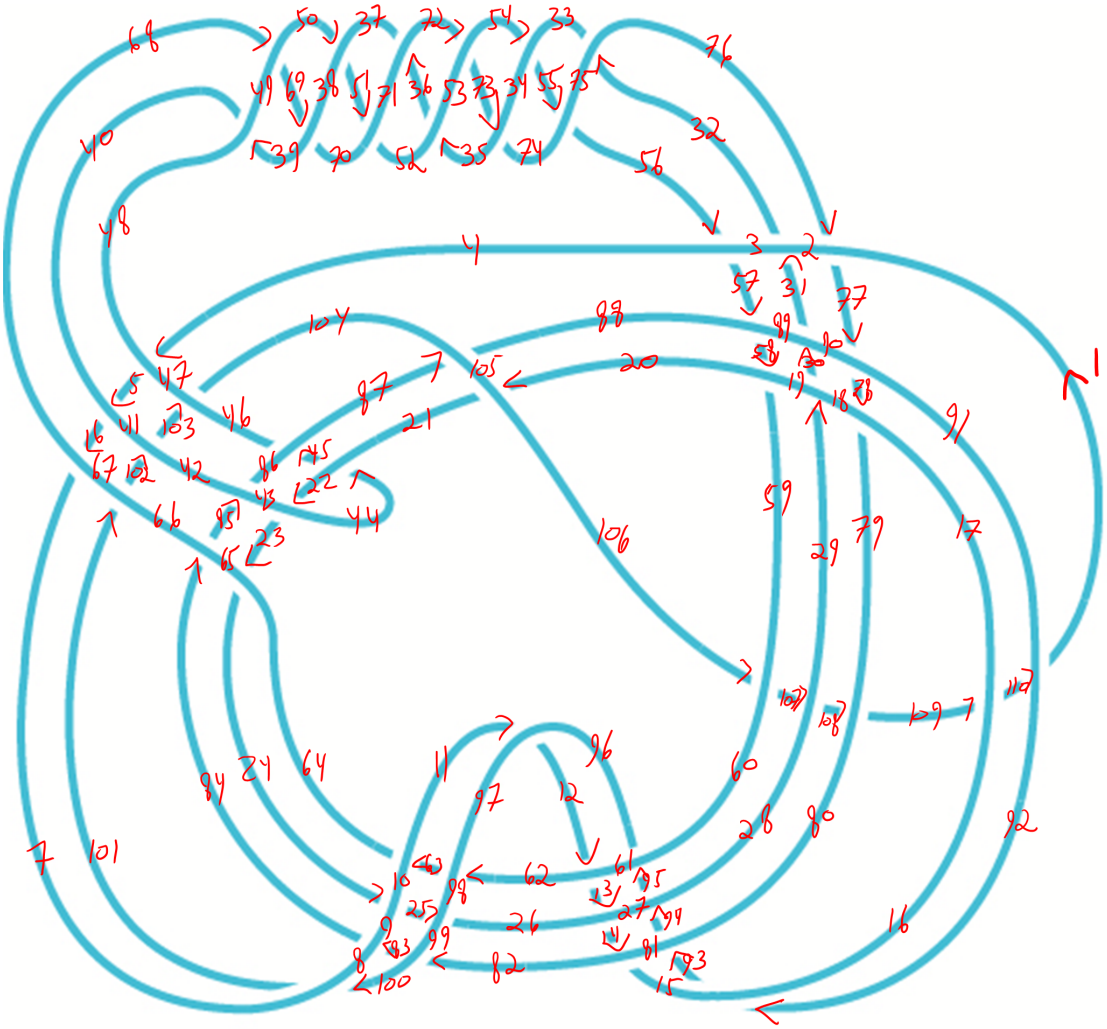
In[•]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Classes\\23-FastComputations"];
Once[<< KnotTheory`]
```

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
```



```
In[•]:= PK = PD[
           X[4, 48, 5, 47], X[5, 40, 6, 41], X[6, 68, 7, 67], X[11, 97, 12, 96],
           X[12, 62, 13, 61], X[13, 26, 14, 27], X[14, 82, 15, 81], X[20, 105, 21, 106],
           X[22, 43, 23, 44], X[23, 65, 24, 64], X[24, 9, 25, 10], X[25, 99, 26, 98],
           X[29, 18, 30, 19], X[30, 90, 31, 89], X[31, 2, 32, 3], X[32, 76, 33, 75],
           X[35, 53, 36, 52], X[36, 72, 37, 71], X[39, 49, 40, 48], X[44, 21, 45, 22],
           X[45, 87, 46, 86], X[50, 38, 51, 37], X[51, 70, 52, 71], X[54, 34, 55, 33],
           X[55, 74, 56, 75], X[56, 4, 57, 3], X[57, 88, 58, 89], X[58, 20, 59, 19],
           X[62, 97, 63, 98], X[63, 11, 64, 10], X[68, 49, 69, 50], X[69, 39, 70, 38],
           X[72, 53, 73, 54], X[73, 35, 74, 34], X[76, 2, 77, 1], X[77, 90, 78, 91],
           X[78, 18, 79, 17], X[82, 99, 83, 100], X[83, 9, 84, 8], X[84, 65, 85, 66],
           X[85, 43, 86, 42], X[87, 105, 88, 104], X[92, 16, 93, 15], X[93, 80, 94, 81],
           X[94, 28, 95, 27], X[95, 60, 96, 61], X[100, 8, 101, 7], X[101, 66, 102, 67],
           X[102, 42, 103, 41], X[103, 46, 104, 47], X[106, 60, 107, 59], X[107, 28, 108, 29],
           X[108, 80, 109, 79], X[109, 16, 110, 17], X[110, 92, 1, 91]
          ];

In[•]:= Crossings[PK]

Out[•]=
55

In[•]:= Jones[PK][q]

Out[•]=
-1 - 1/q^2 + 3/q - q + 2 q^2 - q^3 - q^5 + q^6 - q^8 + q^9 - q^11 + q^12 + q^13 - q^14 + q^16 - q^18 - q^23 + q^24

In[•]:= Alexander[PK][t]

Out[•]=
1
```

In[•]:= **? Kh**

Out[•]=

Symbol

Kh[L][q, t] returns the Poincare polynomial of the
Khovanov Homology of a knot/link L (over a field of characteristic 0)
in terms of the variables q and t. Kh[L, Program –> prog] uses the
program prog to perform the computation. The currently available
programs are "FastKh", written in Mathematica by Dror Bar–Natan in
the winter of 2005, "JavaKh–v1", written in java (java 1.5
required!) by Jeremy Green in the summer of 2005 and "JavaKh–v2" (default),
an update of "JavaKh–v1" (now requiring java 1.6) written by Scott Morrison in 2008.
("JavaKh" is also available, currently an alias for "JavaKh–v2".)
The java programs are several thousand times faster than the Mathematica program, though java
may not be available on some systems. "JavaKh2" also takes the option
"Modulus –> p" which changes the characteristic of the ground field
to p. If p==0 JavaKh works over the rational numbers; if p==Null JavaKh
works over Z (see ?ZMod for the output format).

In[•]:= **Kh[PD@Knot[3, 1]][q, t]**

KnotTheory: Loading precomputed data in PD4Knots`.

KnotTheory: The Khovanov homology program JavaKh–v2 is an update of Jeremy Green's program JavaKh–v1, written by Scott Morrison in 2008 at Microsoft Station Q.

Out[•]=

$$\frac{1}{q^3}+\frac{1}{q}+\frac{1}{q^9\,t^3}+\frac{1}{q^5\,t^2}$$

```
SetOptions[Kh, JavaOptions → "-Xmx1024m"];
kh = Kh[PK][q, t]
```

Out[•]=

$$\frac{1}{q}+2q+q^3+\frac{1}{q^5t^3}+\frac{2}{q^3t^2}+\frac{1}{qt^2}+\frac{2q}{t}+2qt+3q^3t+q^5t+3q^3t^2+3q^5t^2+2q^7t^2+3q^5t^3+$$
$$3q^7t^3+q^9t^3+2q^5t^4+3q^7t^4+3q^9t^4+3q^7t^5+5q^9t^5+3q^{11}t^5+3q^9t^6+5q^{11}t^6+3q^{13}t^6+$$
$$q^9t^7+4q^{11}t^7+4q^{13}t^7+2q^{15}t^7+2q^{11}t^8+5q^{13}t^8+4q^{15}t^8+q^{17}t^8+4q^{13}t^9+5q^{15}t^9+$$
$$4q^{17}t^9+q^{13}t^{10}+3q^{15}t^{10}+6q^{17}t^{10}+3q^{19}t^{10}+2q^{15}t^{11}+4q^{17}t^{11}+4q^{19}t^{11}+2q^{21}t^{11}+$$
$$q^{15}t^{12}+2q^{17}t^{12}+6q^{19}t^{12}+3q^{21}t^{12}+q^{23}t^{12}+q^{17}t^{13}+4q^{19}t^{13}+4q^{21}t^{13}+4q^{23}t^{13}+q^{19}t^{14}+$$
$$4q^{21}t^{14}+4q^{23}t^{14}+2q^{25}t^{14}+q^{19}t^{15}+3q^{21}t^{15}+3q^{23}t^{15}+3q^{25}t^{15}+q^{27}t^{15}+q^{21}t^{16}+4q^{23}t^{16}+$$
$$5q^{25}t^{16}+2q^{27}t^{16}+2q^{23}t^{17}+3q^{25}t^{17}+4q^{27}t^{17}+2q^{29}t^{17}+2q^{25}t^{18}+4q^{27}t^{18}+2q^{29}t^{18}+$$
$$q^{31}t^{18}+q^{25}t^{19}+2q^{27}t^{19}+4q^{29}t^{19}+2q^{31}t^{19}+q^{27}t^{20}+4q^{29}t^{20}+3q^{31}t^{20}+2q^{33}t^{20}+q^{29}t^{21}+$$
$$2q^{31}t^{21}+3q^{33}t^{21}+q^{35}t^{21}+2q^{31}t^{22}+2q^{33}t^{22}+q^{35}t^{22}+q^{31}t^{23}+q^{33}t^{23}+3q^{35}t^{23}+q^{37}t^{23}+$$
$$q^{33}t^{24}+2q^{35}t^{24}+q^{37}t^{24}+q^{39}t^{24}+q^{35}t^{25}+2q^{37}t^{25}+q^{39}t^{25}+q^{35}t^{26}+q^{37}t^{26}+q^{39}t^{26}+q^{41}t^{26}+$$
$$2q^{39}t^{27}+q^{41}t^{27}+q^{39}t^{28}+q^{41}t^{28}+q^{43}t^{28}+q^{41}t^{29}+q^{43}t^{29}+q^{45}t^{29}+q^{45}t^{30}+q^{45}t^{31}+q^{49}t^{32}$$

$$\frac{1}{q}+2q+q^3+\frac{1}{q^5t^3}+\frac{2}{q^3t^2}+\frac{1}{qt^2}+\frac{2q}{t}+2qt+3q^3t+q^5t+3q^3t^2+3q^5t^2+2q^7t^2+3q^5t^3+$$
$$3q^7t^3+q^9t^3+2q^5t^4+3q^7t^4+3q^9t^4+3q^7t^5+5q^9t^5+3q^{11}t^5+3q^9t^6+5q^{11}t^6+3q^{13}t^6+$$
$$q^9t^7+4q^{11}t^7+4q^{13}t^7+2q^{15}t^7+2q^{11}t^8+5q^{13}t^8+4q^{15}t^8+q^{17}t^8+4q^{13}t^9+5q^{15}t^9+$$
$$4q^{17}t^9+q^{13}t^{10}+3q^{15}t^{10}+6q^{17}t^{10}+3q^{19}t^{10}+2q^{15}t^{11}+4q^{17}t^{11}+4q^{19}t^{11}+2q^{21}t^{11}+$$
$$q^{15}t^{12}+2q^{17}t^{12}+6q^{19}t^{12}+3q^{21}t^{12}+q^{23}t^{12}+q^{17}t^{13}+4q^{19}t^{13}+4q^{21}t^{13}+4q^{23}t^{13}+q^{19}t^{14}+$$
$$4q^{21}t^{14}+4q^{23}t^{14}+2q^{25}t^{14}+q^{19}t^{15}+3q^{21}t^{15}+3q^{23}t^{15}+3q^{25}t^{15}+q^{27}t^{15}+q^{21}t^{16}+4q^{23}t^{16}+$$
$$5q^{25}t^{16}+2q^{27}t^{16}+2q^{23}t^{17}+3q^{25}t^{17}+4q^{27}t^{17}+2q^{29}t^{17}+2q^{25}t^{18}+4q^{27}t^{18}+2q^{29}t^{18}+$$
$$q^{31}t^{18}+q^{25}t^{19}+2q^{27}t^{19}+4q^{29}t^{19}+2q^{31}t^{19}+q^{27}t^{20}+4q^{29}t^{20}+3q^{31}t^{20}+2q^{33}t^{20}+q^{29}t^{21}+$$
$$2q^{31}t^{21}+3q^{33}t^{21}+q^{35}t^{21}+2q^{31}t^{22}+2q^{33}t^{22}+q^{35}t^{22}+q^{31}t^{23}+q^{33}t^{23}+3q^{35}t^{23}+q^{37}t^{23}+$$
$$q^{33}t^{24}+2q^{35}t^{24}+q^{37}t^{24}+q^{39}t^{24}+q^{35}t^{25}+2q^{37}t^{25}+q^{39}t^{25}+q^{35}t^{26}+q^{37}t^{26}+q^{39}t^{26}+q^{41}t^{26}+$$
$$2q^{39}t^{27}+q^{41}t^{27}+q^{39}t^{28}+q^{41}t^{28}+q^{43}t^{28}+q^{41}t^{29}+q^{43}t^{29}+q^{45}t^{29}+q^{45}t^{30}+q^{45}t^{31}+q^{49}t^{32}$$

In[ ]:=

```
(CoefficientList[q^5 t^3 kh, {q, t}] /. {0 ..} → Nothing /. 0 → "") // Reverse // MatrixForm
```

Out[ ]//MatrixForm=

																																			1
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																															1	1			
																													1	1	1	1			
																											1	1	1	2	1				
																										1	1	2	1						
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																		1	2	4	4	2	1												
																	2	3	5	3	2	1													
															1	4	4	3	4	2															
														2	3	4	4	3	1																
													3	4	6	4	1	1																	
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