

Pensieve header: Computing the Jones polynomial faster.

The trefoil and the mirror trefoil:

In[]:= **D1 = PD[X[1, 5, 2, 4], X[5, 3, 6, 2], X[3, 1, 4, 6]]**

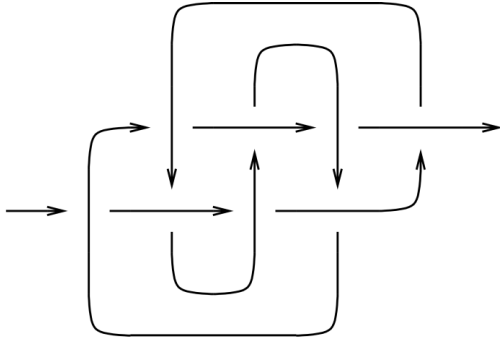
D2 = PD[X[4, 1, 5, 2], X[2, 5, 3, 6], X[6, 3, 1, 4]]

Out[]:=

PD[X[1, 5, 2, 4], X[5, 3, 6, 2], X[3, 1, 4, 6]]

Out[]:=

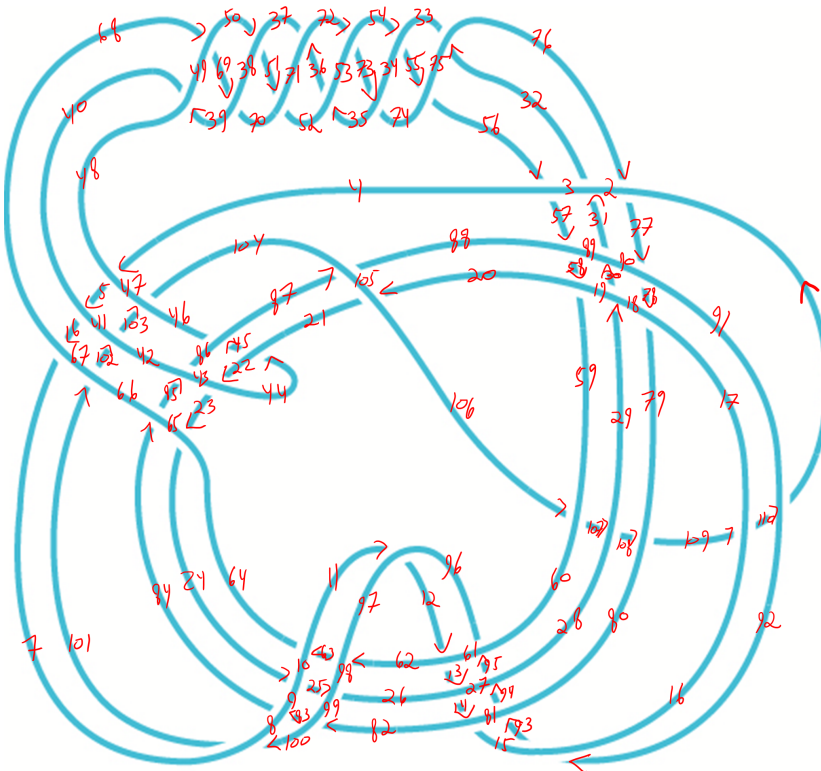
PD[X[4, 1, 5, 2], X[2, 5, 3, 6], X[6, 3, 1, 4]]



In[]:= **K817 = PD[X[6, 2, 7, 1], X[14, 8, 15, 7], X[8, 3, 9, 4], X[2, 13, 3, 14], X[12, 5, 13, 6], X[4, 9, 5, 10], X[16, 12, 1, 11], X[10, 16, 11, 15]]**

Out[]:=

PD[X[6, 2, 7, 1], X[14, 8, 15, 7], X[8, 3, 9, 4], X[2, 13, 3, 14], X[12, 5, 13, 6], X[4, 9, 5, 10], X[16, 12, 1, 11], X[10, 16, 11, 15]]



```
In[*]:= PK = PD[
  X[4, 48, 5, 47], X[5, 40, 6, 41], X[6, 68, 7, 67], X[11, 97, 12, 96],
  X[12, 62, 13, 61], X[13, 26, 14, 27], X[14, 82, 15, 81], X[20, 105, 21, 106],
  X[22, 43, 23, 44], X[23, 65, 24, 64], X[24, 9, 25, 10], X[25, 99, 26, 98],
  X[29, 18, 30, 19], X[30, 90, 31, 89], X[31, 2, 32, 3], X[32, 76, 33, 75],
  X[35, 53, 36, 52], X[36, 72, 37, 71], X[39, 49, 40, 48], X[44, 21, 45, 22],
  X[45, 87, 46, 86], X[50, 38, 51, 37], X[51, 70, 52, 71], X[54, 34, 55, 33],
  X[55, 74, 56, 75], X[56, 4, 57, 3], X[57, 88, 58, 89], X[58, 20, 59, 19],
  X[62, 97, 63, 98], X[63, 11, 64, 10], X[68, 49, 69, 50], X[69, 39, 70, 38],
  X[72, 53, 73, 54], X[73, 35, 74, 34], X[76, 2, 77, 1], X[77, 90, 78, 91],
  X[78, 18, 79, 17], X[82, 99, 83, 100], X[83, 9, 84, 8], X[84, 65, 85, 66],
  X[85, 43, 86, 42], X[87, 105, 88, 104], X[92, 16, 93, 15], X[93, 80, 94, 81],
  X[94, 28, 95, 27], X[95, 60, 96, 61], X[100, 8, 101, 7], X[101, 66, 102, 67],
  X[102, 42, 103, 41], X[103, 46, 104, 47], X[106, 60, 107, 59], X[107, 28, 108, 29],
  X[108, 80, 109, 79], X[109, 16, 110, 17], X[110, 92, 1, 91]
];
```

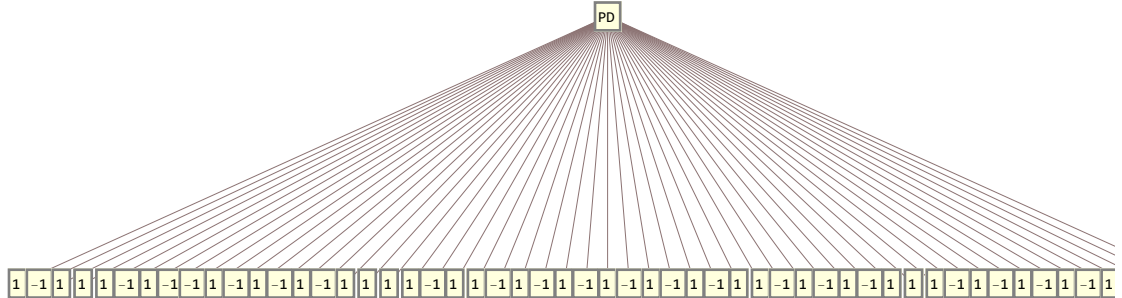
```
J0[pd_, w_] := (
  t1 = pd /. X[i_, j_, k_, l_] => A p[i, j] p[k, l] + B p[i, l] p[j, k];
  t2 = Expand[t1 /. PD -> Times];
  t3 = t2 //. {p[i_, j_] p[j_, k_] => p[i, k],
    p[i_, j_] p[k_, j_] => p[i, k], p[j_, i_] p[j_, k_] => p[i, k]};
  t4 = t3 /. {p[i_, i_] => d, p[i_, j_]^2 => d};
  t5 = Expand[t4 /. {B -> 1 / A, d -> -A^2 - A^-2}];
  Simplify[(-A^3)^-w * t5 / (-A^2 - A^-2) /. A -> q^-1/4]
)
```

```
In[*]:= PK /. {
  X[i_, j_, k_, l_] /; j - l == 1 || l - j > 1 -> 1,
  X[i_, j_, k_, l_] /; l - j == 1 || j - l > 1 -> -1
}
```

```
Out[*]= PD[1, -1, 1, 1, 1, -1, 1, -1, -1, 1, -1, 1, -1, 1, 1, 1, 1, -1, 1, 1, -1, 1, -1, 1, -1, 1,
-1, 1, -1, 1, -1, 1, 1, -1, 1, -1, 1, -1, 1, 1, 1, -1, 1, -1, 1, -1, 1, -1, 1, -1, 1]
```

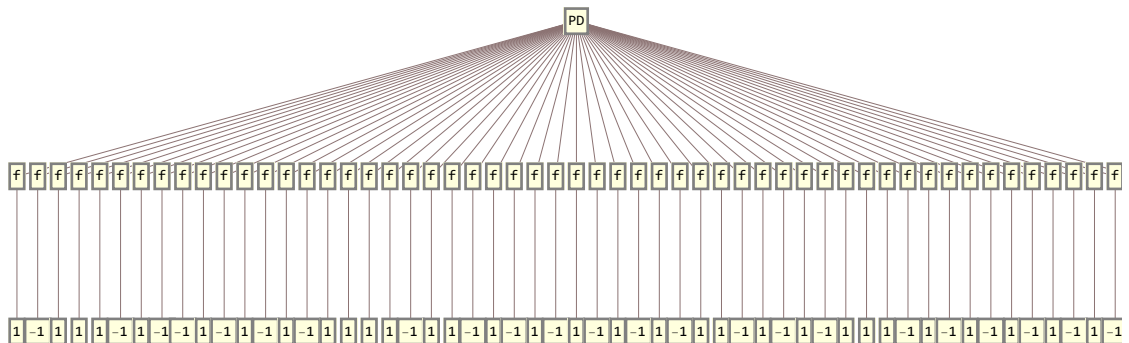
```
In[ ]:= PD[1, -1, 1, 1, 1, -1, 1, -1, -1, 1, -1, 1, -1, 1, 1,
1, 1, -1, 1, 1, -1, 1, -1, 1, -1, 1, -1, 1, -1, 1, 1, -1, 1, -1,
1, -1, 1, 1, 1, -1, 1, -1, 1, -1, 1, -1, 1, -1, 1, -1, 1] // TreeForm
```

Out[]//TreeForm=



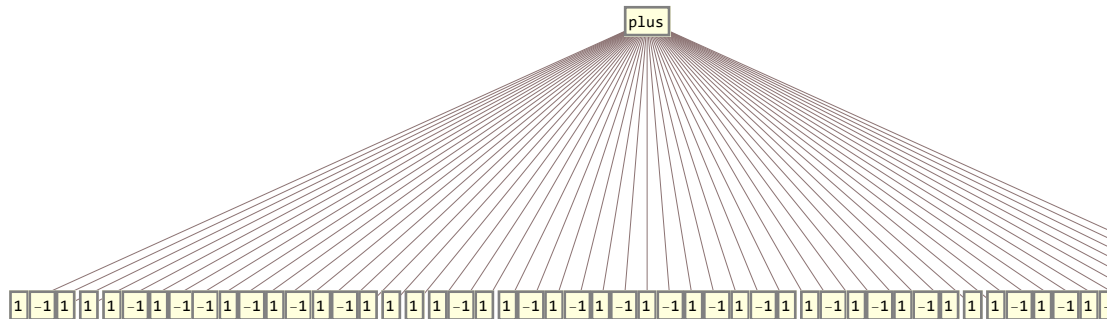
```
In[ ]:= f /@ PD[1, -1, 1, 1, 1, -1, 1, -1, -1, 1, -1, 1, -1, 1, -1,
1, 1, 1, 1, -1, 1, 1, -1, 1, -1, 1, -1, 1, -1, 1, -1, 1, -1, 1,
-1, 1, -1, 1, 1, 1, -1, 1, -1, 1, -1, 1, -1, 1, -1, 1, -1, 1] // TreeForm
```

Out[]//TreeForm=



```
In[ ]:= plus @@ PD[1, -1, 1, 1, 1, -1, 1, -1, -1, 1, -1, 1, -1, 1, -1,
1, 1, 1, 1, -1, 1, 1, -1, 1, -1, 1, -1, 1, -1, 1, -1, 1, -1, 1,
-1, 1, -1, 1, 1, 1, -1, 1, -1, 1, -1, 1, -1, 1, -1, 1, -1, 1] // TreeForm
```

Out[]//TreeForm=



```
In[*]:= Plus @@ PD[1, -1, 1, 1, 1, -1, 1, -1, -1, 1, -1, 1, -1, 1,
  -1, 1, 1, 1, 1, -1, 1, 1, -1, 1, -1, 1, -1, 1, -1, 1, -1, 1, 1,
  -1, 1, -1, 1, -1, 1, 1, 1, -1, 1, -1, 1, -1, 1, -1, 1, -1, 1]
```

```
Out[*]=
9
```

```
In[*]:= Attributes[Plus]
```

```
Out[*]=
{Flat, Listable, NumericFunction, OneIdentity, Orderless, Protected}
```

```
In[*]:= Yusuke = 7
```

```
Out[*]=
7
```

```
In[*]:= Plus = 7
```

 Set: Symbol Plus is Protected.

```
Out[*]=
7
```

```
In[*]:= a + b
```

```
Out[*]=
a + b
```

```
In[*]:= b + a
```

```
Out[*]=
a + b
```

```
In[*]:= J1[pd] := Module[{w, p, t1, t2, t3, t4, t5},
  w = Plus @@ (pd /. {
    X[i_, j_, k_, l_] /; j - l == 1 ∨ l - j > 1 ∩ 1,
    X[i_, j_, k_, l_] /; l - j == 1 ∨ j - l > 1 ∩ -1
  });
  SetAttributes[p, Orderless];
  t1 = pd /. X[i_, j_, k_, l_] ∩ A p[i, j] p[k, l] + B p[i, l] p[j, k];
  t2 = Expand[t1 /. PD ∩ Times];
  t3 = t2 /. p[i_, j_] p[j_, k_] ∩ p[i, k];
  t4 = t3 /. {p[i_, i_] ∩ d, p[i_, j_]^2 ∩ d};
  t5 = Expand[t4 /. {B ∩ 1 / A, d ∩ -A^2 - A^-2}];
  Simplify[(-A^3)^-w  $\frac{t5}{-A^2 - A^{-2}}$  /. A ∩ q^{-1/4}]
]
```

```
In[*]:= J1[D1]
```

```
Out[*]=
q + q^3 - q^4
```

```

In[*]:= J1[D2]
Out[*]=

$$\frac{-1 + q + q^3}{q^4}$$


In[*]:= J1[K817]
Out[*]=

$$7 + \frac{1}{q^4} - \frac{3}{q^3} + \frac{5}{q^2} - \frac{6}{q} - 6q + 5q^2 - 3q^3 + q^4$$


In[*]:= pd = K817
Out[*]=
PD[X[6, 2, 7, 1], X[14, 8, 15, 7], X[8, 3, 9, 4], X[2, 13, 3, 14],
X[12, 5, 13, 6], X[4, 9, 5, 10], X[16, 12, 1, 11], X[10, 16, 11, 15]]

In[*]:= List@@pd // TreeForm

In[*]:= 20!
Out[*]=
2432902008176640000

In[*]:= k = 1; prod = 1;
While[k ≤ 20, prod = prod * k; k = k + 1];
prod
Out[*]=
2432902008176640000

In[*]:= {1, 3, 5, 7, 8} ∩ {1, 8, ayumu, Yusuke}
Out[*]=
{1, 7, 8}

In[*]:= {1, 2, 3, 4, 5, 6} ∩ List@@X[3, 8, 19, 2]
Out[*]=
{2, 3}

In[*]:= Max[{1, 5, 2, 9}]
Out[*]=
9

In[*]:= MaximalBy[{"Ayumu", "Yusuke", "Haruko", "Dror", "Itai", "Reidemeieter"}, StringLength]
Out[*]=
{Reidemeieter}

In[*]:= MinimalBy[{"Ayumu", "Yusuke", "Haruko", "Dror", "Itai", "Reidemeieter"}, StringLength]
Out[*]=
{Dror, Itai}

```

```
In[*]:= First[
  MinimalBy[{"Ayumu", "Yusuke", "Haruko", "Dror", "Itai", "Reidemeieter"}, StringLength]]
```

```
Out[*]=
Dror
```

```
In[*]:= Last[
  MinimalBy[{"Ayumu", "Yusuke", "Haruko", "Dror", "Itai", "Reidemeieter"}, StringLength]]
```

```
Out[*]=
Itai
```

```
In[*]:= RandomChoice[
  MinimalBy[{"Ayumu", "Yusuke", "Haruko", "Dror", "Itai", "Reidemeieter"}, StringLength]]
```

```
Out[*]=
Dror
```

```
In[*]:= J2[pd_] := Module[{w, front, TL, todo, v, x, t1, t2, t3, t4, B, d},
  w = Plus @@ (pd /. {
    X[i_, j_, k_, l_] /; j - l == 1 ∨ l - j > 1 ∴ 1,
    X[i_, j_, k_, l_] /; l - j == 1 ∨ j - l > 1 ∴ -1
  });
  SetAttributes[p, Orderless];
  front = {};
  TL = 1;
  todo = List @@ pd;
  v[x_] := Length[front ∩ List @@ x];
  While[Length[todo] > 0,
    Echo[Short[TL, 10]];
    x = RandomChoice[MaximalBy[todo, v]];
    t1 = TL (x /. X[i_, j_, k_, l_] ∴ A p[i, j] p[k, l] + B p[i, l] p[j, k]);
    t2 = Expand[t1];
    t3 = t2 /. p[i_, j_] p[j_, k_] ∴ p[i, k];
    t4 = t3 /. {p[i_, i_] ∴ d, p[i_, j_]² ∴ d};
    TL = Expand[t4 /. {B → 1 / A, d → -A² - A⁻²}];
    todo = Complement[todo, {x});
    front = Complement[front ∪ List @@ x, front ∩ List @@ x];
  ];
  Simplify[(-A³)⁻w  $\frac{TL}{-A^2 - A^{-2}}$  /. A → q⁻¹/⁴]
```

```
In[*]:= J2[D1]
```

» 1

$$\gg \frac{p[1, 4] p[3, 6]}{A} + A p[1, 3] p[4, 6]$$

$$\gg A^2 p[1, 5] p[2, 4] + p[1, 4] p[2, 5] - \frac{p[1, 4] p[2, 5]}{A^4}$$

Out[*]=

$$q + q^3 - q^4$$

In[*]:= **J2[D2]**

Out[*]=

$$\frac{-1 + q + q^3}{q^4}$$

In[*]:= **J2[K817]**

» 1

» $A p[7, 15] p[8, 14] + \frac{p[7, 14] p[8, 15]}{A}$

» $p[3, 15] p[4, 9] p[7, 14] + \frac{p[3, 9] p[4, 15] p[7, 14]}{A^2} + A^2 p[3, 14] p[4, 9] p[7, 15] + p[3, 9] p[4, 14] p[7, 15]$

» $\frac{p[2, 13] p[4, 15] p[7, 9]}{A} + 2 A p[2, 13] p[4, 9] p[7, 15] - A^5 p[2, 13] p[4, 9] p[7, 15] + \frac{p[2, 7] p[4, 15] p[9, 13]}{A^3} + \frac{p[2, 4] p[7, 15] p[9, 13]}{A} + \frac{p[2, 7] p[4, 9] p[13, 15]}{A}$

» $2 A^2 p[1, 15] p[4, 9] p[6, 13] - A^6 p[1, 15] p[4, 9] p[6, 13] + p[1, 9] p[4, 15] p[6, 13] + p[1, 15] p[4, 6] p[9, 13] - \frac{p[1, 6] p[4, 15] p[9, 13]}{A^6} + \frac{2 p[1, 6] p[4, 15] p[9, 13]}{A^2} + 2 p[1, 6] p[4, 9] p[13, 15] - \frac{p[1, 6] p[4, 9] p[13, 15]}{A^4} - A^4 p[1, 6] p[4, 9] p[13, 15]$

» $\frac{p[1, 15] p[4, 12] p[5, 9]}{A} - \frac{p[1, 12] p[4, 15] p[5, 9]}{A^7} + \frac{2 p[1, 12] p[4, 15] p[5, 9]}{A^3} - \frac{p[1, 15] p[4, 9] p[5, 12]}{A^3} + 3 A p[1, 15] p[4, 9] p[5, 12] - 3 A^5 p[1, 15] p[4, 9] p[5, 12] + A^9 p[1, 15] p[4, 9] p[5, 12] - \frac{p[1, 9] p[4, 15] p[5, 12]}{A^5} + \frac{2 p[1, 9] p[4, 15] p[5, 12]}{A} - A^3 p[1, 9] p[4, 15] p[5, 12] - \frac{p[1, 12] p[4, 9] p[5, 15]}{A^5} + \frac{2 p[1, 12] p[4, 9] p[5, 15]}{A} - A^3 p[1, 12] p[4, 9] p[5, 15]$

» $3 p[1, 15] p[10, 12] - \frac{2 p[1, 15] p[10, 12]}{A^4} - 4 A^4 p[1, 15] p[10, 12] + 3 A^8 p[1, 15] p[10, 12] - A^{12} p[1, 15] p[10, 12] + \frac{p[1, 12] p[10, 15]}{A^{10}} - \frac{3 p[1, 12] p[10, 15]}{A^6} + \frac{3 p[1, 12] p[10, 15]}{A^2} - 3 A^2 p[1, 12] p[10, 15] + A^6 p[1, 12] p[10, 15]$

» $-\frac{2 p[10, 16] p[11, 15]}{A^3} + 3 A p[10, 16] p[11, 15] - 4 A^5 p[10, 16] p[11, 15] + 3 A^9 p[10, 16] p[11, 15] - A^{13} p[10, 16] p[11, 15] - \frac{p[10, 15] p[11, 16]}{A^{13}} + \frac{3 p[10, 15] p[11, 16]}{A^9} - \frac{5 p[10, 15] p[11, 16]}{A^5} + \frac{6 p[10, 15] p[11, 16]}{A} - 5 A^3 p[10, 15] p[11, 16] + 3 A^7 p[10, 15] p[11, 16] - A^{11} p[10, 15] p[11, 16]$

Out[*]=

$$7 + \frac{1}{q^4} - \frac{3}{q^3} + \frac{5}{q^2} - \frac{6}{q} - 6q + 5q^2 - 3q^3 + q^4$$

In[*]:= J2[PK] // Timing

» 1

» $A p[3, 57] p[4, 56] + \frac{p[3, 56] p[4, 57]}{A}$

$$\begin{aligned}
 & \gg \frac{p[2, 32] p[4, 57] p[31, 56]}{A^2} + p[2, 32] p[4, 56] p[31, 57] + \\
 & p[2, 31] p[4, 57] p[32, 56] + A^2 p[2, 31] p[4, 56] p[32, 57] \\
 & \gg \frac{p[2, 32] p[5, 48] p[31, 57] p[47, 56]}{A} + A p[2, 31] p[5, 48] p[32, 57] p[47, 56] + \\
 & \frac{p[2, 32] p[5, 48] p[31, 56] p[47, 57]}{A^3} + \frac{p[2, 31] p[5, 48] p[32, 56] p[47, 57]}{A} + \\
 & A p[2, 32] p[5, 47] p[31, 57] p[48, 56] + A^3 p[2, 31] p[5, 47] p[32, 57] p[48, 56] + \\
 & \frac{p[2, 32] p[5, 47] p[31, 56] p[48, 57]}{A} + A p[2, 31] p[5, 47] p[32, 56] p[48, 57] \\
 & \gg A^2 p[1, 77] p[5, 48] p[31, 76] p[32, 57] p[47, 56] + p[1, 76] p[5, 48] p[31, 77] p[32, 57] p[47, 56] + \\
 & p[1, 77] p[5, 48] p[31, 57] p[32, 76] p[47, 56] + \frac{p[1, 76] p[5, 48] p[31, 57] p[32, 77] p[47, 56]}{A^2} + \\
 & p[1, 77] p[5, 48] p[31, 76] p[32, 56] p[47, 57] + \frac{p[1, 76] p[5, 48] p[31, 77] p[32, 56] p[47, 57]}{A^2} + \\
 & \frac{p[1, 77] \ll 3 \gg p[47, 57]}{A^2} + \ll 1 \gg + \ll 1 \gg + A^2 p[1, 76] \ll 3 \gg p[48, 56] + \\
 & A^2 p[1, 77] p[5, 47] p[31, 57] p[32, 76] p[48, 56] + p[1, 76] p[5, 47] p[31, 57] p[32, 77] p[48, 56] + \\
 & A^2 p[1, 77] p[5, 47] p[31, 76] p[32, 56] p[48, 57] + p[1, 76] p[5, 47] p[31, 77] p[32, 56] p[48, 57] + \\
 & p[1, 77] p[5, 47] p[31, 56] p[32, 76] p[48, 57] + \frac{p[1, 76] p[5, 47] p[31, 56] p[32, 77] p[48, 57]}{A^2} \\
 & \gg \frac{p[1, 77] p[5, 48] p[31, 57] p[33, 75] p[47, 56]}{A} + \\
 & A p[1, 57] p[5, 48] p[31, 77] p[33, 75] p[47, 56] + \frac{p[1, 77] p[5, 48] p[31, 56] p[33, 75] p[47, 57]}{A^3} + \\
 & \frac{p[1, 56] p[5, 48] p[31, 77] p[33, 75] p[47, 57]}{A} + A p[1, 77] p[5, 47] p[31, 57] p[33, 75] p[48, 56] + \\
 & A^3 p[1, 57] p[5, 47] p[31, 77] p[33, 75] p[48, 56] + \ll 8 \gg + \\
 & A^3 p[1, 77] p[5, 47] p[31, 33] p[48, 56] p[57, 75] + A p[1, 33] p[5, 47] p[31, 77] p[48, 56] p[57, 75] + \\
 & \frac{p[1, 33] p[5, 48] p[31, 57] p[47, 56] p[75, 77]}{A^3} + \frac{p[1, 33] p[5, 48] p[31, 56] p[47, 57] p[75, 77]}{A^5} + \\
 & \frac{p[1, 33] p[5, 47] p[31, 57] p[48, 56] p[75, 77]}{A} + \frac{p[1, 33] p[5, 47] p[31, 56] p[48, 57] p[75, 77]}{A^3} \\
 & \gg \frac{p[1, 77] p[5, 48] p[31, 74] p[33, 55] p[47, 57]}{A^4} + \frac{p[1, 74] p[5, 48] p[31, 77] p[33, 55] p[47, 57]}{A^2} + \\
 & \frac{p[1, 77] p[5, 48] p[31, 57] p[33, 55] p[47, 74]}{A^2} + p[1, 57] p[5, 48] p[31, 77] p[33, 55] p[47, 74] + \\
 & \frac{p[1, 77] p[5, 47] p[31, 74] p[33, 55] p[48, 57]}{A^2} + p[1, 74] p[5, 47] p[31, 77] p[33, 55] p[48, 57] + \\
 & p[1, 77] p[5, 47] p[31, 57] p[33, 55] p[48, 74] + \ll 15 \gg + \\
 & A^2 p[1, 33] p[5, 47] p[31, 77] p[48, 57] p[55, 74] + p[1, 33] p[5, 47] p[31, 57] p[48, 77] p[55, 74] + \\
 & \frac{p[1, 33] p[5, 48] p[31, 74] p[47, 57] p[55, 77]}{A^6} + \frac{p[1, 33] p[5, 48] p[31, 57] p[47, 74] p[55, 77]}{A^4} + \\
 & \frac{p[1, 33] p[5, 47] p[31, 74] p[48, 57] p[55, 77]}{A^4} + \frac{p[1, 33] p[5, 47] p[31, 57] p[48, 74] p[55, 77]}{A^2}
 \end{aligned}$$

$$\begin{aligned}
 & \gg \frac{p[1, 77] p[5, 48] p[31, 57] p[34, 74] p[47, 54]}{A} + \\
 & A p[1, 57] p[5, 48] p[31, 77] p[34, 74] p[47, 54] + \frac{p[1, 77] p[5, 48] p[31, 74] p[34, 54] p[47, 57]}{A^5} + \\
 & \frac{p[1, 74] p[5, 48] p[31, 77] p[34, 54] p[47, 57]}{A^3} + \frac{p[1, 77] p[5, 48] p[31, 54] p[34, 74] p[47, 57]}{A^3} + \\
 & \frac{p[1, 54] p[5, 48] p[31, 77] p[34, 74] p[47, 57]}{A^5} + \ll 20 \gg + \\
 & A p[1, 57] p[5, 47] p[31, 77] p[34, 54] p[48, 74] + A p[1, 77] p[5, 47] p[31, 54] p[34, 57] p[48, 74] + \\
 & \frac{p[1, 54] p[5, 47] p[31, 77] p[34, 57] p[48, 74]}{A} + \frac{p[1, 54] p[5, 47] p[31, 57] p[34, 77] p[48, 74]}{A^3} + \\
 & A p[1, 74] p[5, 47] p[31, 57] p[34, 54] p[48, 77] + \frac{p[1, 54] p[5, 47] p[31, 57] p[34, 74] p[48, 77]}{A} \\
 & \gg \frac{p[1, 77] p[5, 48] p[31, 57] p[35, 73] p[47, 54]}{A^2} + p[1, 57] p[5, 48] p[31, 77] p[35, 73] p[47, 54] - \\
 & A^4 p[1, 57] p[5, 48] p[31, 77] p[35, 73] p[47, 54] + \frac{p[1, 77] p[5, 48] p[31, 54] p[35, 73] p[47, 57]}{A^4} + \\
 & \frac{p[1, 54] p[5, 48] p[31, 77] p[35, 73] p[47, 57]}{A^6} + \ll 28 \gg + \\
 & p[1, 77] p[5, 47] p[31, 54] p[35, 48] p[57, 73] + \frac{p[1, 54] p[5, 47] p[31, 77] p[35, 48] p[57, 73]}{A^2} + \\
 & \frac{p[1, 54] p[5, 48] p[31, 57] p[35, 47] p[73, 77]}{A^6} + \frac{p[1, 54] p[5, 47] p[31, 57] p[35, 48] p[73, 77]}{A^4} + \\
 & \frac{p[1, 54] p[5, 48] p[31, 35] p[47, 57] p[73, 77]}{A^8} + \frac{p[1, 54] p[5, 47] p[31, 35] p[48, 57] p[73, 77]}{A^6} \\
 & \gg \frac{p[1, 77] p[5, 48] p[31, 72] p[35, 53] p[47, 57]}{A^5} + \frac{p[1, 72] p[5, 48] p[31, 77] p[35, 53] p[47, 57]}{A^7} + \\
 & \frac{p[1, 72] p[5, 48] p[31, 77] p[35, 53] p[47, 57]}{A^3} - A p[1, 72] p[5, 48] p[31, 77] p[35, 53] p[47, 57] + \\
 & \frac{p[1, 77] p[5, 48] p[31, 57] p[35, 53] p[47, 72]}{A^3} + \ll 46 \gg + \\
 & \frac{p[1, 72] p[5, 48] p[31, 57] p[35, 47] p[53, 77]}{A^7} + \frac{p[1, 72] p[5, 47] p[31, 57] p[35, 48] p[53, 77]}{A^5} + \\
 & \frac{p[1, 72] p[5, 48] p[31, 35] p[47, 57] p[53, 77]}{A^9} + \frac{p[1, 72] p[5, 47] p[31, 35] p[48, 57] p[53, 77]}{A^7} \\
 & \gg \frac{p[1, 77] p[5, 48] p[31, 72] p[36, 57] p[47, 52]}{A^4} + \frac{p[1, 72] p[5, 48] p[31, 77] p[36, 57] p[47, 52]}{A^6} + \\
 & \frac{p[1, 77] p[5, 48] p[31, 57] p[36, 72] p[47, 52]}{A^6} + \frac{p[1, 77] p[5, 48] p[31, 57] p[36, 72] p[47, 52]}{A^2} - \\
 & A^2 p[1, 77] p[5, 48] p[31, 57] p[36, 72] p[47, 52] + \ll 49 \gg + \\
 & p[1, 57] p[5, 47] p[31, 77] p[36, 52] p[48, 72] - A^4 p[1, 57] p[5, 47] p[31, 77] p[36, 52] p[48, 72] + \\
 & \frac{p[1, 72] p[5, 47] p[31, 57] p[36, 52] p[48, 77]}{A^4} - A^4 p[1, 72] p[5, 47] p[31, 57] p[36, 52] p[48, 77] + \\
 & \frac{p[1, 52] p[5, 47] p[31, 57] p[36, 72] p[48, 77]}{A^2} - A^2 p[1, 52] p[5, 47] p[31, 57] p[36, 72] p[48, 77]
 \end{aligned}$$

$$\begin{aligned}
 & \gg \frac{p[1, 77] p[5, 48] p[31, 57] p[37, 71] p[47, 52]}{A^7} + \frac{p[1, 77] p[5, 48] p[31, 57] p[37, 71] p[47, 52]}{A^3} - \\
 & A p[1, 77] p[5, 48] p[31, 57] p[37, 71] p[47, 52] + \frac{p[1, 57] p[5, 48] p[31, 77] p[37, 71] p[47, 52]}{A^5} - \\
 & A^3 p[1, 57] p[5, 48] p[31, 77] p[37, 71] p[47, 52] + A^7 p[1, 57] p[5, 48] p[31, 77] p[37, 71] p[47, 52] + \\
 & \ll 55 \gg + \frac{p[1, 37] p[5, 47] p[31, 77] p[48, 52] p[57, 71]}{A^5} + \\
 & \frac{p[1, 37] p[5, 48] p[31, 57] p[47, 52] p[71, 77]}{A^9} + \frac{p[1, 37] p[5, 48] p[31, 52] p[47, 57] p[71, 77]}{A^{11}} + \\
 & \frac{p[1, 37] p[5, 47] p[31, 57] p[48, 52] p[71, 77]}{A^7} + \frac{p[1, 37] p[5, 47] p[31, 52] p[48, 57] p[71, 77]}{A^9} \\
 & \gg \frac{p[1, 77] p[5, 48] p[31, 70] p[37, 51] p[47, 57]}{A^{10}} - p[1, 70] p[5, 48] p[31, 77] p[37, 51] p[47, 57] + \\
 & \frac{p[1, 70] p[5, 48] p[31, 77] p[37, 51] p[47, 57]}{A^8} + A^4 p[1, 70] p[5, 48] p[31, 77] p[37, 51] p[47, 57] - \\
 & p[1, 77] p[5, 48] p[31, 57] p[37, 51] p[47, 70] + \frac{p[1, 77] p[5, 48] p[31, 57] p[37, 51] p[47, 70]}{A^8} + \\
 & \ll 63 \gg + A^6 p[1, 37] p[5, 47] p[31, 57] p[48, 77] p[51, 70] + \\
 & \frac{p[1, 37] p[5, 48] p[31, 70] p[47, 57] p[51, 77]}{A^{12}} + \frac{p[1, 37] p[5, 48] p[31, 57] p[47, 70] p[51, 77]}{A^{10}} + \\
 & \frac{p[1, 37] p[5, 47] p[31, 70] p[48, 57] p[51, 77]}{A^{10}} + \frac{p[1, 37] p[5, 47] p[31, 57] p[48, 70] p[51, 77]}{A^8} \\
 & \gg \frac{p[1, 77] p[5, 48] p[31, 57] p[38, 70] p[47, 50]}{A^7} - \\
 & A p[1, 77] p[5, 48] p[31, 57] p[38, 70] p[47, 50] + A^5 p[1, 77] p[5, 48] p[31, 57] p[38, 70] p[47, 50] + \\
 & \frac{p[1, 57] p[5, 48] p[31, 77] p[38, 70] p[47, 50]}{A^5} - \frac{p[1, 57] p[5, 48] p[31, 77] p[38, 70] p[47, 50]}{A} + \\
 & A^7 p[1, 57] p[5, 48] p[31, 77] p[38, 70] p[47, 50] + \ll 67 \gg + \\
 & A^7 p[1, 70] p[5, 47] p[31, 57] p[38, 50] p[48, 77] + \frac{p[1, 50] p[5, 47] p[31, 57] p[38, 70] p[48, 77]}{A^7} - \\
 & A p[1, 50] p[5, 47] p[31, 57] p[38, 70] p[48, 77] + A^5 p[1, 50] p[5, 47] p[31, 57] p[38, 70] p[48, 77] \\
 & \gg -p[1, 77] p[5, 48] p[31, 57] p[39, 69] p[47, 50] + \\
 & \frac{p[1, 77] p[5, 48] p[31, 57] p[39, 69] p[47, 50]}{A^8} + A^4 p[1, 77] p[5, 48] p[31, 57] p[39, 69] p[47, 50] + \\
 & \frac{p[1, 57] p[5, 48] p[31, 77] p[39, 69] p[47, 50]}{A^6} - \frac{p[1, 57] p[5, 48] p[31, 77] p[39, 69] p[47, 50]}{A^2} + \\
 & \ll 76 \gg + \frac{p[1, 50] p[5, 47] p[31, 77] p[39, 48] p[57, 69]}{A^8} + \\
 & \frac{p[1, 50] p[5, 48] p[31, 57] p[39, 47] p[69, 77]}{A^{12}} + \frac{p[1, 50] p[5, 47] p[31, 57] p[39, 48] p[69, 77]}{A^{10}} + \\
 & \frac{p[1, 50] p[5, 48] p[31, 39] p[47, 57] p[69, 77]}{A^{14}} + \frac{p[1, 50] p[5, 47] p[31, 39] p[48, 57] p[69, 77]}{A^{12}}
 \end{aligned}$$

$$\begin{aligned}
 & \gg \frac{p[1, 77] p[5, 48] p[31, 68] p[39, 49] p[47, 57]}{A^{11}} + \\
 & \frac{p[1, 68] p[5, 48] p[31, 77] p[39, 49] p[47, 57]}{A^{13}} + \frac{p[1, 68] p[5, 48] p[31, 77] p[39, 49] p[47, 57]}{A^9} - \\
 & \frac{p[1, 68] p[5, 48] p[31, 77] p[39, 49] p[47, 57]}{A^5} + A^3 p[1, 68] p[5, 48] p[31, 77] p[39, 49] p[47, 57] - \\
 & A^7 p[1, 68] p[5, 48] p[31, 77] p[39, 49] p[47, 57] + \ll 89 \gg + \\
 & A^5 p[1, 39] p[5, 47] p[31, 57] p[48, 77] p[49, 68] - A^9 p[1, 39] p[5, 47] p[31, 57] p[48, 77] p[49, 68] + \\
 & \frac{p[1, 68] p[5, 48] p[31, 57] p[39, 47] p[49, 77]}{A^{13}} + \frac{p[1, 68] p[5, 47] p[31, 57] p[39, 48] p[49, 77]}{A^{11}} + \\
 & \frac{p[1, 68] p[5, 48] p[31, 39] p[47, 57] p[49, 77]}{A^{15}} + \frac{p[1, 68] p[5, 47] p[31, 39] p[48, 57] p[49, 77]}{A^{13}} \\
 & \gg \frac{p[1, 77] p[5, 47] p[31, 68] p[40, 57]}{A^{10}} - \frac{p[1, 77] p[5, 47] p[31, 68] p[40, 57]}{A^6} + \\
 & \frac{p[1, 68] p[5, 47] p[31, 77] p[40, 57]}{A^{12}} - \frac{p[1, 68] p[5, 47] p[31, 77] p[40, 57]}{A^4} + \\
 & A^4 p[1, 68] p[5, 47] p[31, 77] p[40, 57] - A^8 p[1, 68] p[5, 47] p[31, 77] p[40, 57] + \\
 & \frac{p[1, 77] p[5, 47] p[31, 57] p[40, 68]}{A^{12}} + \ll 48 \gg + \frac{p[1, 68] p[5, 40] p[31, 57] p[47, 77]}{A^{12}} - \\
 & \frac{p[1, 68] p[5, 40] p[31, 57] p[47, 77]}{A^4} - A^8 p[1, 68] p[5, 40] p[31, 57] p[47, 77] + \\
 & \frac{p[1, 5] p[31, 57] p[40, 68] p[47, 77]}{A^{10}} - \frac{p[1, 5] p[31, 57] p[40, 68] p[47, 77]}{A^6} + \\
 & A^2 p[1, 5] p[31, 57] p[40, 68] p[47, 77] - A^6 p[1, 5] p[31, 57] p[40, 68] p[47, 77] \\
 & \gg \frac{p[1, 77] p[6, 68] p[31, 57] p[41, 47]}{A^{13}} + \\
 & \frac{p[1, 77] p[6, 68] p[31, 57] p[41, 47]}{A^9} - \frac{2 p[1, 77] p[6, 68] p[31, 57] p[41, 47]}{A^5} + \\
 & 2 A^3 p[1, 77] p[6, 68] p[31, 57] p[41, 47] - 2 A^7 p[1, 77] p[6, 68] p[31, 57] p[41, 47] + \\
 & \frac{p[1, 68] p[6, 77] p[31, 57] p[41, 47]}{A^{15}} - \frac{p[1, 68] p[6, 77] p[31, 57] p[41, 47]}{A^3} + \ll 61 \gg + \\
 & \frac{A^3 p[1, 68] p[6, 41] p[31, 57] p[47, 77] - A^7 p[1, 68] p[6, 41] p[31, 57] p[47, 77]}{A^{11}} + \\
 & \frac{p[1, 41] p[6, 68] p[31, 57] p[47, 77]}{A^7} - \frac{p[1, 41] p[6, 68] p[31, 57] p[47, 77]}{A^7} + \\
 & A p[1, 41] p[6, 68] p[31, 57] p[47, 77] - A^5 p[1, 41] p[6, 68] p[31, 57] p[47, 77]
 \end{aligned}$$

$$\begin{aligned}
 & \gg \frac{p[1, 77] p[7, 67] p[31, 57] p[41, 47]}{A^{14}} + \\
 & \frac{p[1, 77] p[7, 67] p[31, 57] p[41, 47]}{A^{10}} - \frac{2 p[1, 77] p[7, 67] p[31, 57] p[41, 47]}{A^6} - \\
 & \frac{p[1, 77] p[7, 67] p[31, 57] p[41, 47]}{A^2} + 2 A^2 p[1, 77] p[7, 67] p[31, 57] p[41, 47] - \\
 & A^6 p[1, 77] p[7, 67] p[31, 57] p[41, 47] - A^{10} p[1, 77] p[7, 67] p[31, 57] p[41, 47] + \\
 & A^{14} p[1, 77] p[7, 67] p[31, 57] p[41, 47] + \ll 60 \gg + A^2 p[1, 7] p[31, 77] p[41, 47] p[57, 67] - \\
 & A^6 p[1, 7] p[31, 77] p[41, 47] p[57, 67] + p[1, 7] p[31, 57] p[41, 47] p[67, 77] + \\
 & \frac{p[1, 7] p[31, 57] p[41, 47] p[67, 77]}{A^{16}} - \frac{p[1, 7] p[31, 57] p[41, 47] p[67, 77]}{A^4} - \\
 & A^8 p[1, 7] p[31, 57] p[41, 47] p[67, 77] + \frac{p[1, 7] p[31, 41] p[47, 57] p[67, 77]}{A^{18}} \\
 & \gg \frac{p[1, 89] p[7, 67] p[31, 77] p[41, 47] p[58, 88]}{A^{13}} - \frac{2 p[1, 89] p[7, 67] p[31, 77] p[41, 47] p[58, 88]}{A^5} + \\
 & \frac{p[1, 89] p[7, 67] p[31, 77] p[41, 47] p[58, 88]}{A} + A^3 p[1, 89] p[7, 67] p[31, 77] p[41, 47] p[58, 88] - \\
 & 2 A^7 p[1, 89] p[7, 67] p[31, 77] p[41, 47] p[58, 88] + \\
 & A^{11} p[1, 89] p[7, 67] p[31, 77] p[41, 47] p[58, 88] + \ll 150 \gg + \\
 & \frac{p[1, 7] p[31, 77] p[41, 47] p[58, 88] p[67, 89]}{A^{15}} - \frac{p[1, 7] p[31, 77] p[41, 47] p[58, 88] p[67, 89]}{A^7} + \\
 & A p[1, 7] p[31, 77] p[41, 47] p[58, 88] p[67, 89] - A^5 p[1, 7] p[31, 77] p[41, 47] p[58, 88] p[67, 89] \\
 & p[1, 77] p[7, 67] p[30, 90] p[41, 58] p[47, 88] \\
 & \gg \frac{p[1, 77] p[7, 67] p[30, 90] p[41, 58] p[47, 88]}{A^{14}} + \frac{p[1, 77] p[7, 90] p[30, 58] p[41, 67] p[47, 88]}{A^{14}} - \\
 & \frac{p[1, 77] p[7, 90] p[30, 58] p[41, 67] p[47, 88]}{A^2} + \frac{p[1, 77] p[7, 58] p[30, 90] p[41, 67] p[47, 88]}{A^{12}} - \\
 & \frac{p[1, 77] p[7, 58] p[30, 90] p[41, 67] p[47, 88]}{A^6} + \ll 200 \gg + \\
 & \frac{p[1, 77] p[7, 58] p[30, 90] p[41, 67] p[47, 88]}{A^4} + \\
 & A^2 p[1, 7] p[30, 58] p[41, 67] p[47, 77] p[88, 90] - A^6 p[1, 7] p[30, 58] p[41, 67] p[47, 77] p[88, 90] + \\
 & p[1, 7] p[30, 58] p[41, 47] p[67, 77] p[88, 90] + \frac{p[1, 7] p[30, 58] p[41, 47] p[67, 77] p[88, 90]}{A^{16}} - \\
 & \frac{p[1, 7] p[30, 58] p[41, 47] p[67, 77] p[88, 90]}{A^4} - A^8 p[1, 7] p[30, 58] p[41, 47] p[67, 77] p[88, 90] \\
 & \gg \frac{p[1, 91] p[7, 67] p[30, 78] p[41, 58] p[47, 88]}{A^{15}} - \frac{p[1, 91] p[7, 67] p[30, 78] p[41, 58] p[47, 88]}{A^3} + \\
 & \frac{p[1, 91] p[7, 78] p[30, 58] p[41, 67] p[47, 88]}{A^{15}} - \frac{p[1, 91] p[7, 78] p[30, 58] p[41, 67] p[47, 88]}{A^7} + \\
 & \frac{p[1, 91] p[7, 58] p[30, 78] p[41, 67] p[47, 88]}{A^{13}} - \frac{p[1, 91] p[7, 58] p[30, 78] p[41, 67] p[47, 88]}{A^5} + \\
 & \ll 199 \gg + \frac{p[1, 7] p[30, 58] p[41, 47] p[67, 88] p[78, 91]}{A^{11}} - \\
 & \frac{p[1, 7] p[30, 58] p[41, 47] p[67, 88] p[78, 91]}{A^7} - \frac{p[1, 7] p[30, 58] p[41, 47] p[67, 88] p[78, 91]}{A^3} + \\
 & A p[1, 7] p[30, 58] p[41, 47] p[67, 88] p[78, 91] - A^9 p[1, 7] p[30, 58] p[41, 47] p[67, 88] p[78, 91]
 \end{aligned}$$

$$\begin{aligned}
 & \gg \frac{p[7, 92] p[30, 78] p[41, 67] p[47, 110] p[58, 88]}{A^{18}} + \frac{p[7, 92] p[30, 78] p[41, 67] p[47, 110] p[58, 88]}{A^{14}} - \\
 & \frac{p[7, 92] p[30, 78] p[41, 67] p[47, 110] p[58, 88]}{A^{10}} - \\
 & \frac{p[7, 92] p[30, 78] p[41, 67] p[47, 110] p[58, 88]}{A^6} - \\
 & \frac{A^2 p[7, 92] p[30, 78] p[41, 67] p[47, 110] p[58, 88] + p[7, 47] p[30, 58] p[41, 67] p[78, 88] p[92, 110]}{A^{14}} - \\
 & \frac{\ll 202 \gg + p[7, 47] p[30, 58] p[41, 67] p[78, 88] p[92, 110]}{A^6} + \\
 & A^2 p[7, 47] p[30, 58] p[41, 67] p[78, 88] p[92, 110] - \\
 & A^6 p[7, 47] p[30, 58] p[41, 67] p[78, 88] p[92, 110] \\
 & \gg \frac{p[7, 92] p[30, 78] p[42, 103] p[47, 102] p[58, 110] p[67, 88]}{A^{15}} - \\
 & \frac{p[7, 92] p[30, 78] p[42, 103] p[47, 102] p[58, 110] p[67, 88]}{A^7} + \\
 & A p[7, 92] p[30, 78] p[42, 103] p[47, 102] p[58, 110] p[67, 88] - \\
 & A^5 p[7, 92] p[30, 78] p[42, 103] p[47, 102] p[58, 110] p[67, 88] + \\
 & \ll 423 \gg + \frac{p[7, 67] p[30, 58] p[42, 102] p[47, 88] p[78, 103] p[92, 110]}{A^{17}} + \\
 & \frac{p[7, 67] p[30, 58] p[42, 102] p[47, 88] p[78, 103] p[92, 110]}{A^9} - \\
 & \frac{p[7, 67] p[30, 58] p[42, 102] p[47, 88] p[78, 103] p[92, 110]}{A^5} + \\
 & \frac{p[7, 67] p[30, 47] p[42, 102] p[58, 88] p[78, 103] p[92, 110]}{A^{19}} \\
 & \gg \frac{p[7, 92] p[30, 78] p[42, 102] p[46, 104] p[58, 110] p[67, 88] + p[7, 92] p[30, 78] p[42, 102] p[46, 104] p[58, 110] p[67, 88]}{A^{16}} + \\
 & \ll 567 \gg + \frac{p[7, 67] p[30, 78] p[42, 46] p[58, 88] p[92, 102] p[104, 110]}{A^{16}} - \\
 & \frac{p[7, 67] p[30, 78] p[42, 46] p[58, 88] p[92, 102] p[104, 110]}{A^8} + \\
 & \frac{p[7, 67] p[30, 58] p[42, 46] p[78, 88] p[92, 102] p[104, 110]}{A^{14}} - \\
 & \frac{p[7, 67] p[30, 58] p[42, 46] p[78, 88] p[92, 102] p[104, 110]}{A^{10}}
 \end{aligned}$$

$$\begin{aligned}
 & \gg \frac{p[7, 101] p[30, 78] p[42, 110] p[46, 104] p[58, 88] p[66, 92]}{A^{19}} - \\
 & \frac{p[7, 101] p[30, 78] p[42, 110] p[46, 104] p[58, 88] p[66, 92]}{A^{11}} + \\
 & \frac{p[7, 101] p[30, 78] p[42, 88] p[46, 104] p[58, 110] p[66, 92]}{A^{17}} - \\
 & \frac{p[7, 101] p[30, 78] p[42, 88] p[46, 104] p[58, 110] p[66, 92]}{A^9} + \\
 & \ll 585 \gg + \frac{p[7, 92] p[30, 58] p[42, 46] p[66, 101] p[78, 88] p[104, 110]}{A} - \\
 & \frac{A^7 p[7, 92] p[30, 58] p[42, 46] p[66, 101] p[78, 88] p[104, 110] + p[7, 42] p[30, 58] p[46, 92] p[66, 101] p[78, 88] p[104, 110]}{A^{11}} - \\
 & \frac{p[7, 42] p[30, 58] p[46, 92] p[66, 101] p[78, 88] p[104, 110]}{A^7} \\
 & \gg \frac{p[8, 100] p[30, 104] p[42, 110] p[46, 78] p[58, 88] p[66, 92]}{A^{18}} + \\
 & \frac{p[8, 100] p[30, 78] p[42, 110] p[46, 104] p[58, 88] p[66, 92]}{A^{20}} + \\
 & \frac{p[8, 100] p[30, 78] p[42, 110] p[46, 104] p[58, 88] p[66, 92]}{A^{16}} + \\
 & \frac{p[8, 100] p[30, 78] p[42, 88] p[46, 104] p[58, 110] p[66, 92]}{A^{18}} + \\
 & \ll 625 \gg + \frac{p[8, 46] p[30, 58] p[42, 66] p[78, 88] p[92, 100] p[104, 110] + p[8, 46] p[30, 58] p[42, 66] p[78, 88] p[92, 100] p[104, 110]}{A^{16}} - \\
 & \frac{p[8, 46] p[30, 58] p[42, 66] p[78, 88] p[92, 100] p[104, 110]}{A^8} - \\
 & \frac{A^4 p[8, 46] p[30, 58] p[42, 66] p[78, 88] p[92, 100] p[104, 110]}{p[8, 100] p[30, 87] p[42, 110] p[46, 78] p[58, 105] p[66, 92]} + \\
 & \gg \frac{p[8, 100] p[30, 78] p[42, 110] p[46, 87] p[58, 105] p[66, 92]}{A^{19}} + \\
 & \frac{p[8, 100] p[30, 78] p[42, 110] p[46, 87] p[58, 105] p[66, 92]}{A^{21}} + \\
 & \frac{p[8, 100] p[30, 78] p[42, 105] p[46, 87] p[58, 110] p[66, 92]}{A^{17}} + \\
 & \frac{p[8, 30] p[42, 66] p[46, 58] p[78, 100] p[87, 105] p[92, 110]}{\ll 761 \gg + A^{19}} - \\
 & \frac{p[8, 30] p[42, 66] p[46, 58] p[78, 100] p[87, 105] p[92, 110]}{A^{15}} + \\
 & \frac{2 p[8, 30] p[42, 46] p[58, 66] p[78, 100] p[87, 105] p[92, 110]}{A^{11}} - \\
 & \frac{p[8, 30] p[42, 46] p[58, 66] p[78, 100] p[87, 105] p[92, 110]}{A^{17}} \\
 & \frac{p[8, 30] p[42, 46] p[58, 66] p[78, 100] p[87, 105] p[92, 110]}{A^{13}}
 \end{aligned}$$

$$\begin{aligned}
 & \gg \frac{p[8, 100] p[30, 78] p[42, 110] p[45, 105] p[58, 86] p[66, 92]}{A^{18}} + \\
 & \frac{p[8, 100] p[30, 78] p[42, 110] p[45, 105] p[58, 86] p[66, 92]}{A^{14}} - \\
 & \frac{p[8, 100] p[30, 78] p[42, 110] p[45, 105] p[58, 86] p[66, 92]}{A^{10}} + \\
 & \ll 1014 \gg + \frac{p[8, 66] p[30, 58] p[42, 86] p[45, 100] p[78, 105] p[92, 110]}{A^{12}} - \\
 & \frac{p[8, 66] p[30, 58] p[42, 86] p[45, 100] p[78, 105] p[92, 110]}{A^8} - \\
 & \frac{p[8, 66] p[30, 58] p[42, 86] p[45, 100] p[78, 105] p[92, 110]}{A^4} - \\
 & A^8 p[8, 66] p[30, 58] p[42, 86] p[45, 100] p[78, 105] p[92, 110] \\
 & \gg \frac{p[8, 100] p[30, 78] p[43, 92] p[45, 110] p[58, 105] p[66, 85]}{A^{17}} + \\
 & \frac{p[8, 100] p[30, 78] p[43, 92] p[45, 110] p[58, 105] p[66, 85]}{A^{13}} - \\
 & \frac{p[8, 100] p[30, 78] p[43, 92] p[45, 110] p[58, 105] p[66, 85]}{A^9} - \\
 & \frac{p[8, 100] p[30, 78] p[43, 92] p[45, 110] p[58, 105] p[66, 85]}{A^5} + \ll 1165 \gg + \\
 & A^{11} p[8, 66] p[30, 58] p[43, 45] p[78, 100] p[85, 105] p[92, 110] - \\
 & \frac{p[8, 30] p[43, 45] p[58, 66] p[78, 100] p[85, 105] p[92, 110]}{A^{23}} + \\
 & \frac{2 p[8, 30] p[43, 45] p[58, 66] p[78, 100] p[85, 105] p[92, 110]}{A^{19}} - \\
 & \frac{p[8, 30] p[43, 45] p[58, 66] p[78, 100] p[85, 105] p[92, 110]}{A^{15}} \\
 & \gg - \frac{p[8, 100] p[30, 78] p[43, 92] p[45, 110] p[58, 105] p[65, 84]}{A^{22}} + \\
 & \frac{2 p[8, 100] p[30, 78] p[43, 92] p[45, 110] p[58, 105] p[65, 84]}{A^{18}} + \\
 & \ll 1290 \gg + \frac{2 p[8, 84] p[30, 58] p[43, 45] p[65, 100] p[78, 105] p[92, 110]}{A^4} - \\
 & A^4 p[8, 84] p[30, 58] p[43, 45] p[65, 100] p[78, 105] p[92, 110] \\
 & \gg - \frac{p[9, 83] p[30, 78] p[43, 92] p[45, 110] p[58, 105] p[65, 100]}{A^{21}} + \\
 & \frac{2 p[9, 83] p[30, 78] p[43, 92] p[45, 110] p[58, 105] p[65, 100]}{A^{17}} + \\
 & \ll 1419 \gg + \frac{2 p[9, 45] p[30, 58] p[43, 65] p[78, 100] p[83, 105] p[92, 110]}{A^3} + \\
 & 2 A p[9, 45] p[30, 58] p[43, 65] p[78, 100] p[83, 105] p[92, 110] - \\
 & 4 A^5 p[9, 45] p[30, 58] p[43, 65] p[78, 100] p[83, 105] p[92, 110] + \\
 & 2 A^9 p[9, 45] p[30, 58] p[43, 65] p[78, 100] p[83, 105] p[92, 110]
 \end{aligned}$$

$$\begin{aligned}
 & \gg - \frac{p[9, 99] p[30, 78] p[43, 92] p[45, 110] p[58, 105] p[65, 82]}{A^{22}} + \\
 & \frac{2 p[9, 99] p[30, 78] p[43, 92] p[45, 110] p[58, 105] p[65, 82]}{A^{18}} + \\
 & \ll 1508 \gg + \frac{p[9, 45] p[30, 58] p[43, 65] p[78, 105] p[82, 92] p[99, 110]}{A^8} + \\
 & \frac{2 p[9, 45] p[30, 58] p[43, 65] p[78, 105] p[82, 92] p[99, 110]}{A^4} + \\
 & A^4 p[9, 45] p[30, 58] p[43, 65] p[78, 105] p[82, 92] p[99, 110] \\
 & \gg - \frac{p[9, 99] p[17, 110] p[18, 79] p[30, 92] p[43, 65] p[45, 105] p[58, 82]}{A^{23}} + \\
 & \frac{2 p[9, 99] p[17, 110] p[18, 79] p[30, 92] p[43, 65] p[45, 105] p[58, 82]}{A^{19}} + \\
 & \ll 3022 \gg + \frac{p[9, 17] p[18, 79] p[30, 58] p[43, 45] p[65, 105] p[82, 92] p[99, 110]}{A^9} - \\
 & \frac{p[9, 17] p[18, 79] p[30, 58] p[43, 45] p[65, 105] p[82, 92] p[99, 110]}{A} \\
 & \gg 9 p[9, 99] p[16, 109] p[18, 79] p[30, 92] p[43, 65] p[45, 105] p[58, 82] - \\
 & \frac{p[9, 99] p[16, 109] p[18, 79] p[30, 92] p[43, 65] p[45, 105] p[58, 82]}{A^{24}} + \\
 & \frac{p[9, 99] p[16, 109] p[18, 79] p[30, 92] p[43, 65] p[45, 105] p[58, 82]}{A^{20}} + \ll 4243 \gg + \\
 & \frac{2 p[9, 65] p[16, 45] p[18, 79] p[30, 58] p[43, 92] p[82, 99] p[105, 109]}{A^4} - \\
 & A^4 p[9, 65] p[16, 45] p[18, 79] p[30, 58] p[43, 92] p[82, 99] p[105, 109] + \\
 & A^8 p[9, 65] p[16, 45] p[18, 79] p[30, 58] p[43, 92] p[82, 99] p[105, 109] \\
 & \gg - \frac{p[9, 99] p[16, 92] p[18, 108] p[30, 80] p[43, 65] p[45, 105] p[58, 82]}{A^{27}} + \\
 & \frac{p[9, 99] p[16, 92] p[18, 108] p[30, 80] p[43, 65] p[45, 105] p[58, 82]}{A^{23}} + \\
 & \ll 4381 \gg + \frac{p[9, 18] p[16, 80] p[30, 58] p[43, 45] p[65, 105] p[82, 92] p[99, 108]}{A^7} - \\
 & A p[9, 18] p[16, 80] p[30, 58] p[43, 45] p[65, 105] p[82, 92] p[99, 108] \\
 & \gg - \frac{p[9, 99] p[16, 92] p[19, 80] p[29, 108] p[43, 65] p[45, 105] p[58, 82]}{A^{26}} + \\
 & \frac{p[9, 99] p[16, 92] p[19, 80] p[29, 108] p[43, 65] p[45, 105] p[58, 82]}{A^{22}} + \\
 & \ll 4970 \gg + \frac{p[9, 29] p[16, 80] p[19, 58] p[43, 45] p[65, 105] p[82, 92] p[99, 108]}{A^6} - \\
 & A^2 p[9, 29] p[16, 80] p[19, 58] p[43, 45] p[65, 105] p[82, 92] p[99, 108]
 \end{aligned}$$

$$\begin{aligned}
 & \gg - \frac{p[9, 99] p[16, 92] p[20, 105] p[29, 108] p[43, 65] p[45, 82] p[59, 80]}{A^{27}} + \\
 & \frac{4 p[9, 99] p[16, 92] p[20, 105] p[29, 108] p[43, 65] p[45, 82] p[59, 80]}{A^{19}} + \\
 & \ll 4928 \gg + \frac{p[9, 20] p[16, 80] p[29, 59] p[43, 45] p[65, 105] p[82, 92] p[99, 108]}{A^7} - \\
 & A p[9, 20] p[16, 80] p[29, 59] p[43, 45] p[65, 105] p[82, 92] p[99, 108] \\
 & \gg - \frac{p[9, 99] p[15, 105] p[20, 93] p[29, 108] p[43, 65] p[45, 82] p[59, 80]}{A^{22}} + \\
 & \frac{2 p[9, 99] p[15, 105] p[20, 93] p[29, 108] p[43, 65] p[45, 82] p[59, 80]}{A^{18}} + \\
 & \ll 5199 \gg + \frac{p[9, 20] p[15, 82] p[29, 59] p[43, 45] p[65, 105] p[80, 93] p[99, 108]}{A^8} \\
 & \gg - \frac{p[9, 99] p[15, 105] p[20, 93] p[28, 107] p[43, 65] p[45, 82] p[59, 80]}{A^{23}} + \\
 & \frac{2 p[9, 99] p[15, 105] p[20, 93] p[28, 107] p[43, 65] p[45, 82] p[59, 80]}{A^{19}} + \\
 & \ll 5338 \gg + \frac{2 p[9, 65] p[15, 82] p[20, 59] p[28, 45] p[43, 80] p[93, 99] p[105, 107]}{A^{33}} \\
 & \gg - 4 p[9, 99] p[15, 82] p[20, 105] p[28, 107] p[43, 65] p[45, 81] p[59, 94] - \\
 & \frac{p[9, 99] p[15, 82] p[20, 105] p[28, 107] p[43, 65] p[45, 81] p[59, 94]}{A^{32}} + \\
 & \frac{4 p[9, 99] p[15, 82] p[20, 105] p[28, 107] p[43, 65] p[45, 81] p[59, 94]}{A^{24}} + \\
 & \ll 5704 \gg + \frac{3 p[9, 15] p[20, 59] p[28, 45] p[43, 65] p[81, 94] p[82, 99] p[105, 107]}{A^4} - \\
 & A^4 p[9, 15] p[20, 59] p[28, 45] p[43, 65] p[81, 94] p[82, 99] p[105, 107] + \\
 & A^8 p[9, 15] p[20, 59] p[28, 45] p[43, 65] p[81, 94] p[82, 99] p[105, 107] \\
 & \gg \frac{p[9, 107] p[14, 99] p[20, 105] p[28, 94] p[43, 65] p[45, 59]}{A^{37}} - \\
 & \frac{2 p[9, 107] p[14, 99] p[20, 105] p[28, 94] p[43, 65] p[45, 59]}{A^{33}} - \\
 & \frac{p[9, 107] p[14, 99] p[20, 105] p[28, 94] p[43, 65] p[45, 59]}{A^{29}} + \ll 2562 \gg + \\
 & \frac{4 p[9, 65] p[14, 45] p[20, 59] p[28, 94] p[43, 99] p[105, 107]}{A^5} - \\
 & \frac{p[9, 65] p[14, 45] p[20, 59] p[28, 94] p[43, 99] p[105, 107]}{A} - \\
 & 2 A^3 p[9, 65] p[14, 45] p[20, 59] p[28, 94] p[43, 99] p[105, 107] + \\
 & A^7 p[9, 65] p[14, 45] p[20, 59] p[28, 94] p[43, 99] p[105, 107]
 \end{aligned}$$

$$\begin{aligned}
 & \gg \frac{p[9, 107] p[14, 99] p[20, 105] p[27, 95] p[43, 65] p[45, 59]}{A^{38}} - \\
 & \frac{2 p[9, 107] p[14, 99] p[20, 105] p[27, 95] p[43, 65] p[45, 59]}{A^{34}} - \\
 & \frac{p[9, 107] p[14, 99] p[20, 105] p[27, 95] p[43, 65] p[45, 59]}{A^{30}} + \ll 2691 \gg + \\
 & \frac{4 p[9, 45] p[14, 27] p[20, 59] p[43, 65] p[95, 99] p[105, 107]}{A^8} + \\
 & \frac{3 p[9, 45] p[14, 27] p[20, 59] p[43, 65] p[95, 99] p[105, 107]}{A^4} - \\
 & A^4 p[9, 45] p[14, 27] p[20, 59] p[43, 65] p[95, 99] p[105, 107] + \\
 & A^8 p[9, 45] p[14, 27] p[20, 59] p[43, 65] p[95, 99] p[105, 107] \\
 & \gg - \frac{2 p[9, 99] p[14, 65] p[20, 106] p[27, 95] p[43, 60] p[45, 105]}{A^{33}} - \\
 & \frac{2 p[9, 99] p[14, 65] p[20, 106] p[27, 95] p[43, 60] p[45, 105]}{A^{29}} + \\
 & \frac{3 p[9, 99] p[14, 65] p[20, 106] p[27, 95] p[43, 60] p[45, 105]}{A^{25}} + \\
 & \ll 2688 \gg + \frac{5 p[9, 20] p[14, 27] p[43, 45] p[60, 95] p[65, 105] p[99, 106]}{A^5} - \\
 & \frac{4 p[9, 20] p[14, 27] p[43, 45] p[60, 95] p[65, 105] p[99, 106]}{A} + \\
 & 2 A^{11} p[9, 20] p[14, 27] p[43, 45] p[60, 95] p[65, 105] p[99, 106] - \\
 & A^{15} p[9, 20] p[14, 27] p[43, 45] p[60, 95] p[65, 105] p[99, 106] \\
 & \gg - \frac{p[9, 99] p[14, 65] p[21, 60] p[27, 95] p[43, 45]}{A^{46}} + \frac{p[9, 99] p[14, 65] p[21, 60] p[27, 95] p[43, 45]}{A^{42}} + \\
 & \ll 995 \gg + 7 A^8 p[9, 45] p[14, 27] p[21, 60] p[43, 65] p[95, 99] + \\
 & 3 A^{12} p[9, 45] p[14, 27] p[21, 60] p[43, 65] p[95, 99] - \\
 & 4 A^{16} p[9, 45] p[14, 27] p[21, 60] p[43, 65] p[95, 99] + \\
 & A^{20} p[9, 45] p[14, 27] p[21, 60] p[43, 65] p[95, 99] \\
 & \gg - \frac{p[9, 99] p[14, 96] p[21, 65] p[27, 61] p[43, 45]}{A^{45}} + \frac{2 p[9, 99] p[14, 96] p[21, 65] p[27, 61] p[43, 45]}{A^{41}} + \\
 & \frac{p[9, 99] p[14, 96] p[21, 65] p[27, 61] p[43, 45]}{A^{37}} - \frac{2 p[9, 99] p[14, 96] p[21, 65] p[27, 61] p[43, 45]}{A^{33}} - \\
 & \frac{4 p[9, 99] p[14, 96] p[21, 65] p[27, 61] p[43, 45]}{A^{29}} + \ll 1011 \gg + \\
 & 8 A^7 p[9, 65] p[14, 27] p[21, 45] p[43, 96] p[61, 99] + \\
 & A^{11} p[9, 65] p[14, 27] p[21, 45] p[43, 96] p[61, 99] - \\
 & 3 A^{15} p[9, 65] p[14, 27] p[21, 45] p[43, 96] p[61, 99] + \\
 & A^{19} p[9, 65] p[14, 27] p[21, 45] p[43, 96] p[61, 99]
 \end{aligned}$$

$$\begin{aligned}
 & \gg -17 p[9, 99] p[13, 61] p[21, 96] p[26, 65] p[43, 45] - \\
 & \quad \frac{p[9, 99] p[13, 61] p[21, 96] p[26, 65] p[43, 45]}{A^{48}} + \frac{p[9, 99] p[13, 61] p[21, 96] p[26, 65] p[43, 45]}{A^{44}} + \\
 & \quad \ll 1030 \gg + 9 A^{10} p[9, 65] p[13, 26] p[21, 45] p[43, 96] p[61, 99] - \\
 & \quad 3 A^{14} p[9, 65] p[13, 26] p[21, 45] p[43, 96] p[61, 99] - \\
 & \quad 2 A^{18} p[9, 65] p[13, 26] p[21, 45] p[43, 96] p[61, 99] + \\
 & \quad A^{22} p[9, 65] p[13, 26] p[21, 45] p[43, 96] p[61, 99] \\
 & \gg - \frac{p[9, 99] p[13, 61] p[22, 44] p[26, 96] p[43, 65]}{A^{47}} + \frac{4 p[9, 99] p[13, 61] p[22, 44] p[26, 96] p[43, 65]}{A^{39}} + \\
 & \quad \frac{p[9, 99] p[13, 61] p[22, 44] p[26, 96] p[43, 65]}{A^{35}} - \frac{8 p[9, 99] p[13, 61] p[22, 44] p[26, 96] p[43, 65]}{A^{31}} - \\
 & \quad \frac{3 p[9, 99] p[13, 61] p[22, 44] p[26, 96] p[43, 65]}{A^{27}} + \ll 1051 \gg + \\
 & \quad 4 A^7 p[9, 22] p[13, 26] p[43, 65] p[44, 96] p[61, 99] + \\
 & \quad 10 A^{11} p[9, 22] p[13, 26] p[43, 65] p[44, 96] p[61, 99] - \\
 & \quad 6 A^{15} p[9, 22] p[13, 26] p[43, 65] p[44, 96] p[61, 99] - \\
 & \quad A^{19} p[9, 22] p[13, 26] p[43, 65] p[44, 96] p[61, 99] + \\
 & \quad A^{23} p[9, 22] p[13, 26] p[43, 65] p[44, 96] p[61, 99] \\
 & \gg -12 p[9, 99] p[13, 61] p[23, 96] p[26, 65] + \\
 & \quad \frac{p[9, 99] p[13, 61] p[23, 96] p[26, 65]}{A^{52}} - \frac{p[9, 99] p[13, 61] p[23, 96] p[26, 65]}{A^{48}} - \\
 & \quad \frac{2 p[9, 99] p[13, 61] p[23, 96] p[26, 65]}{A^{44}} + \frac{2 p[9, 99] p[13, 61] p[23, 96] p[26, 65]}{A^{40}} + \ll 385 \gg + \\
 & \quad 12 A^8 p[9, 65] p[13, 26] p[23, 96] p[61, 99] + 10 A^{12} p[9, 65] p[13, 26] p[23, 96] p[61, 99] - \\
 & \quad 14 A^{16} p[9, 65] p[13, 26] p[23, 96] p[61, 99] + 3 A^{20} p[9, 65] p[13, 26] p[23, 96] p[61, 99] + \\
 & \quad 2 A^{24} p[9, 65] p[13, 26] p[23, 96] p[61, 99] - A^{28} p[9, 65] p[13, 26] p[23, 96] p[61, 99] \\
 & \gg \frac{p[9, 96] p[13, 98] p[23, 65] p[25, 61]}{A^{49}} - \frac{2 p[9, 96] p[13, 98] p[23, 65] p[25, 61]}{A^{45}} + \\
 & \quad \frac{2 p[9, 96] p[13, 98] p[23, 65] p[25, 61]}{A^{37}} - \frac{p[9, 96] p[13, 98] p[23, 65] p[25, 61]}{A^{29}} - \\
 & \quad \frac{4 p[9, 96] p[13, 98] p[23, 65] p[25, 61]}{A^{25}} + \frac{5 p[9, 96] p[13, 98] p[23, 65] p[25, 61]}{A^{21}} + \ll 407 \gg + \\
 & \quad 13 A^7 p[9, 25] p[13, 61] p[23, 65] p[96, 98] + 18 A^{11} p[9, 25] p[13, 61] p[23, 65] p[96, 98] - \\
 & \quad 18 A^{15} p[9, 25] p[13, 61] p[23, 65] p[96, 98] + 4 A^{19} p[9, 25] p[13, 61] p[23, 65] p[96, 98] + \\
 & \quad 2 A^{23} p[9, 25] p[13, 61] p[23, 65] p[96, 98] - A^{27} p[9, 25] p[13, 61] p[23, 65] p[96, 98] \\
 & \gg -24 p[9, 65] p[12, 96] p[23, 62] p[25, 98] - \frac{p[9, 65] p[12, 96] p[23, 62] p[25, 98]}{A^{56}} + \\
 & \quad \frac{3 p[9, 65] p[12, 96] p[23, 62] p[25, 98]}{A^{52}} - \frac{p[9, 65] p[12, 96] p[23, 62] p[25, 98]}{A^{48}} - \\
 & \quad \frac{5 p[9, 65] p[12, 96] p[23, 62] p[25, 98]}{A^{44}} + \frac{4 p[9, 65] p[12, 96] p[23, 62] p[25, 98]}{A^{40}} + \\
 & \quad \ll 406 \gg + 17 A^{10} p[9, 25] p[12, 62] p[23, 65] p[96, 98] - \\
 & \quad 8 A^{14} p[9, 25] p[12, 62] p[23, 65] p[96, 98] - 5 A^{18} p[9, 25] p[12, 62] p[23, 65] p[96, 98] + \\
 & \quad 5 A^{22} p[9, 25] p[12, 62] p[23, 65] p[96, 98] - A^{26} p[9, 25] p[12, 62] p[23, 65] p[96, 98]
 \end{aligned}$$

$$\begin{aligned}
 & \gg - \frac{p[9, 96] p[12, 62] p[24, 64] p[25, 98]}{A^{57}} + \frac{3 p[9, 96] p[12, 62] p[24, 64] p[25, 98]}{A^{53}} - \\
 & \frac{p[9, 96] p[12, 62] p[24, 64] p[25, 98]}{A^{49}} - \frac{5 p[9, 96] p[12, 62] p[24, 64] p[25, 98]}{A^{45}} + \\
 & \frac{4 p[9, 96] p[12, 62] p[24, 64] p[25, 98]}{A^{41}} + \ll 413 \gg + \\
 & 14 A^7 p[9, 24] p[12, 62] p[25, 64] p[96, 98] + 2 A^{11} p[9, 24] p[12, 62] p[25, 64] p[96, 98] - \\
 & 8 A^{15} p[9, 24] p[12, 62] p[25, 64] p[96, 98] + 2 A^{19} p[9, 24] p[12, 62] p[25, 64] p[96, 98] + \\
 & 2 A^{23} p[9, 24] p[12, 62] p[25, 64] p[96, 98] - A^{27} p[9, 24] p[12, 62] p[25, 64] p[96, 98] \\
 & \gg - \frac{p[10, 98] p[12, 96] p[62, 64]}{A^{58}} + \frac{2 p[10, 98] p[12, 96] p[62, 64]}{A^{54}} - \\
 & \frac{3 p[10, 98] p[12, 96] p[62, 64]}{A^{46}} + \frac{2 p[10, 98] p[12, 96] p[62, 64]}{A^{42}} + \frac{2 p[10, 98] p[12, 96] p[62, 64]}{A^{38}} - \\
 & \frac{2 p[10, 98] p[12, 96] p[62, 64]}{A^{34}} - \frac{p[10, 98] p[12, 96] p[62, 64]}{A^{30}} + \frac{2 p[10, 98] p[12, 96] p[62, 64]}{A^{22}} + \\
 & \ll 131 \gg + 16 A^6 p[10, 64] p[12, 62] p[96, 98] - 2 A^{10} p[10, 64] p[12, 62] p[96, 98] - \\
 & 10 A^{14} p[10, 64] p[12, 62] p[96, 98] + 8 A^{18} p[10, 64] p[12, 62] p[96, 98] - \\
 & 3 A^{26} p[10, 64] p[12, 62] p[96, 98] + A^{30} p[10, 64] p[12, 62] p[96, 98] \\
 & \gg - \frac{p[10, 98] p[11, 97] p[62, 64]}{A^{59}} + \frac{2 p[10, 98] p[11, 97] p[62, 64]}{A^{55}} - \\
 & \frac{3 p[10, 98] p[11, 97] p[62, 64]}{A^{47}} + \frac{2 p[10, 98] p[11, 97] p[62, 64]}{A^{43}} + \frac{2 p[10, 98] p[11, 97] p[62, 64]}{A^{39}} - \\
 & \frac{2 p[10, 98] p[11, 97] p[62, 64]}{A^{35}} - \frac{p[10, 98] p[11, 97] p[62, 64]}{A^{31}} + \frac{2 p[10, 98] p[11, 97] p[62, 64]}{A^{23}} + \\
 & \ll 129 \gg + \frac{7 p[10, 64] p[11, 97] p[62, 98]}{A^5} - \frac{19 p[10, 64] p[11, 97] p[62, 98]}{A} + \\
 & 2 A^3 p[10, 64] p[11, 97] p[62, 98] + 21 A^7 p[10, 64] p[11, 97] p[62, 98] - \\
 & 19 A^{11} p[10, 64] p[11, 97] p[62, 98] + 11 A^{19} p[10, 64] p[11, 97] p[62, 98] - \\
 & 7 A^{23} p[10, 64] p[11, 97] p[62, 98] + A^{27} p[10, 64] p[11, 97] p[62, 98] \\
 & \gg - 8 p[10, 64] p[11, 63] + \frac{p[10, 64] p[11, 63]}{A^{64}} - \frac{2 p[10, 64] p[11, 63]}{A^{60}} + \frac{p[10, 64] p[11, 63]}{A^{56}} + \\
 & \frac{p[10, 64] p[11, 63]}{A^{52}} - \frac{2 p[10, 64] p[11, 63]}{A^{48}} + \frac{p[10, 64] p[11, 63]}{A^{44}} - \frac{p[10, 64] p[11, 63]}{A^{36}} + \\
 & \frac{2 p[10, 64] p[11, 63]}{A^{32}} - \frac{p[10, 64] p[11, 63]}{A^{28}} - \frac{2 p[10, 64] p[11, 63]}{A^{24}} + \ll 37 \gg + \frac{3 p[10, 63] p[11, 64]}{A^{18}} - \\
 & \frac{5 p[10, 63] p[11, 64]}{A^{14}} + \frac{p[10, 63] p[11, 64]}{A^{10}} + \frac{5 p[10, 63] p[11, 64]}{A^6} - \frac{6 p[10, 63] p[11, 64]}{A^2} - \\
 & A^2 p[10, 63] p[11, 64] + 9 A^6 p[10, 63] p[11, 64] - 9 A^{10} p[10, 63] p[11, 64] - A^{14} p[10, 63] p[11, 64] + \\
 & 10 A^{18} p[10, 63] p[11, 64] - 10 A^{22} p[10, 63] p[11, 64] + 4 A^{26} p[10, 63] p[11, 64]
 \end{aligned}$$

Out[4]=

$$\left\{ 20.9375, - \frac{1 - 3 q + q^2 + q^3 - 2 q^4 + q^5 + q^7 - q^8 + q^{10} - q^{11} + q^{13} - q^{14} - q^{15} + q^{16} - q^{18} + q^{20} + q^{25} - q^{26}}{q^2} \right\}$$