 2, 3, 3, 3] → -8, FFs[4, 2, 2, 3, 3, 2, 3, 2, 3, 3] → 88, FFs[4, 2, 2, 3, 3, 2, 3, 3, 2, 3] → -37, ≪6≫}.

{3, 0, 2, 2, 6, 8, 18, 28, 56}
```