Initialization

```
SetDirectory["/home/drorbn/AcademicPensieve/People/Ens/171205"];
<< FreeLie.m;
$SeriesShowDegree = 1;
$IterationLimit = 2^20;
Arb = Arbitrator → (Replace[#, _ :↦ RandomInteger[{-100, 100}], 1] &);
```

```
FreeLie` implements / extends
 {*, +, **, $SeriesShowDegree, ⟨⟩, ∫, ≡, ad, Ad, adSeries, AllCyclicWords, AllLyndonWords,
  AllWords, Arbitrator, ASeries, AW, b, BCH, BooleanSequence, BracketForm, BS, CC, Crop, cw,
  CW, CWS, CWSeries, D, Deg, DegreeScale, DerivationSeries, div, DK, DKS, DKSeries, EulerE,
  Exp, Inverse, j, J, JA, LieDerivation, LieMorphism, LieSeries, LS, LW, LyndonFactorization,
  Morphism, New, RandomCWSeries, Randomizer, RandomLieSeries, RC, SeriesSolve, Support,
  t, tb, TopBracketForm, tr, UndeterminedCoefficients, αMap, Γ, ι, Λ, σ, ħ, ⎼, ⏜}.

FreeLie` is in the public domain. Dror Bar-Natan is committed
  to support it within reason until July 15, 2022. This is version 150814.
```

Testing The Reidemeister III Syzygy

```
FF = DKS[4, FFs];
R3Info = SeriesSolve[FF, FF^σ[1,3,4,5] + FF^σ[13,2,4,5] - FF^σ[1,2,4,5] - FF^σ[12,3,4,5] ≡ DKS[0], Arb];
```

```
Do[FF@{k}; Print[Length[Last[#]] & /@ Read[R3Info]], {k, 12}]
```

```
SeriesSolve: In degree 1 arbitrarily setting {FFs[2, 1] → -92, FFs[3, 2] → 57, FFs[4, 2] → 0, FFs[4, 3] → 2}.

{4}

SeriesSolve: In degree 2 arbitrarily setting {FFs[4, 2, 3] → -45}.

{4, 1}

SeriesSolve: In degree 3 arbitrarily setting {FFs[4, 1, 2, 3] → 11, FFs[4, 2, 2, 3] → 54, FFs[4, 2, 3, 3] → -95}.

... General: Further output of SeriesSolve::ArbitrarilySetting will be suppressed during this calculation.

{4, 1, 3}

{4, 1, 3, 5}

{4, 1, 3, 5, 11}

{4, 1, 3, 5, 11, 17}

{4, 1, 3, 5, 11, 17, 35}

{4, 1, 3, 5, 11, 17, 35, 59}

{4, 1, 3, 5, 11, 17, 35, 59, 111}

{4, 1, 3, 5, 11, 17, 35, 59, 111, 197}
```

```
ArbR3 = Length[Last[#]] & /@ Read[R3Info]
```

{4, 1, 3, 5, 11, 17, 35, 59, 111, 0}