Homework Assignment 4

Due: Friday, October 15, 2021 11:59 pm (Eastern Daylight Time)

Assignment description

Solve and submit your solutions of the following problems. They are taken from Spivak's book, pages 23, 33, and 34. Note that the late policy remains strict - you will lose 10% for each hour that you are late. In other words, please submit on time!

Submit your assignment

Help

After you have completed the assignment, please save, scan, or take photos of your work and upload your files to the questions below. Crowdmark accepts PDF, JPG, and PNG file formats.

Q1 (10 points)

Spivak's 2-18. Find the partial derivatives of the following functions (where $g\colon \mathbb{R} \to \mathbb{R}$ is continuous):

(a) $f(x,y) = \int_a^{x+y} g$.

(b) $f(x,y) = \int_y^{x} g$.

(c) $f(x,y) = \int_a^{xy} g$.

(d) $f(x,y) = \int_a^{\int_b^y g} g$.

Q2 (10 points)

Spivak's 2-19. If $f(x,y) = x^{x^{x^{x^y}}} + (\log x)(\arctan(\arctan(\arctan(\sin(\cos xy) - \log(x+y)))))$ find $D_2 f(1,y)$. (Hint in textbook).

Q3 (10 points)

Spivak's 2-21. Let $g_1, g_2 \colon \mathbb{R}^2 \to \mathbb{R}$ be continuous. Define $f \colon \mathbb{R}^2 \to \mathbb{R}$ by

$$f(x,y) = \int_0^x g_1(t,0)dt + \int_0^y g_2(x,t)dt.$$

(a) Show that $D_2 f(x,y) = g_2(x,y)$.

(b) How would you change the definition of f so that $D_1 f(x,y) = g_1(x,y)$?

(c) Find a function $f_c \colon \mathbb{R}^2 \to \mathbb{R}$ such that $D_1 f_c(x,y) = x$ and $D_2 f_c(x,y) = y$.

(c) Find a function $f_d \colon \mathbb{R}^2 \to \mathbb{R}$ such that $D_1 f_d(x,y) = y$ and $D_2 f_d(x,y) = x$.

Q4 (10 points)

Spivak's 2-29. Let $f \colon \mathbb{R}^n \to \mathbb{R}$. For $x \in \mathbb{R}^n$, the limit

$$\lim_{t \to 0} \frac{f(a+tx) - f(a)}{t},$$

if it exists, is denoted $D_x f(a)$, and called the **directional derivative** of f at a, in the direction x.

(a) Show that $D_{e_i} f(a) = D_i f(a)$ (recall that e_i is the ith standard basis vector of $\mathbb{R}^n$).

(b) Show that $D_{tx} f(a) = t D_x f(a)$.

(c) If f is diffable at a, show that $D_x f(a) = Df(a)(x)$ and therefore $D_{x+y} f(a) = D_x f(a) + D_y f(a)$.

Q5 (10 points)

Spivak's 2-34. A function $f \colon \mathbb{R}^n \to \mathbb{R}$ is **homogeneous** of degree m if $f(tx) = t^m f(x)$ for all x. If f is also diffable, show that

$$\sum_{i=1}^n x_i D_i f(x) = m f(x).$$

(Hint in textbook).

Q6 (10 points)

Spivak's 2-35. If $f \colon \mathbb{R}^n \to \mathbb{R}$ is diffable and $f(0) = 0$, prove that there exist $g_i \colon \mathbb{R}^n \to \mathbb{R}$ such that

$$f(x) = \sum_{i=1}^n x_i g_i(x).$$

(Hint in textbook).