

Pensieve header: Computing the Jones polynomial faster.

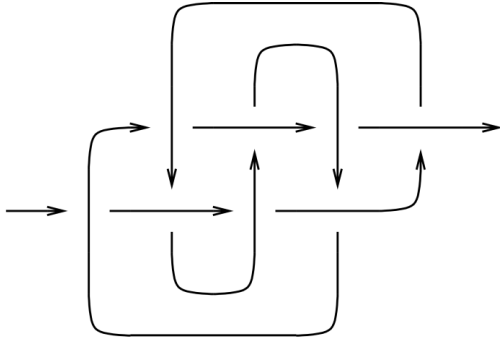
The trefoil and the mirror trefoil:

```
In[ ]:= D1 = PD[X[1, 5, 2, 4], X[5, 3, 6, 2], X[3, 1, 4, 6]]
```

```
D2 = PD[X[4, 1, 5, 2], X[2, 5, 3, 6], X[6, 3, 1, 4]]
```

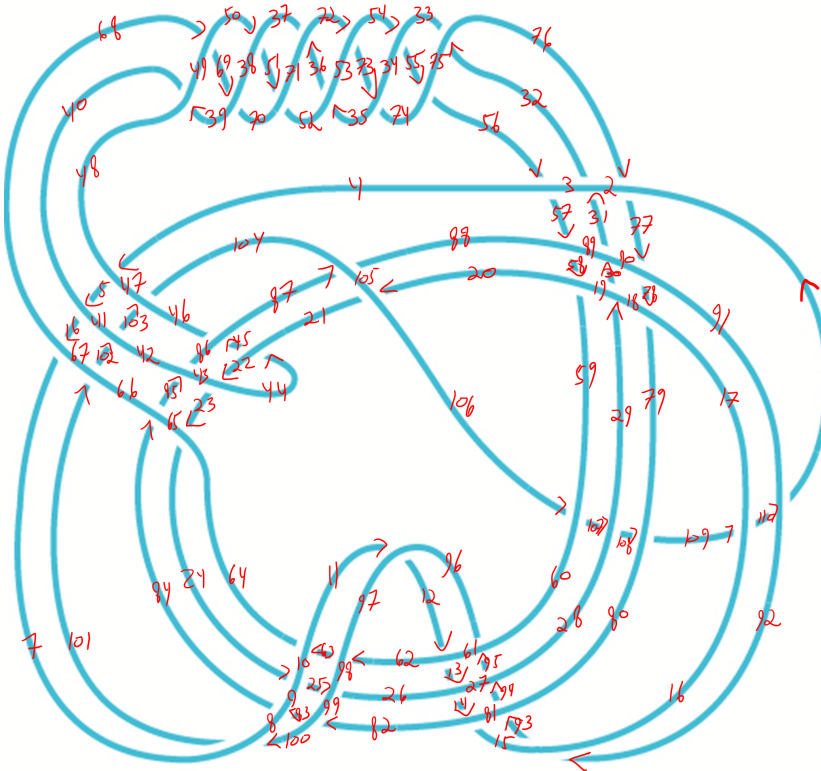
```
Out[ ]:= PD[X[1, 5, 2, 4], X[5, 3, 6, 2], X[3, 1, 4, 6]]
```

```
Out[ ]:= PD[X[4, 1, 5, 2], X[2, 5, 3, 6], X[6, 3, 1, 4]]
```



```
In[ ]:= K817 = PD[X[6, 2, 7, 1], X[14, 8, 15, 7], X[8, 3, 9, 4], X[2, 13, 3, 14], X[12, 5, 13, 6], X[4, 9, 5, 10], X[16, 12, 1, 11], X[10, 16, 11, 15]]
```

```
Out[ ]:= PD[X[6, 2, 7, 1], X[14, 8, 15, 7], X[8, 3, 9, 4], X[2, 13, 3, 14], X[12, 5, 13, 6], X[4, 9, 5, 10], X[16, 12, 1, 11], X[10, 16, 11, 15]]
```



```
In[*]:= PK = PD[
  X[4, 48, 5, 47], X[5, 40, 6, 41], X[6, 68, 7, 67], X[11, 97, 12, 96],
  X[12, 62, 13, 61], X[13, 26, 14, 27], X[14, 82, 15, 81], X[20, 105, 21, 106],
  X[22, 43, 23, 44], X[23, 65, 24, 64], X[24, 9, 25, 10], X[25, 99, 26, 98],
  X[29, 18, 30, 19], X[30, 90, 31, 89], X[31, 2, 32, 3], X[32, 76, 33, 75],
  X[35, 53, 36, 52], X[36, 72, 37, 71], X[39, 49, 40, 48], X[44, 21, 45, 22],
  X[45, 87, 46, 86], X[50, 38, 51, 37], X[51, 70, 52, 71], X[54, 34, 55, 33],
  X[55, 74, 56, 75], X[56, 4, 57, 3], X[57, 88, 58, 89], X[58, 20, 59, 19],
  X[62, 97, 63, 98], X[63, 11, 64, 10], X[68, 49, 69, 50], X[69, 39, 70, 38],
  X[72, 53, 73, 54], X[73, 35, 74, 34], X[76, 2, 77, 1], X[77, 90, 78, 91],
  X[78, 18, 79, 17], X[82, 99, 83, 100], X[83, 9, 84, 8], X[84, 65, 85, 66],
  X[85, 43, 86, 42], X[87, 105, 88, 104], X[92, 16, 93, 15], X[93, 80, 94, 81],
  X[94, 28, 95, 27], X[95, 60, 96, 61], X[100, 8, 101, 7], X[101, 66, 102, 67],
  X[102, 42, 103, 41], X[103, 46, 104, 47], X[106, 60, 107, 59], X[107, 28, 108, 29],
  X[108, 80, 109, 79], X[109, 16, 110, 17], X[110, 92, 1, 91]
];
```

```
In[*]:= J0[pd_, w_] := (
  t1 = pd /. X[i_, j_, k_, l_] => A p[i, j] p[k, l] + B p[i, l] p[j, k];
  t2 = Expand[t1 /. PD -> Times];
  t3 = t2 /. {p[i_, j_] p[j_, k_] => p[i, k], p[i_, j_] p[k_, j_] => p[i, k]};
  t4 = t3 /. {p[i_, i_] => d, p[i_, j_]^2 => d};
  t5 = Expand[t4 /. {B -> 1/A, d -> -A^2 - A^-2}];
  Simplify[(-A^3)^-w * t5 / (-A^2 - A^-2) /. A -> q^-1/4]
)
```

```
In[*]:= J1[pd_] := Module[{w, p, t1, t2, t3, t4, t5},
  w = Plus@@(pd /. {
    X[i_, j_, k_, l_] /; j - l == 1 || l - j > 1 -> 1,
    X[i_, j_, k_, l_] /; l - j == 1 || j - l > 1 -> -1
  });
  SetAttributes[p, Orderless];
  t1 = pd /. X[i_, j_, k_, l_] => A p[i, j] p[k, l] + B p[i, l] p[j, k];
  t2 = Expand[t1 /. PD -> Times];
  t3 = t2 /. p[i_, j_] p[j_, k_] => p[i, k];
  t4 = t3 /. {p[i_, i_] => d, p[i_, j_]^2 => d};
  t5 = Expand[t4 /. {B -> 1/A, d -> -A^2 - A^-2}];
  Simplify[(-A^3)^-w * t5 / (-A^2 - A^-2) /. A -> q^-1/4]
]
```

In[*]:= **J1 [D1]**

Out[*]=

$$q + q^3 - q^4$$

In[*]:= **J1 [D2]**

Out[*]=

$$\frac{-1 + q + q^3}{q^4}$$

In[*]:= **J1 [K817]**

Out[*]=

$$7 + \frac{1}{q^4} - \frac{3}{q^3} + \frac{5}{q^2} - \frac{6}{q} - 6q + 5q^2 - 3q^3 + q^4$$