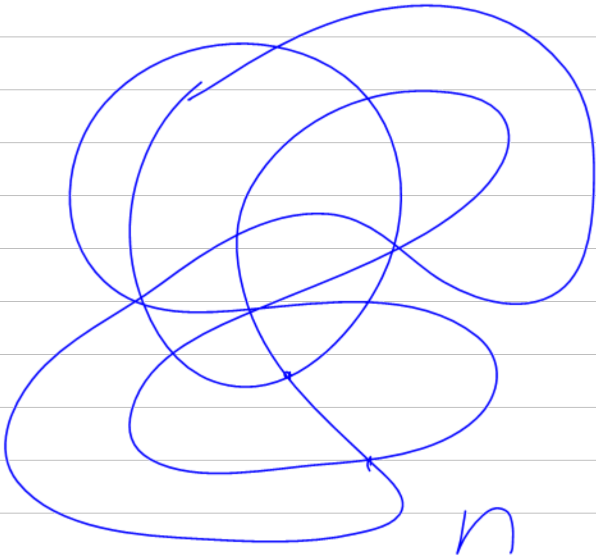
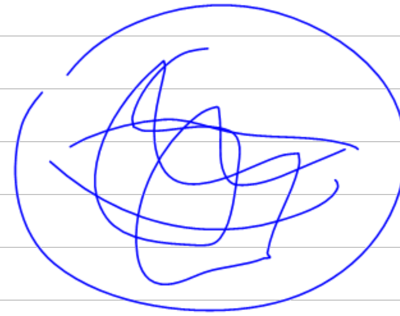
yarn-ball knots.



n

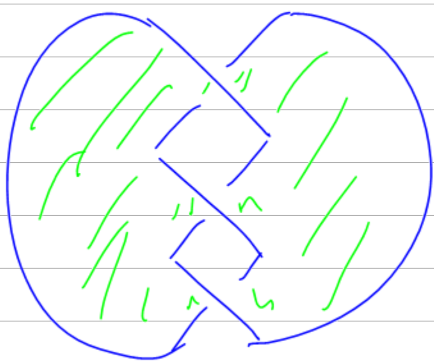


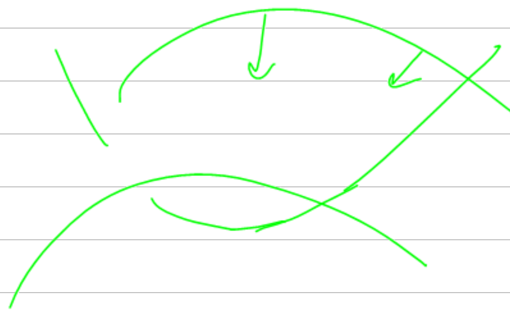
$V = L^3$

$n = V^{4/3} = L^2 \cdot L^2$

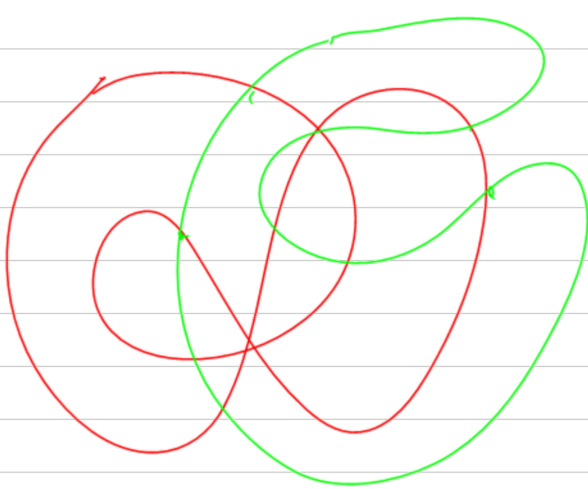
<u>Def</u> ϕ is 3D

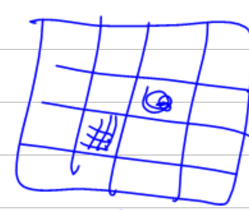
$$C_{YB}(V) \ll C_{PD}(V^{4/3})$$





$C_{PD}(\psi K) = n$



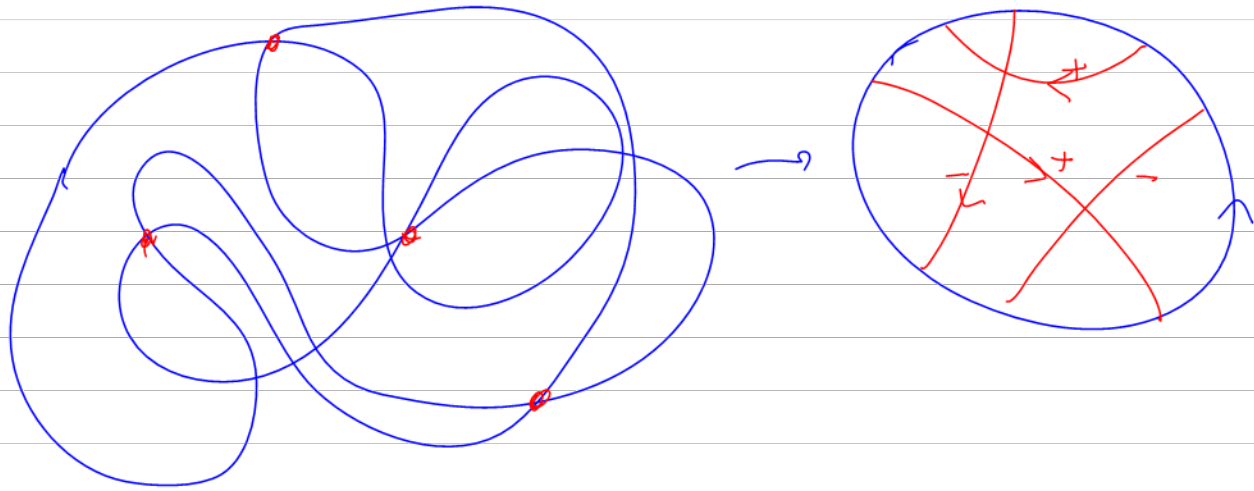


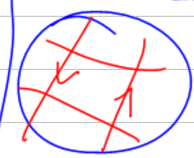
L

L

$L^2 \cdot L = L^3$

Gauss Diagram invts: of type p

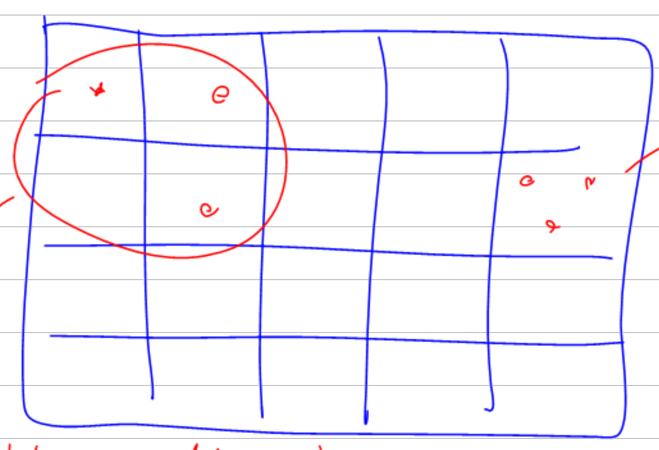


$\{\text{knots}\} \xrightarrow{\Psi} \mathbb{Z}\Big[$  $\Big] \xrightarrow{\varphi} \mathbb{Z}$

$$C_{PD}(\Psi) = n^p$$

$$C_{YB}(\Psi)$$

$p = 3$



what I asked

the following:

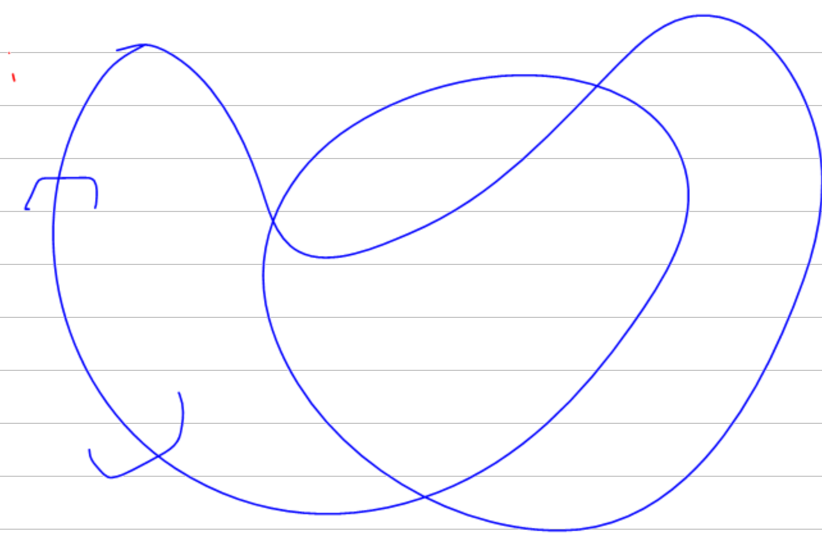
$$\overbrace{\underbrace{\alpha_1\ \beta_1 \text{ times}}_{L}\quad \underbrace{\alpha_2\ t^{\beta_2}}_{L}\quad \cdots \quad \underbrace{\alpha_p\ \beta_p}_{L}}$$

$$h(\alpha_i) < h(\beta_i)$$

$$\alpha_1 < \beta_1 < \alpha_2 < \alpha_3 < \beta_2 < \alpha_4 \ldots$$

do that in less than L^{2p}

can do $L^{p+1} (\log \ldots)$



Source: the two handouts at http://drorbn.net/la19/

The PBW Principle Lots of algebras are isomorphic as vector spaces to polynomial algebras. So we want to understand arbitrary linear maps between polynomial algebras.

Gentle Agreement. Everything converges!

Convention. For a finite set A, let $z_A := \{z_i\}_{i\in A}$ and let $\zeta_A := \{z_i^* = \zeta_i\}_{i\in A}$. $\qquad (y,b,a,x)^* = (\eta,\beta,\alpha,\xi)$

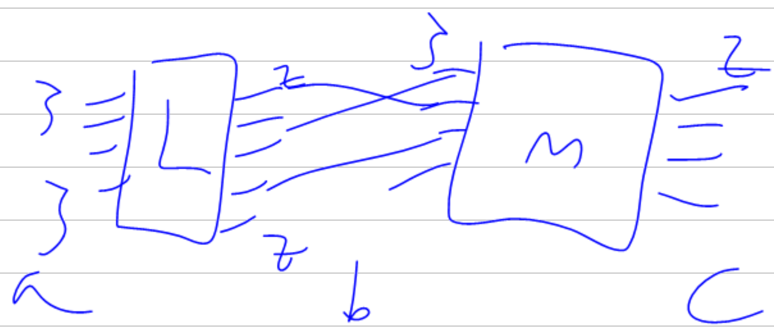
The Generating Series $\mathcal{G}$: $\mathrm{Hom}(\mathbb{Q}[z_A]\to\mathbb{Q}[z_B]) \to \mathbb{Q}[\![\zeta_A, z_B]\!]$.

Claim. $L \in \mathrm{Hom}(\mathbb{Q}[z_A]\to\mathbb{Q}[z_B]) \xrightarrow[\mathcal{G}]{\sim} \mathbb{Q}[z_B][\![\zeta_A]\!] \ni \mathcal{L}$ via

$$\mathcal{G}(L) := \sum_{n\in\mathbb{N}^A} \frac{\zeta_A^n}{n!} L(z_A^n) = L\left(\mathbb{e}^{\sum_{a\in A}\zeta_a z_a}\right) = \mathcal{L} = {}_{\text{greek}}\mathcal{L}_{\text{latin}},$$

$$\mathcal{G}^{-1}(\mathcal{L})(p) = \left(p|_{z_a\to\partial_{\zeta_a}}\mathcal{L}\right)_{\zeta_a=0} \quad \text{for } p\in\mathbb{Q}[z_A].$$

Claim. If $L \in \mathrm{Hom}(\mathbb{Q}[z_A]\to\mathbb{Q}[z_B])$, $M \in \mathrm{Hom}(\mathbb{Q}[z_B]\to\mathbb{Q}[z_C])$, then $\mathcal{G}(L/\!\!/M) = \left(\mathcal{G}(L)|_{z_b\to\partial_{\zeta_b}}\mathcal{G}(M)\right)_{\zeta_b=0}$.



DoPeGDO := The category with objects finite sets[†1] and

$$\mathrm{mor}(A \to B) = \{\mathcal{L} = \omega \exp(Q+P)\} \subset \mathbb{Q}[\![\zeta_A, z_B, \epsilon]\!],$$

where: • ω is a scalar.[†2] • Q is a "small" ϵ-free quadratic in $\zeta_A \cup z_B$.[†3] • P is a "docile perturbation": $P = \sum_{k\geq 1} \epsilon^k P^{(k)}$, where $\deg P^{(k)} \leq 2k+2$.[†4] • Compositions:[†6] $\mathcal{L} /\!\!/ \mathcal{M} := \left(\mathcal{L}|_{z_i \to \partial_{\zeta_i}} \mathcal{M} \right)_{\zeta_i = 0}$.

e^Q

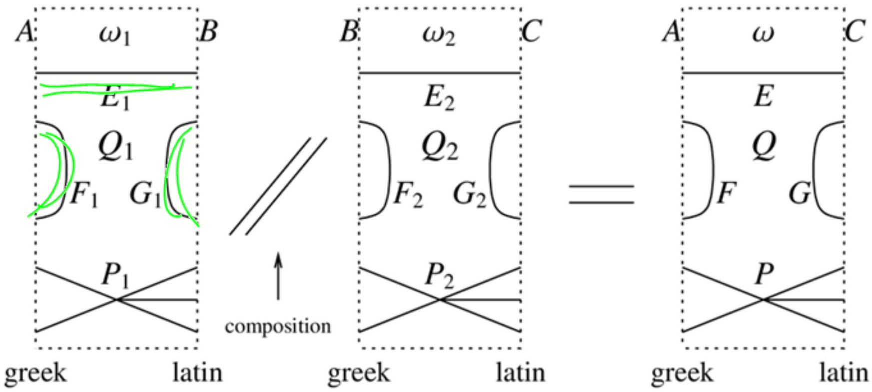
e^{Q+P}

ζ

Compositions. In $\mathrm{mor}(A \to B)$,

$$Q = \sum_{i\in A, j \in B} E_{ij}\zeta_i z_j + \frac{1}{2}\sum_{i,j\in A} F_{ij}\zeta_i\zeta_j + \frac{1}{2}\sum_{i,j\in B} G_{ij} z_i z_j,$$

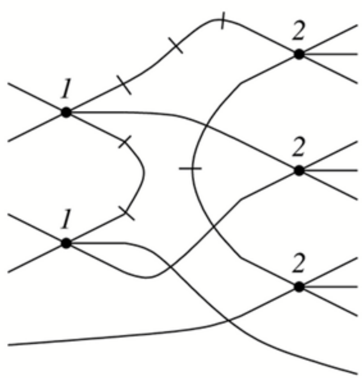
and so



$E_1E_2 + \dots$

$= E_1 \sum_{r\geq 0} (F_2G_1)^r E_2$

where • $E = E_1(I - F_2G_1)^{-1}E_2$.
• $F = F_1 + E_1F_2(I - G_1F_2)^{-1}E_1^T$.
• $G = G_2 + E_2^T G_1(I - F_2G_1)^{-1}E_2$.
• $\omega = \omega_1\omega_2 \det(I - F_2G_1)^{-1}$.
• P is computed as the solution of a messy PDE or using "connected Feynman diagrams" (yet we're still in pure algebra!). Docility is preserved.



Compositions (2). Recall that with all indices i running in some set B,

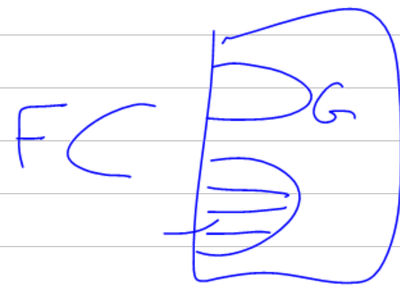
$$\mathcal{F}/\!/\mathcal{G} = \left(\mathcal{F}|_{z_i\to\partial_{\zeta_i}}\mathcal{G}\right)_{\zeta_i=0} \overset{(1)}{=} \mathbb{e}^{\sum \partial_{z_i}\partial_{\zeta_i}}(\mathcal{F}\mathcal{G})\Big|_{z_i=\zeta_i=0},$$

(1) Strictly speaking, true only when $B\cap(A\cup C)=\emptyset$.

so in general we wish to understand

$$[F:\mathcal{E}]_B \coloneqq \mathbb{e}^{\frac{1}{2}\sum_{i,j\in B}F_{ij}\partial_{z_i}\partial_{z_j}}\mathcal{E} \quad\text{and}\quad \langle F:\mathcal{E}\rangle_B \coloneqq [F:\mathcal{E}]_B|_{z_B\to 0},$$

where $\mathcal{E}$ is a docile perturbed Gaussian. The following lemma allows us to restrict to the case where $\mathcal{E}$ has no B-B quadratic part:



Lemma 1. With convergences left to the reader,

$$\left\langle F:\mathcal{E}\,\mathbb{e}^{\frac{1}{2}\sum_{i,j\in B}G_{ij}z_iz_j}\right\rangle_B = \det(1-GF)^{-1/2}\left\langle F(1-GF)^{-1}:\mathcal{E}\right\rangle_B.$$

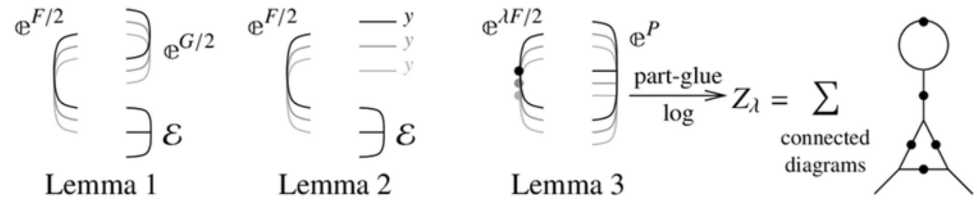
The next lemma dispatches the case where $\mathcal{E}$ has a B-linear part:

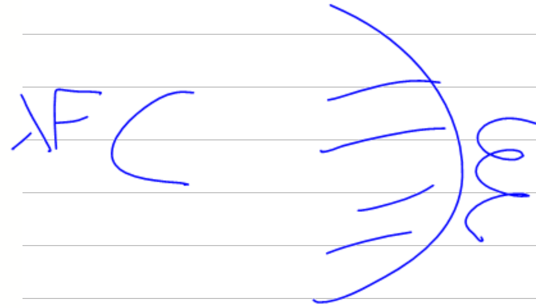
Lemma 2. $\left\langle F:\mathcal{E}\,\mathbb{e}^{\sum_{i\in B}y_iz_i}\right\rangle_B = \mathbb{e}^{\frac{1}{2}\sum_{i,j\in B}F_{ij}y_iy_j}\left\langle F:\ \mathcal{E}|_{z_B\to z_B+Fy_B}\right\rangle_B.$

Finally, we deal with the docile perturbation case:

Lemma 3. With an extra variable λ, $Z_\lambda := \log[\lambda F : e^P]_B$ satisfies and is determined by the following PDE / IVP:

$$Z_0 = P \quad \text{and} \quad \partial_\lambda Z_\lambda = \frac{1}{2} \sum_{i,j \in B} F_{ij} \left(\partial_{z_i} \partial_{z_j} Z_\lambda + (\partial_{z_i} Z_\lambda)(\partial_{z_j} Z_\lambda) \right).$$





$$\partial_\lambda Z_\lambda = F_{ij} (\partial_{z_i} Z_\lambda)(\partial_{z_j} Z_\lambda)$$

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