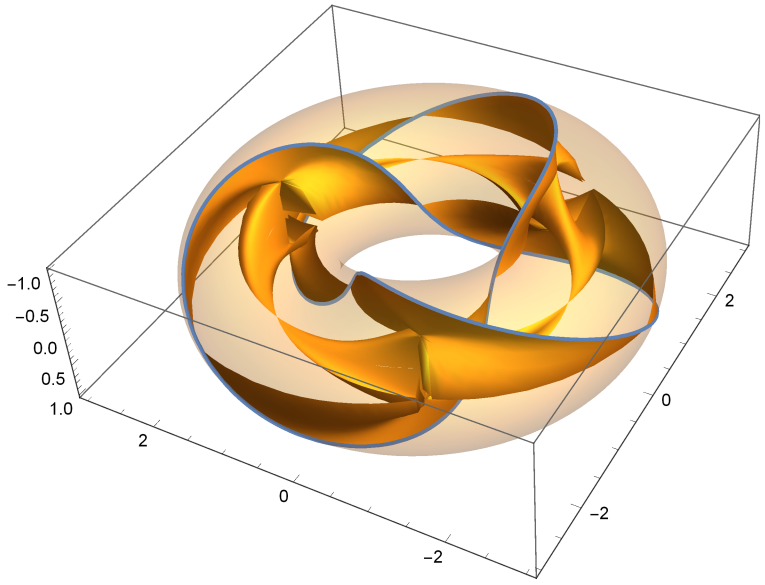
# Defining Rolfsen Screw

```
twist = {r {x, Sin[3/2 x], Cos[3/2 x]} +
    (1 - r) {2 π/3 - x, Sin[3/2 (2 π/3 - x)], Cos[3/2 (2 π/3 - x)]} +
    {2/3 π n, 0, 0}, r {x, -Sin[3/2 x], -Cos[3/2 x]} +
    (1 - r) {2 π/3 - x, -Sin[3/2 (2 π/3 - x)], -Cos[3/2 (2 π/3 - x)]} + {2/3 π n, 0, 0}}

ParametricPlot3D[twist /. n → 0, {x, 0, 2 π/3}, {r, 0, 1}]

trefoiltwist = Flatten[Table[twist, {n, 0, 2}], 1]
ParametricPlot3D[trefoiltwist, {x, 0, 2 π/3}, {r, 0, 1}, Exclusions → None]


R[{x_, y_, z_}, θ_] := {x + θ/3, y Cos[θ/2] + z Sin[θ/2], -y Sin[θ/2] + z Cos[θ/2]}

R[#, π/2] & /@ trefoiltwist // Simplify
ParametricPlot3D[R[#, π/2] & /@ trefoiltwist // Simplify, {x, 0, 2 π/3}, {r, 0, 1}]
```

# Different Torus Projection Attempt

```
onetwist
ParametricPlot3D[onetwist /. n → 0, {x, 0, 2 π/3}, {r, 0, 1}]

Theta[{x_, y_, z_}] := ArcTan[y, z]
Phi[{x_, y_, z_}] := x

AnotherTorus[leaf_] := Module[{θ = Theta[leaf], ϕ = Phi[leaf]},
  {(2 + r Cos[θ]) Cos[ϕ], (2 + r Cos[θ]) Sin[ϕ], r Sin[θ]} // Simplify]


ParametricPlot3D[AnotherTorus /@ trefoiltwist, {x, 0, 2 π/3}, {r, 0, 1}]


Show[
 ParametricPlot3D[{(2 + Cos[θ]) Cos[ϕ], (2 + Cos[θ]) Sin[ϕ], Sin[θ]}, {θ, 0, 2 π},
  {ϕ, 0, 2 π}, PlotStyle → Directive[Opacity[0.2]], Mesh → None, PlotPoints → 100],
 ParametricPlot3D[AnotherTorus /@ trefoiltwist, {x, 0, 2 π/3}, {r, 0, 1}, Mesh → None],
 ParametricPlot3D[RotationMatrix[π/3, {0, 0, 1}].{(2 - Cos[3 t]) Cos[2 t],
    (2 - Cos[3 t]) Sin[2 t], Sin[3 t]}, {t, 0, 2 π}, PlotStyle → {Thick}]

]
```

```
Show[
 ParametricPlot3D[{(2 + Cos[θ]) Cos[ϕ], (2 + Cos[θ]) Sin[ϕ], Sin[θ]}, {θ, 0, 2 π},
  {ϕ, 0, 2 π}, PlotStyle → Directive[Opacity[0.2]], Mesh → None, PlotPoints → 100],
 ParametricPlot3D[AnotherTorus /@ ((R[#, Pi] & /@ trefoiltwist) // Simplify) // Simplify,
  {x, 0, 2 π / 3}, {r, 0, 1}, Mesh → None],
 ParametricPlot3D[RotationMatrix[π / 3, {0, 0, 1}].{(2 - Cos[3 t]) Cos[2 t],
    (2 - Cos[3 t]) Sin[2 t], Sin[3 t]}, {t, 0, 2 π}, PlotStyle → {Thick}]

]
```




```
trefoiltwist
```

```
R[#, 0] & /@ trefoiltwist
```

```
AnotherTorus /@ ((R[#, 0] & /@ trefoiltwist) // Simplify) // Simplify
```

# Projecting to Torus

```
onetwist = twist[[1]] /. n → 1
zRot[θ_] := RotationMatrix[θ, {0, 0, 1}]
TorusProject[leaf_] := Module[
  {θ = {1, 0, 0}.leaf},
  zRot[θ].({0, 1, 1} * leaf) + {Cos[θ], Sin[θ], 0}
 ]
```

```
ParametricPlot3D[zRot[π / 2].({0, 1, 1} * onetwist) +
  {Cos[{1, 0, 0}.onetwist], Sin[{1, 0, 0}.onetwist], 0}, {x, 0, 2 π / 3}, {r, 0, 1}]
```

```
trefoiltwist
```

```
trefoilscrew = ((Table[Map[RotateFunction, trefoiltwist], {n, 0, 2}]) // Simplify)
```

```
ParametricPlot3D[trefoilscrew, {x, 0, 2 π / 3}, {r, 0, 1}]
```

# Badlands

```
Manipulate[
 Show[ParametricPlot3D[
   RotationMatrix[θ, {0, 0, 1}].Table[twist, {n, 0, 0}][[1, 1]], {x, 0, 2 π / 3}, {r, 0, 1}],
  Graphics3D[Arrow[{{0, 0, 0}, twist[[1]] /. {r → 0.5, x → 0.5, n → 0}}]]], {θ, 0, 2 π}]
```

```
twist[[1]] /. {r → 0.5, x → 0.5, n → 0}
```

```
Graphics3D[Arrow[{{0, 0, 0}, twist[[1, 1]] /. {r → 0.5, x → 0.5}}]]
```

```
Table[twist, {n, 0, 0}][[1, 1]]
```

```
ParametricPlot3D[Table[twist, {n, 0, 2}], {x, 0, 2 π / 3}, {r, 0.8, 1}]
```

```
ParametricPlot3D[Table[twist, {n, 0, 2}], {x, 0, 2 π / 3}, {r, 0, 1}]
```

```
R[{x_, y_, z_}, θ_] := {x + θ / 3, y Cos[θ / 2] + z Sin[θ / 2], -y Sin[θ / 2] + z Cos[θ / 2]}
```

```
ParametricPlot3D[Table[q@R[#, 0] & /@ twist, {n, 0, 3}], {x, 0, 2 π / 3}, {r, 0, 1}]
```

```
q[{x_, y_, z_}, r_ : 1, R_ : 2] := {R + r Sin[x], R + r Cos[x] y, z}
```

```
Torus[r_ : 1, R_ : 2] := {(R + r Cos[θ]) Cos[ϕ], (R + r Cos[θ]) Sin[ϕ], r Sin[θ]}
```

```
ParametricPlot3D[Torus[], {θ, 0, 2 Pi}, {ϕ, 0, 2 Pi}]
```

$$\theta = r x + (1 - r)\left(2\frac{\pi}{3} - x\right) + 2/3\,\pi n$$

```
torustwist = {RotationMatrix[θ, {0, 0, 1}].
    (r {x, Sin[3 / 2 x], Cos[3 / 2 x]} + (1 - r) {2 π / 3 - x, Sin[3 / 2 (2 π / 3 - x)],
        Cos[3 / 2 (2 π / 3 - x)]} - {θ, 0, 0} + {2 / 3 π n, 0, 0} + 2 {Cos[θ], Sin[θ], 0}),
  RotationMatrix[θ, {0, 0, 1}].(r {x, -Sin[3 / 2 x], -Cos[3 / 2 x]} +
      (1 - r) {2 π / 3 - x, -Sin[3 / 2 (2 π / 3 - x)], -Cos[3 / 2 (2 π / 3 - x)]}) +
    {2 / 3 π n, 0, 0} - {θ, 0, 0} + 2 {Cos[θ], Sin[θ], 0}}
```

```
ParametricPlot3D[Table[torustwist, {n, 0, 2}], {x, 0, 2 π / 3}, {r, 0, 1}]
```

```
CoordinateTransform["Cartesian" -> "Cylindrical", {1, 1, 1}]
```