```
M_k_[ℰ_] := Expand[ℰ /. {e_k → (1 - t) e_k + e_(k+1), e_(k+1) → t e_k}]

Table[Coefficient[M_3[e_j], e_i], {i, 7}, {j, 7}] // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1-t & t & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
Table[e_i // M_2 // M_5, {i, 7}]
```

$\{e_1, e_2 - t e_2 + e_3, t e_2, e_4, e_5 - t e_5 + e_6, t e_5, e_7\}$

```
Table[e_i // M_5 // M_2, {i, 7}]
```

$\{e_1, e_2 - t e_2 + e_3, t e_2, e_4, e_5 - t e_5 + e_6, t e_5, e_7\}$

```
lhs = Table[e_i // M_3 // M_4 // M_3, {i, 7}]
```

$\{e_1, e_2, e_3 - t e_3 + e_4 - t e_4 + e_5, t e_3 - t^2 e_3 + t e_4, t^2 e_3, e_6, e_7\}$

```
rhs = Table[e_i // M_4 // M_3 // M_4, {i, 7}]
```

$\{e_1, e_2, e_3 - t e_3 + e_4 - t e_4 + e_5, t e_3 - t^2 e_3 + t e_4, t^2 e_3, e_6, e_7\}$

```
lhs == rhs
```

True

```
δ_(i_,j_) := Which[i == j, 1, i ≠ j, 0]

δ_(2,2)
```

1

```
N_(i_,j_)[A_(r_,s_)] := Which[s < i || i < r, δ_(i,j), s == i, (1 - t_i) δ_(i,r) + t_r δ_(i,j), r == i,
  (1 - t_i) (δ_(i,j) + t_i δ_(s,j)) + t_i t_s δ_(i,j), r < i < s, (1 - t_i) (1 - t_s) δ_(r,j) - (1 - t_r) δ_(s,j) + δ_(i,j)]

N_(2,2)[A_(1,2)]
```

t_1

```
Table[N_(i,j)[A_(1,2)], {i, 3}, {j, 3}] // MatrixForm
```

$$\begin{pmatrix} 1 - t_1 + t_1 t_2 & (1 - t_1) t_1 & 0 \\ 0 & t_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
Table[N_{i,j}[A_{3,7}], {i, 9}, {j, 9}] // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 - t_3 + t_3 t_7 & 0 & 0 & 0 & (1 - t_3)\, t_3 & 0 & 0 \\ 0 & 0 & (1 - t_4)\,(1 - t_7) & 1 & 0 & 0 & -1 + t_3 & 0 & 0 \\ 0 & 0 & (1 - t_5)\,(1 - t_7) & 0 & 1 & 0 & -1 + t_3 & 0 & 0 \\ 0 & 0 & (1 - t_6)\,(1 - t_7) & 0 & 0 & 1 & -1 + t_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & t_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$