2021-22 MAT257 Term Test 2 Rejects

The following questions were a part of a question pool for the 2021-22 MAT257 Term Test 2, but at the end, they were not included.

1. Prove that the set of irrational numbers is not of measure-0.

2. Prove that the collection of all finite sequences of rational numbers is countable.

3. Given a set A, an "accumulation point" for A is a point x such that every open neighbourhood of x contains infinitely many elements of A. Show that if A is bounded and has finitely many accumulation points, then A is of content 0.

4. Prove that every closed set is the intersection of countably many open sets.

5. Give an example of two functions that differ only on a bounded set of measure 0, yet such that one is integrable and the other is not.

6. Use Fubini's Theorem to compute the volume of the set $\{x \in \mathbb{R}^5 : 0 \le x_1 \le x_2 \le x_3 \le x_4 \le x_5 \le 1\}$.

7. Show that there is a smooth function on $\mathbb{R}^3$ whose support is precisely the cube $[-1, 1]^3$.

8. Find an example of a continuous function on $\mathbb{R}$ for which there is a constant M such that $\int_I f \le M$ for every internal $I \subset \mathbb{R}$, but yet such that f is not integrable (NT).

9. We've shown in class that $\int_{\mathbb{R}^n} e^{-|x|^2/2} dx = (2\pi)^{n/2}$. Let λ be a positive real number. Compute $\int_{\mathbb{R}^n} e^{-\lambda|x|^2/2} dx$.