

```

Mobius[z_] := I  $\frac{1+z}{1-z}$ ;
ReIm := {Re[#], Im[#]} &

ParametricPlot[ReIm[Mobius[r Exp[I  $\theta$ ]]], {r, 0, 1}, { $\theta$ , 0, 2  $\pi$ }]

ComPolar[{z_, f_}] := {Re[z], Im[z], f}

ParametricPlot3D[Table[ComPolar[{r Exp[I  $\theta$ ], 2  $\pi$  n / 4}], {n, 0, 4}], {r, 0, 1}, { $\theta$ , 0, 2  $\pi$ }]

ParametricPlot3D[
  Table[ComPolar[{Mobius[r Exp[I  $\theta$ ]], 2  $\pi$  n / 4}], {n, 0, 4}], {r, 0, 1}, { $\theta$ , 0, 2  $\pi$ }]

SterProj[{{x_, y_}, {z_, w_}}] :=  $\left\{ \frac{x}{1-w}, \frac{y}{1-w}, \frac{z}{1-w} \right\}$ 
vertRot = RotationTransform[ $\pi/2$ , {1, 0, 0}]
radRot = RotationTransform[ $\pi/2$ , {0, 0, 1}]
Manipulate[
  Show[ParametricPlot3D[
    Table[ComPolar[{r Exp[I  $\theta$ ], 2  $\pi$  n / 4}], {n, 0, 4}], {r, 0, 1}, { $\theta$ , 0, 2  $\pi$ }],
    ParametricPlot3D[vertRot@radRot@SterProj[ReIm /@ {r Exp[I  $\theta$ ], Exp[I  $\phi$ ]}],
    {r, 0, 1}, { $\theta$ , 0, 2  $\pi$ }], { $\phi$ , 0, 2  $\pi$ }]

ParametricPlot3D[
  Table[ComPolar[{Mobius[r Exp[I  $\theta$ ]], 2  $\pi$  n / 4}], {n, 0, 4}], {r, 0, 1}, { $\theta$ , 0, 2  $\pi$ }]

SterProj[{{x_, y_}, {z_, w_}}] :=  $\left\{ \frac{x}{1-w}, \frac{y}{1-w}, \frac{z}{1-w} \right\}$ 
vertRot = RotationTransform[ $\pi/2$ , {1, 0, 0}];
radRot = RotationTransform[ $\pi/2$ , {0, 0, 1}];

Show[ParametricPlot3D[
  Table[ComPolar[{r Exp[I  $\theta$ ], 2  $\pi$  n / 4}], {n, 0, 4}], {r, 0, 1}, { $\theta$ , 0, 2  $\pi$ }],
  ParametricPlot3D[Table[radRot@vertRot@ComPolar[{r Exp[I  $\theta$ ], 2  $\pi$  n / 4}], {n, 0, 4}],
  {r, 0, 1}, { $\theta$ , 0, 2  $\pi$ }]

Torus[{a_, b_,  $\phi$ _}] := {(1+a) Sin[ $\phi$ ], (1+a) Cos[ $\phi$ ], b}
Torus[ComPolar[{r Exp[I  $\theta$ ], 2  $\pi$  n / 4}]]

Show[ParametricPlot3D[
  Table[Torus@ComPolar[{r Exp[I  $\theta$ ], 2  $\pi$  n / 4}], {n, 0, 4}], {r, 0, 1}, { $\theta$ , 0, 2  $\pi$ }],
  ParametricPlot3D[Table[radRot@vertRot@ComPolar[{r Exp[I  $\theta$ ], 2  $\pi$  n / 4}], {n, 0, 4}],
  {r, 0, 1}, { $\theta$ , 0, 2  $\pi$ }]

Rot[ $\omega$ ] := RotationTransform[ $\omega$ , {0, 0, 1}]

f = Table[Torus@Rot[- $\pi/2$ ]@ComPolar[{Mobius@{r Exp[I  $\theta$ ]], 2  $\pi$  n / 4}] + {0, 0,  $\pi$ },
  {n, 0, 4}] // ComplexExpand;

ParametricPlot3D[f, {r, 0, 1}, { $\theta$ , 0, 2  $\pi$ }, PlotRange -> {{-10, 10}, {-10, 10}, {-10, 10}}]

```

```
Show[
  ParametricPlot3D[f, {r, 0, 1}, {θ, 0, 2π}],
  ParametricPlot3D[Table[ComPolar[{r Exp[I θ], 2π n / 4}], {n, 0, 4}],
    {r, 0, 1}, {θ, 0, 2π}, PlotRange → {{-3, 3}, {-3, 3}, {-3, 3}}]
]
```

```
ParametricPlot3D[
  Table[ComPolar[{r Exp[I θ], r Sin[θ] + 2π n / 4}], {n, 0, 4}], {r, 0, 1}, {θ, 0, 2π}]
f = Table[Torus@Rot[-π/2]@ComPolar[{Mobius@ (r Exp[I θ]), r Sin[θ] + 2π n / 4}] + {0, 0, π},
  {n, 0, 4}] // ComplexExpand;
```

```
f = Table[Torus@Rot[0]@ComPolar2[{Unwrap@ (r Exp[I θ]), r Sin[θ] + 2π n / 4}] + {0, 0, π},
  {n, 0, 4}] // ComplexExpand;
ParametricPlot3D[f, {r, 0, 1}, {θ, 0, 2π}]
```

```
ParametricPlot3D[
  Table[ComPolar[{r Exp[I θ], r Sin[θ] + 2π n / 4}], {n, 0, 4}], {r, 0, 1}, {θ, 0, 2π}]
```

```
ParametricPlot[Unwrap[r Exp[I θ]], {r, 0, 1}, {θ, 0, 2π}, PlotPoints → 100]
```

```
Unwrap[z_] := {Abs[z] - 3, τArg[z]}
ComPolar2[{{a_, b_}, f_] := {a, b, f};
```

```
τArg[r Exp[I θ]]
```

```
Arg[0.1 Exp[I * -3]]
```