

```

ReIm := {Re@#, Im@#} &
SterProj[{{z1_, z2_}, {w1_, w2_}}] := { $\frac{z1}{1-w2}$ ,  $\frac{z2}{1-w2}$ ,  $\frac{w1}{1-w2}$ };
T2 = { $\alpha \text{Exp}[I \theta] / \text{Sqrt}[2]$ ,  $\text{Exp}[I \phi] / \text{Sqrt}[2]$ };
ReIm@ (ComplexExpand /@ T2)
T = ReIm /@ {Sin[ $\alpha$ ] Exp[I  $\theta$ ], Cos[ $\alpha$ ] Exp[I  $\phi$ ]}

SterProj[T]
Manipulate[ParametricPlot3D[SterProj[T] /.  $\alpha \rightarrow A$ , { $\theta$ , 0, 2 Pi}, { $\phi$ , 0, 2 Pi}], {A, 0, Pi/2}]
ParametricPlot3D[SterProj[T] /.  $\alpha \rightarrow \pi/8$ , { $\theta$ , 0, 2 Pi}, { $\phi$ , 0, 2 Pi}]
ParametricPlot3D[knot, { $\theta$ , 0, 2  $\pi$ }, PlotStyle -> {Thick, Blue}]
ParametricPlot3D[
  {r Sin[ $\theta$ ], r Cos[ $\theta$ ],  $1/3 \text{Abs}[\text{Cos}[\theta] + i \text{Sin}[\theta]]^3 \text{Arg}[-(\text{Cos}[\theta] + i \text{Sin}[\theta])^2]}$ },
  { $\theta$ , 0, 2 Pi}, {r, 0, 1}, PlotPoints -> 50]
Abs[x + I y]^3 Arg[(x + I y)^3] / 2
Plot3D[ $\frac{1}{2} \text{Abs}[x + i y]^3 \text{Arg}[(x + i y)^3]$ , {x, -20., 20.}, {y, -20., 20.}]
surface = Table[{r Sin[ $\theta$ ], r Cos[ $\theta$ ],  $\pi n + \frac{1}{3} e^{-3 \text{Im}[\theta]} \text{Abs}[r]^3 \text{Arg}[e^{2 i \theta} r^2]$ }, {n, 0, 2}]
knot = {{Sin[ $\frac{3 \theta}{2}$ ], Cos[ $\frac{3 \theta}{2}$ ],  $\frac{2 n \pi}{3} + \theta$ }, {-Sin[ $\frac{3 \theta}{2}$ ], -Cos[ $\frac{3 \theta}{2}$ ],  $\frac{2 n \pi}{3} + \theta$ }} /. n -> 0
Abs[x + I y]^3 Arg[(x + I y)^2] / 3 /. (x + I y) -> r Exp[I  $\theta$ ]
Abs[x + I y]^3 Arg[(x + I y)^2] / 3 /. (x + I y) -> r Exp[I  $\theta$ ] // Simplify
Plot3D[Abs[x + I y]^3 Arg[(x + I y)^2] / 3, {x, -1, 1},
  {y, -1, 1}, Exclusions -> None, RegionFunction -> ((#1^2 + #2^2) < 1 &)]
ParametricPlot3D[{r Sin[ $\theta$ ], r Cos[ $\theta$ ],
  Abs[x + I y]^3 Arg[(x + I y)^2] / 3 /. (x + I y) -> r Exp[I  $\theta$ ]}, { $\theta$ , 0, 2 Pi}, {r, 0, 1}]
ParametricPlot3D[surface, { $\theta$ , 0, 2 Pi}, {r, 0, 1}]
outsidesurface = Table[{1/r Sin[ $\theta$ ], 1/r Cos[ $\theta$ ],
  ( $\pi n + \frac{1}{3} \text{Abs}[x + i y]^3 \text{Arg}[\text{Exp}[n I \pi] (x + i y)^2]$ ) /. {x -> Cos[ $\theta$ ], y -> Sin[ $\theta$ ]}}, {n, 0, 2}]

```

```
Show[
  ParametricPlot3D[{ $\frac{\text{Sin}[\theta + \pi/2]}{r}$ ,  $\frac{\text{Cos}[\theta + \pi/2]}{r}$ ,  $2\pi/3 + \frac{1}{3} \text{Abs}[\text{Exp}[I\theta]]^3 \text{Arg}[-\text{Exp}[I\theta]^2]$ },
    { $\theta$ ,  $\theta$ ,  $\text{Pi}/2$ }, { $r$ ,  $\theta$ , 1}, PlotRange → {{-3, 3}, {-3, 3}, { $\theta$ , 7}}, PlotPoints → 20],
  ParametricPlot3D[{ $\frac{\text{Sin}[\theta + 3\pi/2]}{r}$ ,  $\frac{\text{Cos}[\theta + 3\pi/2]}{r}$ ,
     $2\pi/3 + \frac{1}{3} \text{Abs}[\text{Exp}[I\theta]]^3 \text{Arg}[-\text{Exp}[I\theta]^2]$ }, { $\theta$ ,  $\theta$ ,  $\text{Pi}/2$ },
    { $r$ ,  $\theta$ , 1}, PlotRange → {{-3, 3}, {-3, 3}, { $\theta$ , 7}}, PlotPoints → 20],
  ParametricPlot3D[{ $\frac{\text{Sin}[\theta + \pi]}{r}$ ,  $\frac{\text{Cos}[\theta + \pi]}{r}$ ,  $5\pi/3 + \frac{1}{3} \text{Abs}[\text{Exp}[I\theta]]^3 \text{Arg}[-\text{Exp}[I\theta]^2]$ },
    { $\theta$ ,  $\theta$ ,  $\text{Pi}/2$ }, { $r$ ,  $\theta$ , 1}, PlotRange → {{-3, 3}, {-3, 3}, { $\theta$ , 7}}, PlotPoints → 20],
  ParametricPlot3D[{ $\frac{\text{Sin}[\theta + 2\pi]}{r}$ ,  $\frac{\text{Cos}[\theta + 2\pi]}{r}$ ,  $5\pi/3 + \frac{1}{3} \text{Abs}[\text{Exp}[I\theta]]^3 \text{Arg}[-\text{Exp}[I\theta]^2]$ },
    { $\theta$ ,  $\theta$ ,  $\text{Pi}/2$ }, { $r$ ,  $\theta$ , 1}, PlotRange → {{-3, 3}, {-3, 3}, { $\theta$ , 7}}, PlotPoints → 20],
  ParametricPlot3D[{{ $r \text{Sin}[\theta]$ ,  $r \text{Cos}[\theta]$ ,  $\frac{1}{3} e^{-3 \text{Im}[\theta]} \text{Abs}[r]^3 \text{Arg}[e^{2 \pm i \theta} r^2]$ },
    { $r \text{Sin}[\theta + \pi/2]$ ,  $r \text{Cos}[\theta + \pi/2]$ ,  $\pi + \frac{1}{3} e^{-3 \text{Im}[\theta]} \text{Abs}[r]^3 \text{Arg}[e^{2 \pm i \theta} r^2]$ },
    { $r \text{Sin}[\theta]$ ,  $r \text{Cos}[\theta]$ ,  $2\pi + \frac{1}{3} e^{-3 \text{Im}[\theta]} \text{Abs}[r]^3 \text{Arg}[e^{2 \pm i \theta} r^2]$ }},
    { $\theta$ ,  $\theta$ ,  $2\text{Pi}$ }, { $r$ ,  $\theta$ , 1}, Exclusions → None],
  ParametricPlot3D[knot, { $\theta$ ,  $\theta$ ,  $2\pi$ }, PlotStyle → {Thick, Blue}]
]
```

```
Show[
  ParametricPlot3D[{ $\frac{\text{Sin}[\theta + \pi/2]}{r}$ ,  $\frac{\text{Cos}[\theta + \pi/2]}{r}$ ,  $2\pi/3 + \frac{1}{3} \text{Abs}[\text{Exp}[I\theta]]^3 \text{Arg}[-\text{Exp}[I\theta]^2]$ },
    { $\theta$ ,  $\theta$ ,  $\text{Pi}/2$ }, { $r$ ,  $\theta$ , 1}, PlotRange → {{-3, 3}, {-3, 3}, { $\theta$ , 7}}, PlotPoints → 20],
  ParametricPlot3D[{ $\frac{\text{Sin}[\theta + 3\pi/2]}{r}$ ,  $\frac{\text{Cos}[\theta + 3\pi/2]}{r}$ ,
     $2\pi/3 + \frac{1}{3} \text{Abs}[\text{Exp}[I\theta]]^3 \text{Arg}[-\text{Exp}[I\theta]^2]$ }, { $\theta$ ,  $\theta$ ,  $\text{Pi}/2$ },
    { $r$ ,  $\theta$ , 1}, PlotRange → {{-3, 3}, {-3, 3}, { $\theta$ , 7}}, PlotPoints → 20],
  ParametricPlot3D[{ $\frac{\text{Sin}[\theta + \pi]}{r}$ ,  $\frac{\text{Cos}[\theta + \pi]}{r}$ ,  $5\pi/3 + \frac{1}{3} \text{Abs}[\text{Exp}[I\theta]]^3 \text{Arg}[-\text{Exp}[I\theta]^2]$ },
    { $\theta$ ,  $\theta$ ,  $\text{Pi}/2$ }, { $r$ ,  $\theta$ , 1}, PlotRange → {{-3, 3}, {-3, 3}, { $\theta$ , 7}}, PlotPoints → 20],
  ParametricPlot3D[{ $\frac{\text{Sin}[\theta + 2\pi]}{r}$ ,  $\frac{\text{Cos}[\theta + 2\pi]}{r}$ ,  $5\pi/3 + \frac{1}{3} \text{Abs}[\text{Exp}[I\theta]]^3 \text{Arg}[-\text{Exp}[I\theta]^2]$ },
    { $\theta$ ,  $\theta$ ,  $\text{Pi}/2$ }, { $r$ ,  $\theta$ , 1}, PlotRange → {{-3, 3}, {-3, 3}, { $\theta$ , 7}}, PlotPoints → 20],
  ParametricPlot3D[{{ $r \text{Sin}[\theta]$ ,  $r \text{Cos}[\theta]$ ,  $\frac{1}{3} e^{-3 \text{Im}[\theta]} \text{Abs}[r]^3 \text{Arg}[e^{2 \pm i \theta} r^2]$ },
    { $r \text{Sin}[\theta + \pi/2]$ ,  $r \text{Cos}[\theta + \pi/2]$ ,  $\pi + \frac{1}{3} e^{-3 \text{Im}[\theta]} \text{Abs}[r]^3 \text{Arg}[e^{2 \pm i \theta} r^2]$ },
    { $r \text{Sin}[\theta]$ ,  $r \text{Cos}[\theta]$ ,  $2\pi + \frac{1}{3} e^{-3 \text{Im}[\theta]} \text{Abs}[r]^3 \text{Arg}[e^{2 \pm i \theta} r^2]$ }}, { $\theta$ ,  $\theta$ ,  $2\text{Pi}$ }, { $r$ ,  $\theta$ , 1}],
  ParametricPlot3D[knot, { $\theta$ ,  $\theta$ ,  $2\pi$ }, PlotStyle → {Thick, Blue}]
]
```

```

Theta[{x_, y_, z_}] := ArcTan[x, y]
Phi[{x_, y_, z_}] := z
Torus[α_, θ_, φ_] := ReIm /@ {Sin[α] Exp[I θ], Cos[α] Exp[I φ]}

Show[
  ParametricPlot3D[SterProj@(Torus[r, Theta[#], Phi[#]]) &@
    {
       $\frac{\text{Sin}[\theta + \pi/2]}{r}$ ,  $\frac{\text{Cos}[\theta + \pi/2]}{r}$ ,  $2\pi/3 + \frac{1}{3} \text{Abs}[\text{Exp}[I\theta]]^3 \text{Arg}[-\text{Exp}[I\theta]^2]$ 
    },
    {θ, 0, 2 Pi}, {r, 0, 1}, PlotPoints → 100],
  ParametricPlot3D[SterProj@(Torus[r, Theta[#], Phi[#]]) &@
    {
       $\frac{\text{Sin}[\theta + 3\pi/2]}{r}$ ,  $\frac{\text{Cos}[\theta + 3\pi/2]}{r}$ ,  $2\pi/3 + \frac{1}{3} \text{Abs}[\text{Exp}[I\theta]]^3 \text{Arg}[-\text{Exp}[I\theta]^2]$ 
    },
    {θ, 0, 2 Pi}, {r, 0, 1}, PlotPoints → 100],

  ParametricPlot3D[SterProj@(Torus[r, Theta[#], Phi[#]]) &@
    {
       $\frac{\text{Sin}[\theta + \pi]}{r}$ ,  $\frac{\text{Cos}[\theta + \pi]}{r}$ ,  $5\pi/3 + \frac{1}{3} \text{Abs}[\text{Exp}[I\theta]]^3 \text{Arg}[-\text{Exp}[I\theta]^2]$ 
    },
    {θ, 0, 2 Pi}, {r, 0, 1}, PlotPoints → 100],

  ParametricPlot3D[SterProj@(Torus[r, Theta[#], Phi[#]]) &@
    {
       $\frac{\text{Sin}[\theta + 2\pi]}{r}$ ,  $\frac{\text{Cos}[\theta + 2\pi]}{r}$ ,  $5\pi/3 + \frac{1}{3} \text{Abs}[\text{Exp}[I\theta]]^3 \text{Arg}[-\text{Exp}[I\theta]^2]$ 
    },
    {θ, 0, 2 Pi}, {r, 0, 1}, PlotPoints → 100]
]

Theta@{r Sin[θ], r Cos[θ],  $\pi + \frac{1}{3} \text{Abs}[\text{Cos}[\theta] + \text{i Sin}[\theta]]^3 \text{Arg}[-(\text{Cos}[\theta] + \text{i Sin}[\theta])^2]$ }

ParametricPlot3D[SterProj /@ (Torus[r, Theta[#], Phi[#]] & /@ surface),
  {θ, 0, 2 Pi}, {r, 0.9, 1}, PlotPoints → 100]

ParametricPlot3D[SterProj /@ (Torus[r, Theta[#], Phi[#]] & /@ knot),
  {θ, 0, 2 Pi}, {r, 0.99, 1}, PlotPoints → 50]

```

```
Show[
  ParametricPlot3D[SterProj /@ (Torus[r, Theta[#], Phi[#]] & /@knot),
    {θ, 0, 2 Pi}, {r, 0.99, 1}, PlotPoints → 50, MeshStyle → None,
    PlotStyle → Red, PlotRange → {{-5, 5}, {-5, 5}, {-5, 5}},
  ParametricPlot3D[SterProj /@ (Torus[r, Theta[#], Phi[#]] & /@surface),
    {θ, 0, 2 Pi}, {r, 0, 1}, PlotPoints → 50, MeshStyle → None, PlotStyle → Blue],
  ParametricPlot3D[SterProj@ (Torus[r, Theta[#], Phi[#]]) &@
    { $\frac{\sin[\theta + \pi/2]}{r}$ ,  $\frac{\cos[\theta + \pi/2]}{r}$ ,  $2\pi/3 + \frac{1}{3} \text{Abs}[\text{Exp}[I \theta]]^3 \text{Arg}[-\text{Exp}[I \theta]^2]$ },
    {θ, 0, 2 Pi}, {r, 0, 1}, PlotPoints → 100],
  ParametricPlot3D[SterProj@ (Torus[r, Theta[#], Phi[#]]) &@
    { $\frac{\sin[\theta + 3\pi/2]}{r}$ ,  $\frac{\cos[\theta + 3\pi/2]}{r}$ ,  $2\pi/3 + \frac{1}{3} \text{Abs}[\text{Exp}[I \theta]]^3 \text{Arg}[-\text{Exp}[I \theta]^2]$ },
    {θ, 0, 2 Pi}, {r, 0, 1}, PlotPoints → 100],

  ParametricPlot3D[SterProj@ (Torus[r, Theta[#], Phi[#]]) &@
    { $\frac{\sin[\theta + \pi]}{r}$ ,  $\frac{\cos[\theta + \pi]}{r}$ ,  $5\pi/3 + \frac{1}{3} \text{Abs}[\text{Exp}[I \theta]]^3 \text{Arg}[-\text{Exp}[I \theta]^2]$ },
    {θ, 0, 2 Pi}, {r, 0, 1}, PlotPoints → 100],

  ParametricPlot3D[SterProj@ (Torus[r, Theta[#], Phi[#]]) &@
    { $\frac{\sin[\theta + 2\pi]}{r}$ ,  $\frac{\cos[\theta + 2\pi]}{r}$ ,  $5\pi/3 + \frac{1}{3} \text{Abs}[\text{Exp}[I \theta]]^3 \text{Arg}[-\text{Exp}[I \theta]^2]$ },
    {θ, 0, 2 Pi}, {r, 0, 1}, PlotPoints → 100]
]
```

```
Show[
  ParametricPlot3D[SterProj /@ (Torus[r, Theta[#], Phi[#]] & /@knot),
    {θ, 0, 2 Pi}, {r, 0.99, 1}, PlotPoints → 50, MeshStyle → None,
    PlotStyle → Red, PlotRange → {{-2, 2}, {-2, 2}, {-2, 2}},
  ParametricPlot3D[SterProj /@ (Torus[r, Theta[#], Phi[#]] & /@surface),
    {θ, 0, 2 Pi}, {r, 0, 1}, PlotPoints → 50, MeshStyle → None, PlotStyle → Blue],

  ParametricPlot3D[SterProj@ (Torus[r, Theta[#], Phi[#]]) &@
    {  $\frac{\sin[\theta + \pi/2]}{r}$ ,  $\frac{\cos[\theta + \pi/2]}{r}$ ,  $2\pi/3 + \frac{1}{3} \text{Abs}[\text{Exp}[I \theta]]^3 \text{Arg}[-\text{Exp}[I \theta]^2]$  },
    {θ, 0, 2 Pi}, {r, 0, 1}, PlotPoints → 100],
  ParametricPlot3D[SterProj@ (Torus[r, Theta[#], Phi[#]]) &@
    {  $\frac{\sin[\theta + 3\pi/2]}{r}$ ,  $\frac{\cos[\theta + 3\pi/2]}{r}$ ,  $2\pi/3 + \frac{1}{3} \text{Abs}[\text{Exp}[I \theta]]^3 \text{Arg}[-\text{Exp}[I \theta]^2]$  },
    {θ, 0, 2 Pi}, {r, 0, 1}, PlotPoints → 100],

  ParametricPlot3D[SterProj@ (Torus[r, Theta[#], Phi[#]]) &@
    {  $\frac{\sin[\theta + \pi]}{r}$ ,  $\frac{\cos[\theta + \pi]}{r}$ ,  $5\pi/3 + \frac{1}{3} \text{Abs}[\text{Exp}[I \theta]]^3 \text{Arg}[-\text{Exp}[I \theta]^2]$  },
    {θ, 0, 2 Pi}, {r, 0, 1}, PlotPoints → 100],

  ParametricPlot3D[SterProj@ (Torus[r, Theta[#], Phi[#]]) &@
    {  $\frac{\sin[\theta + 2\pi]}{r}$ ,  $\frac{\cos[\theta + 2\pi]}{r}$ ,  $5\pi/3 + \frac{1}{3} \text{Abs}[\text{Exp}[I \theta]]^3 \text{Arg}[-\text{Exp}[I \theta]^2]$  },
    {θ, 0, 2 Pi}, {r, 0, 1}, PlotPoints → 100]
]
```