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SterProj[{{z1_, z2_}, {w1_, w2_}}] := {
   $\frac{z1}{\text{Sqrt}[2] - w2}$ ,  $\frac{z2}{\text{Sqrt}[2] - w2}$ ,  $\frac{w1}{\text{Sqrt}[2] - w2}$ 
};
knotpairs = {
   $\xi \text{Exp}[q \text{I} \theta]$ ,  $\eta \text{Exp}[p \text{I} \theta + \frac{\pi \text{I}}{q}]$ 
};
Factor[ComplexExpand[{Re[#], Im[#]} & /@ (knotpairs /. {q → 2, p → 3, ξ → 1, η → 1})]];
SterProj[%];
knotplot = ParametricPlot3D[%, {θ, 0, 2 π}, Boxed → False, Axes → False]

knotpoly = z^2 + w^3 /. {z → zr + I zi, w → wr + I wi}
ComplexExpand[{Re[knotpoly], Im[knotpoly]}]
# == 0 & /@ ComplexExpand[{Re[knotpoly], Im[knotpoly]}]

InverseStereographicProjection = {
  zr →  $\frac{4 zr}{zr^2 + zi^2 + wr^2 + 2}$ , zi →  $\frac{4 zi}{zr^2 + zi^2 + wr^2 + 2}$ ,
  wr →  $\frac{4 wr}{zr^2 + zi^2 + wr^2 + 2}$ , wi →  $\frac{\text{Sqrt}[2] (zr^2 + zi^2 + wr^2 - 2)}{zr^2 + zi^2 + wr^2 + 2}$ 
}
(*InverseStereographicProjection = {
  zr →  $\frac{2zr}{zr^2 + zi^2 + wr^2 + 1}$ ,
  zi →  $\frac{2zi}{zr^2 + zi^2 + wr^2 + 1}$ , wr →  $\frac{2wr}{zr^2 + zi^2 + wr^2 + 1}$ , wi →  $\frac{zr^2 + zi^2 + wr^2 - 1}{zr^2 + zi^2 + wr^2 + 1}$ 
} *)
Factor[ComplexExpand[{Re[knotpoly], Im[knotpoly]}] /. InverseStereographicProjection]
{f, g} = Simplify[% * (2 + wr^2 + zi^2 + zr^2)^3]
function = Cos[θ] f + Sin[θ] g /. θ → Pi / 2
Show[ContourPlot3D[{function == 0}, {zr, -4, 4},
  {zi, -4, 4}, {wr, -4, 4}, PlotPoints → 40, Mesh → False,
  Boxed → False, Axes → False], knotplot, ViewPoint → {5, 0, 0}]

zr^2 + zi^2 + wr^2 + wi^2 == 1
{zr^2 + zi^2 + wr^2 + wi^2 == 1, -3 wi^2 wr + wr^3 - zi^2 + zr^2 == 0, -wi^3 + 3 wi wr^2 + 2 zi zr == 0}
Solve[{wi^2 + wr^2 + zi^2 + zr^2 == 1,
  -3 wi^2 wr + wr^3 - zi^2 + zr^2 == 0, -wi^3 + 3 wi wr^2 + 2 zi zr == 0}, {wi, wr, zi, zr}]

{wi^2 + wr^2 + zi^2 + zr^2 == Sqrt[2], -3 wi^2 wr + wr^3 - zi^2 + zr^2 == 0, -wi^3 + 3 wi wr^2 + 2 zi zr == 0} /.
  InverseStereographicProjection
Solve[%23, {wr, zi, zr}]
Solve[%14, {wr, zi, zr}]
RegionPlot3D[cond, {zr, -100, 100}, {zi, -100, 100}, {wr, -100, 100}]
cond = (wi^2 + wr^2 + zi^2 + zr^2 == 1 && -3 wi^2 wr + wr^3 - zi^2 + zr^2 == 0 && -wi^3 + 3 wi wr^2 + 2 zi zr == 0) /.
  InverseStereographicProjection
Solve[cond, {wr, zi, zr}]

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Simplify[cond]

Solve[%43, {wr, zi, zr}]

Simplify[Factor[# == 0 & /@ ComplexExpand[{Re[knotpoly], Im[knotpoly]}] /. InverseStereographicProjection]]

cond =

$$\frac{1}{1 + wr^2 + zi^2 + zr^2} \left(3 wr^5 + 2 wr^2 (zi^2 - zr^2) + 3 wr (-1 + zi^2 + zr^2)^2 + 2 wr^3 (-5 + 3 zi^2 + 3 zr^2) + 2 (zi^2 + zi^4 - zr^2 (1 + zr^2)) \right) == 0 \&\& \frac{1}{1 + wr^2 + zi^2 + zr^2} \left(wr^6 + zi^6 - 8 zi^3 zr + 3 zi^4 (-1 + zr^2) + 3 zi^2 (-1 + zr^2)^2 + (-1 + zr^2)^3 + 3 wr^4 (-5 + zi^2 + zr^2) - 8 zi (zr + zr^3) + wr^2 (3 zi^4 - 8 zi zr + 6 zi^2 (-3 + zr^2) + 3 (5 - 6 zr^2 + zr^4)) \right) == 0 \&\& (wi^2 + wr^2 + zi^2 + zr^2) == 1 /. InverseStereographicProjection$$

cond = {wi^2 + wr^2 + zi^2 + zr^2 == Sqrt[2], -3 wi^2 wr + wr^3 - zi^2 + zr^2 == 0, -wi^3 + 3 wi wr^2 + 2 zi zr == 0} /. InverseStereographicProjection

== 0 & /@ eqns

== 0 & /@ eqns

$$-8 \left(3 wr^5 + 2 wr^2 (zi^2 - zr^2) + 3 wr (-2 + zi^2 + zr^2)^2 + wr^3 (-20 + 6 zi^2 + 6 zr^2) + 2 (2 zi^2 + zi^4 - zr^2 (2 + zr^2)) \right) == 0 \&\& -2 \left(\sqrt{2} wr^6 + \sqrt{2} zi^6 - 16 zi^3 zr + 3 \sqrt{2} zi^4 (-2 + zr^2) + 3 \sqrt{2} zi^2 (-2 + zr^2)^2 + \sqrt{2} (-2 + zr^2)^3 - 16 zi zr (2 + zr^2) + 3 \sqrt{2} wr^4 (-10 + zi^2 + zr^2) + wr^2 (3 \sqrt{2} zi^4 - 16 zi zr + 6 \sqrt{2} zi^2 (-6 + zr^2) + 3 \sqrt{2} (20 - 12 zr^2 + zr^4)) \right) == 0$$

RegionPlot3D[-8 (3 wr^5 + 2 wr^2 (zi^2 - zr^2) + 3 wr (-2 + zi^2 + zr^2)^2 + wr^3 (-20 + 6 zi^2 + 6 zr^2) + 2 (2 zi^2 + zi^4 - zr^2 (2 + zr^2))) == 0 && -2 (sqrt(2) wr^6 + sqrt(2) zi^6 - 16 zi^3 zr + 3 sqrt(2) zi^4 (-2 + zr^2) + 3 sqrt(2) zi^2 (-2 + zr^2)^2 + sqrt(2) (-2 + zr^2)^3 - 16 zi zr (2 + zr^2) + 3 sqrt(2) wr^4 (-10 + zi^2 + zr^2) + wr^2 (3 sqrt(2) zi^4 - 16 zi zr + 6 sqrt(2) zi^2 (-6 + zr^2) + 3 sqrt(2) (20 - 12 zr^2 + zr^4))) == 0, {zr, -1, 1}, {zi, -1, 1}, {wr, -1, 1}]

ParametricPlot3D[{ 0.5 Cos[2 theta] / (1 - 0.5 Cos[3 theta]), 0.5 Sin[2 theta] / (1 - 0.5 Cos[3 theta]), -0.5 Sin[3 theta] / (1 - 0.5 Cos[3 theta]) }, {theta, -26.3894, 26.3894}]

RegionPlot3D@ImplicitRegion[0.8 < x^2 + y^2 + z^2 < 1 && x > 0, {x, y, z}]

ExpToTrig[Exp[I x]]